Computational Fluid Dynamics

an Overview of Methods

...just the color figures for those who got the B&W printed book...

by D. James Benton

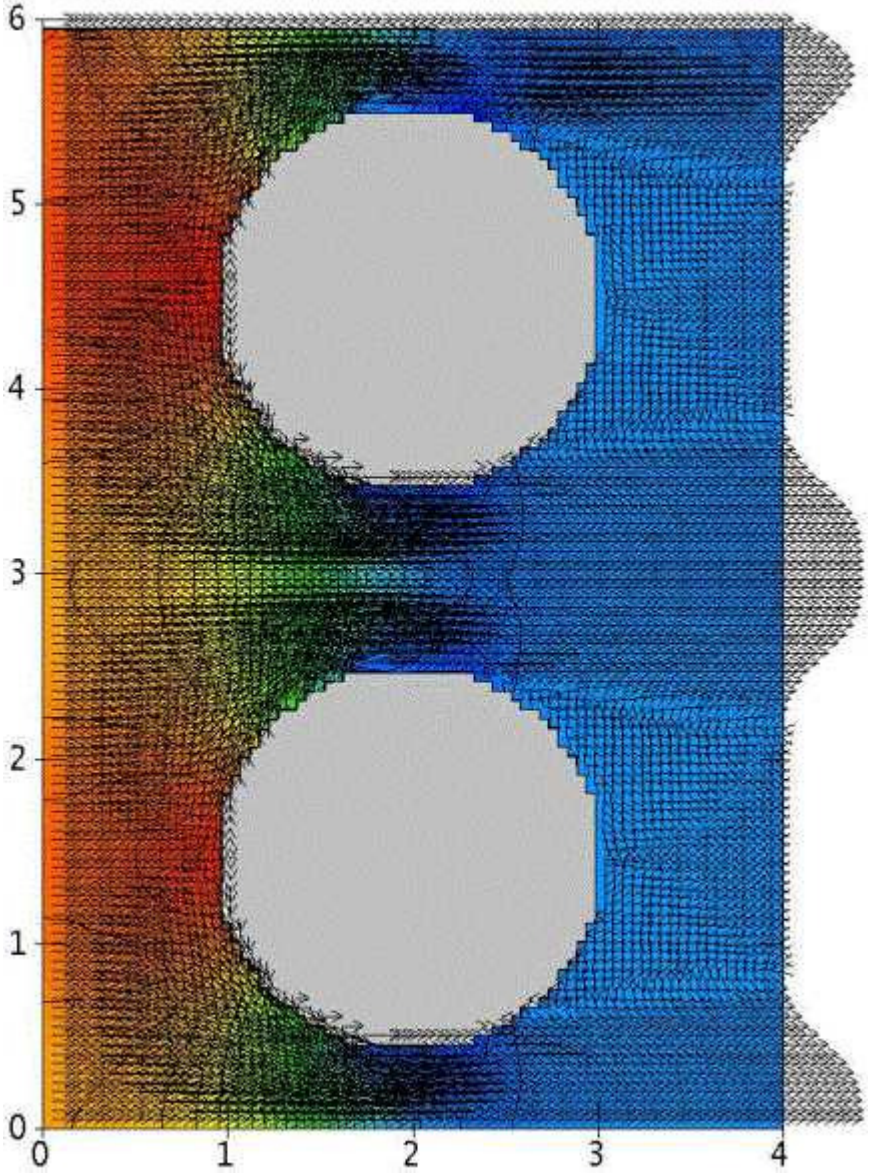
Forward

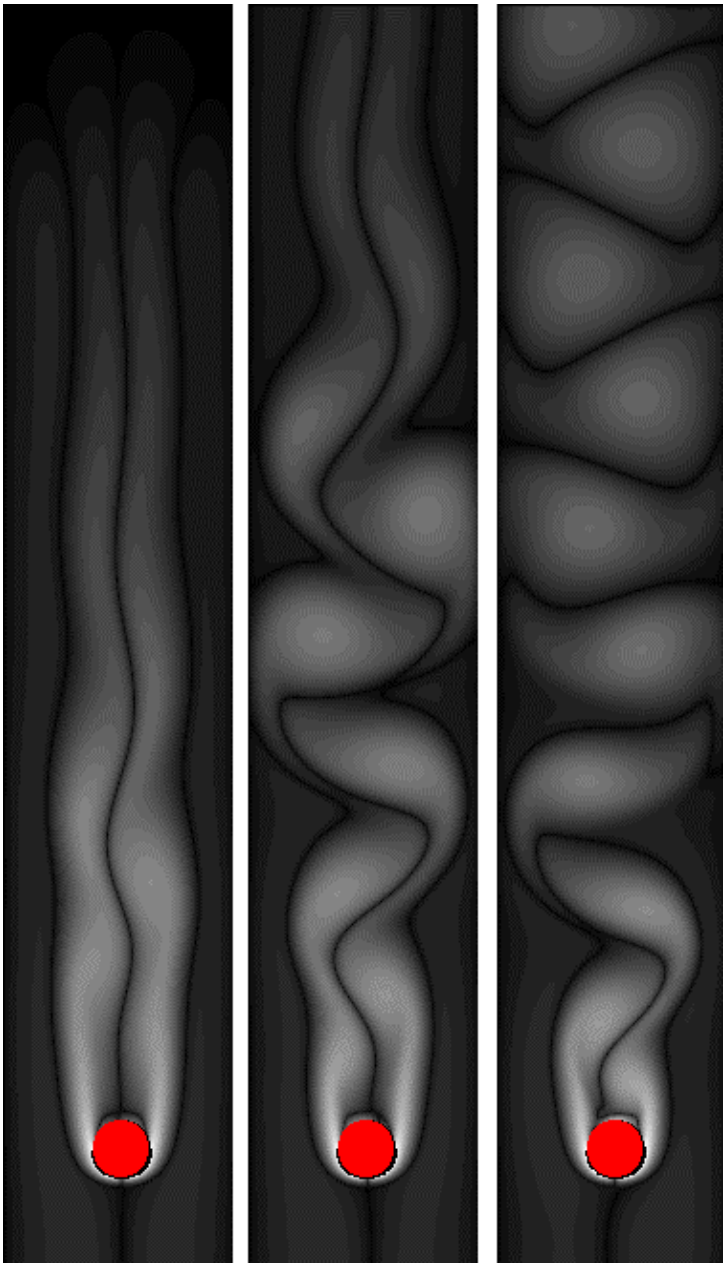
Fluid flow is a very broad and complicated topic because of the many different aspects and types of applications. I have covered inviscid flow, which occurs when the viscosity is negligible or the velocities are very large, in two previous texts: *Differential Equations* and *Numerical Calculus*. I may at some point cover creeping flow, which occurs when the viscosity dominates momentum or the velocities are very small. In this text we will cover the range between these two extremes: when viscosity and momentum are both important, at least in some significant part of the flow field.

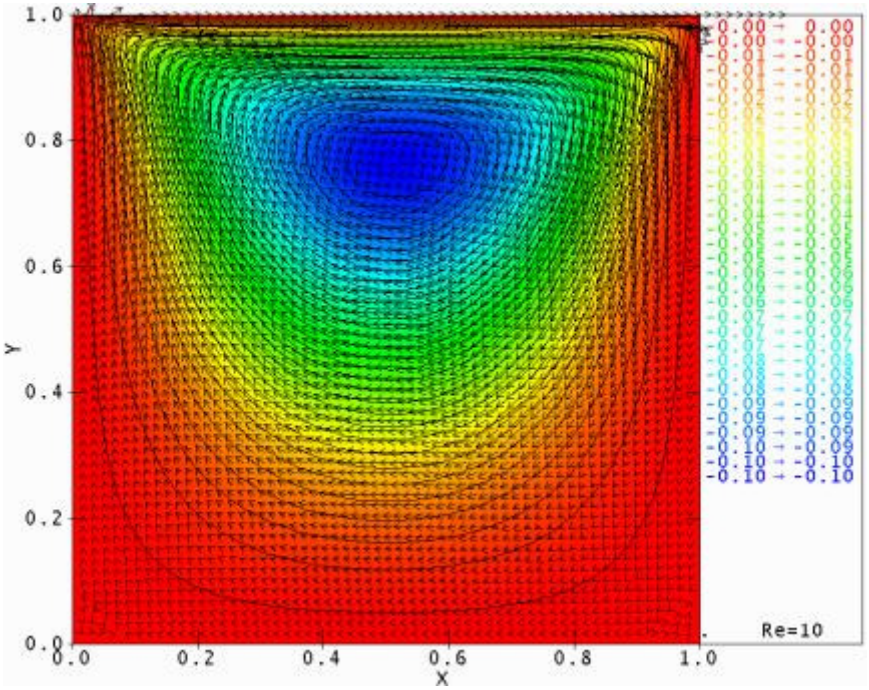
This is by no means an exhaustive reference on the subject of computational fluid dynamics; rather, it is an introduction and overview, going just far enough to get the reader started along this path with some helpful direction based on years of experience. I have striven to make this a clear presentation, particularly of finite elements, avoiding the traditional esoteric derivations that never seem to arrive at any useful destination. The impetus to undertake this project has arisen from my interacting with many graduate students on Research Gate, who are pursuing CFD and struggling with traditional presentations. You must make decisions when you arrive at a fork in the proverbial road (such as FDM, FVM, or FEM) and I hope this text will provide you with enough information to do that without me imposing my personal preference.

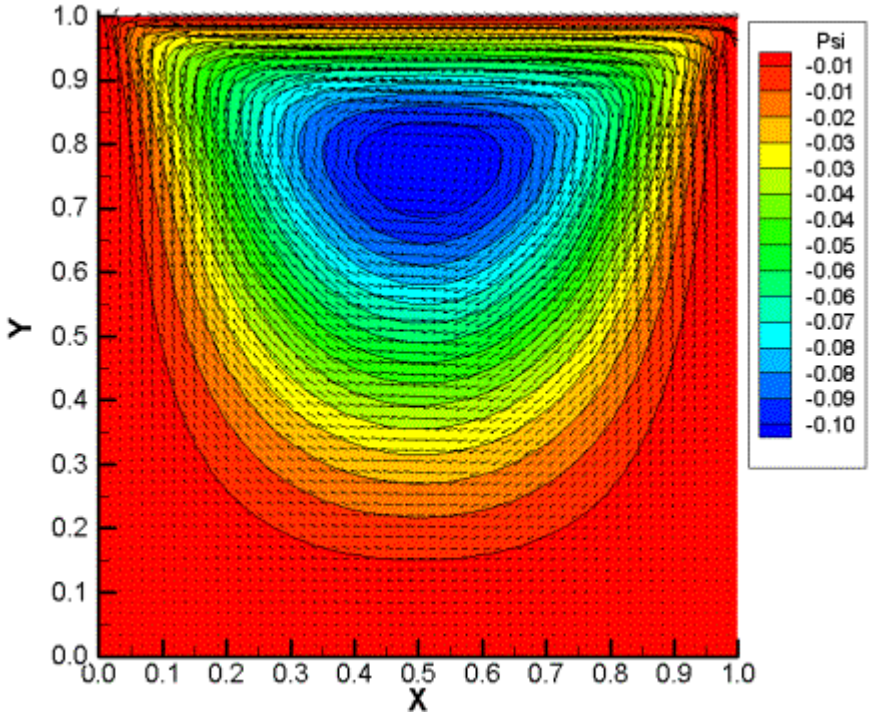
All of the examples in this text are written in the C language. CFD codes continue to be written in FORTRAN, even long after the language has fallen into disuse in most every other field of programming except complex variables. It is often said that FORTRAN is more efficient—even faster—for such applications, but I have found this to be more a result of inefficient C than a benefit of FORTRAN. Ultimately, the processor executes machine language instructions, which become the limiting factor with efficiently written code. The C software presented here is ANSI standard and uses no compiler or operating system specific features, except for being compatible with the Microsoft® C compiler. When thus compiled, these examples will run on ***any*** version of Windows®, including: 95, 98, ME, 2K, XP, Vista, 7, 8, 9, and 10 (32-bit and 64-bit).

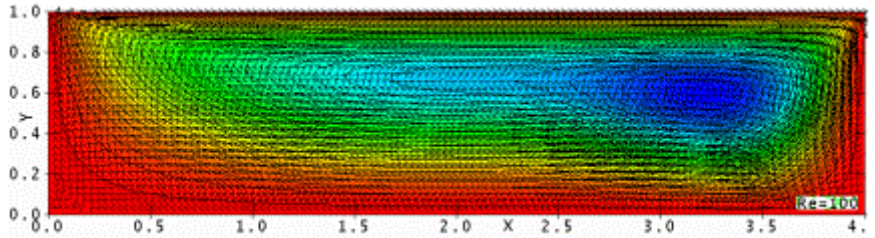
All of the examples contained in this book,
(as well as a lot of free programs) are available at…
http://www.dudleybenton.altervista.org/software/index.html

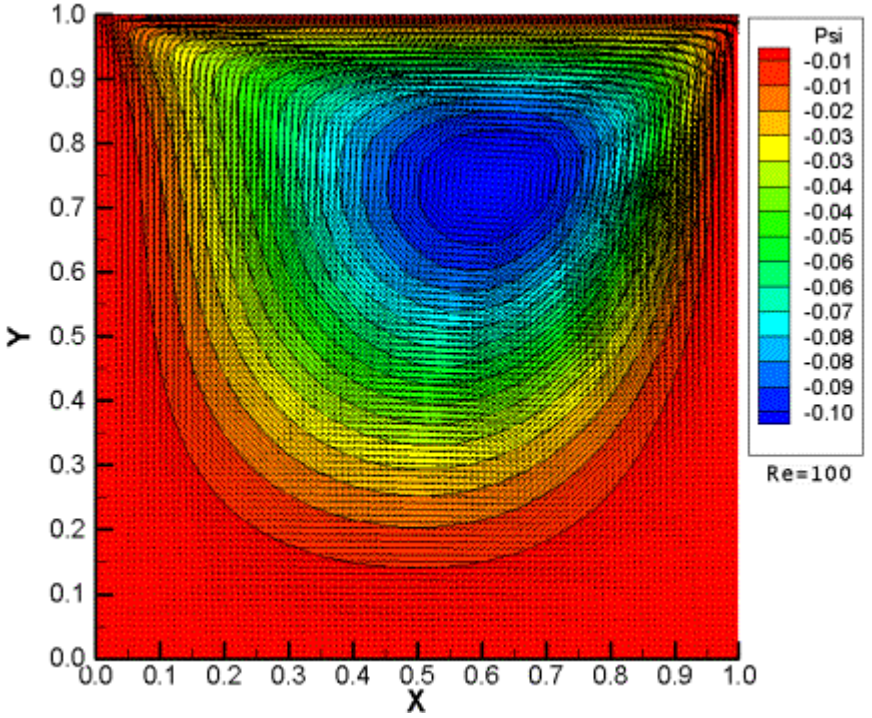

6
5
4
3
2
1
0
0
1
2
3
4

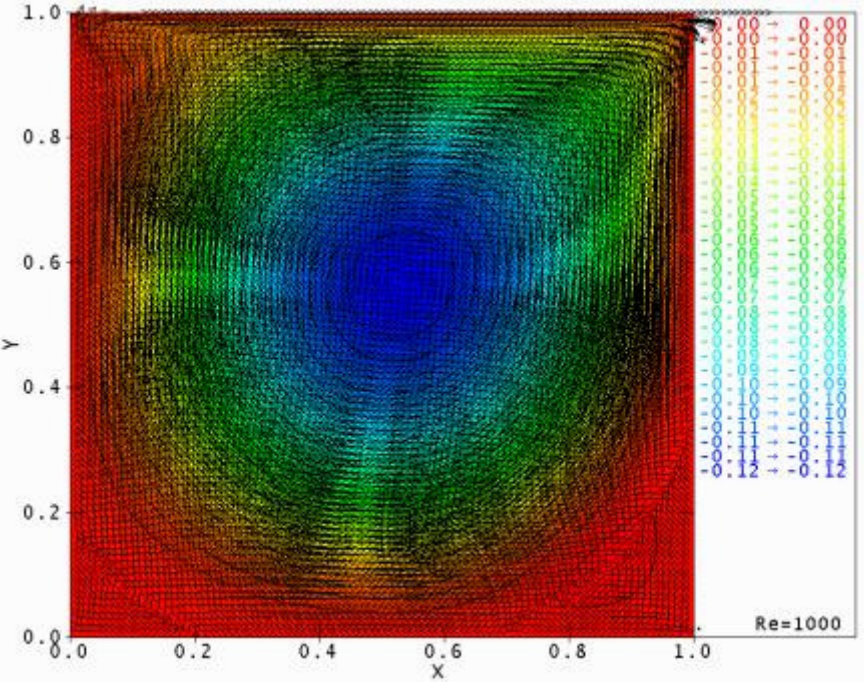


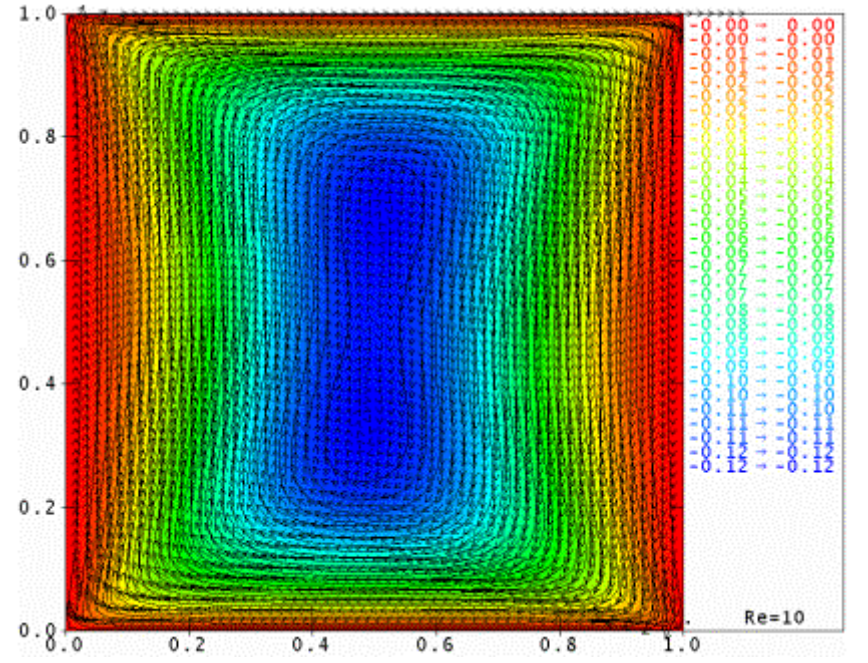

1.0
0.8
0.6
0.4
0.2
0.0
Y
0.0
0.2
0.4
0.6
0.8
1.0
X
-0.00 → -0.00
-0.00 → -0.00
-0.01 → -0.01
-0.01 → -0.01
-0.02 → -0.02
-0.02 → -0.02
-0.03 → -0.03
-0.03 → -0.03
-0.04 → -0.04
-0.04 → -0.04
-0.05 → -0.05
-0.05 → -0.05
-0.06 → -0.06
-0.06 → -0.06
-0.07 → -0.07
-0.07 → -0.07
-0.08 → -0.08
-0.08 → -0.08
-0.09 → -0.09
-0.09 → -0.09
-0.09 → -0.09
-0.10 → -0.09
-0.10 → -0.10
-0.10 → -0.10
Re=10

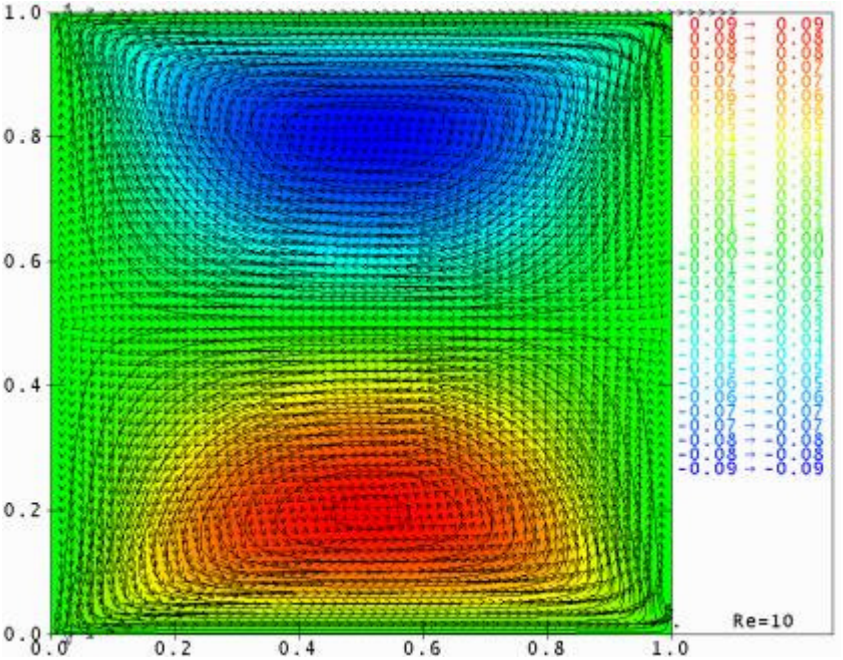

Psi
-0.01
-0.01
-0.02
-0.03
-0.03
-0.04
-0.04
-0.05
-0.06
-0.06
-0.07
-0.08
-0.08
-0.09
-0.10
Y
X
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

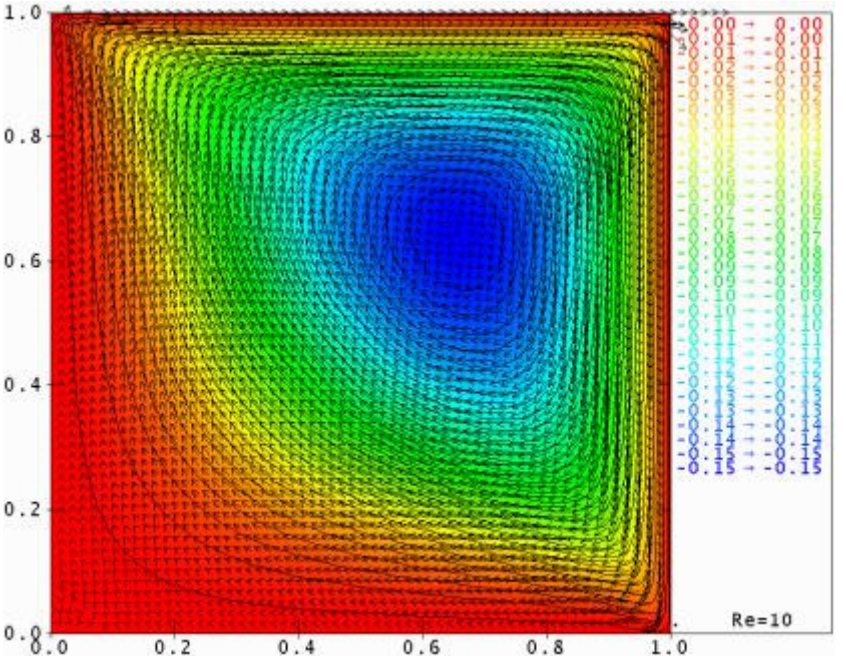

Y
X
Re=100
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.5
2.0
2.5
3.0
3.5
4.

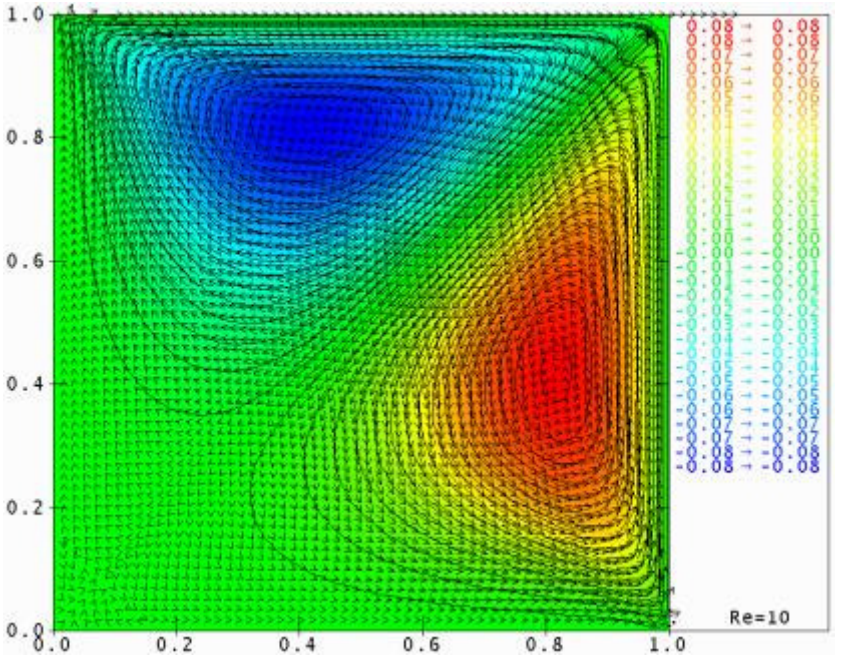

Psi
-0.01
-0.01
-0.02
-0.03
-0.03
-0.04
-0.04
-0.05
-0.06
-0.06
-0.07
-0.08
-0.08
-0.09
-0.10
Re=100
Y
X
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

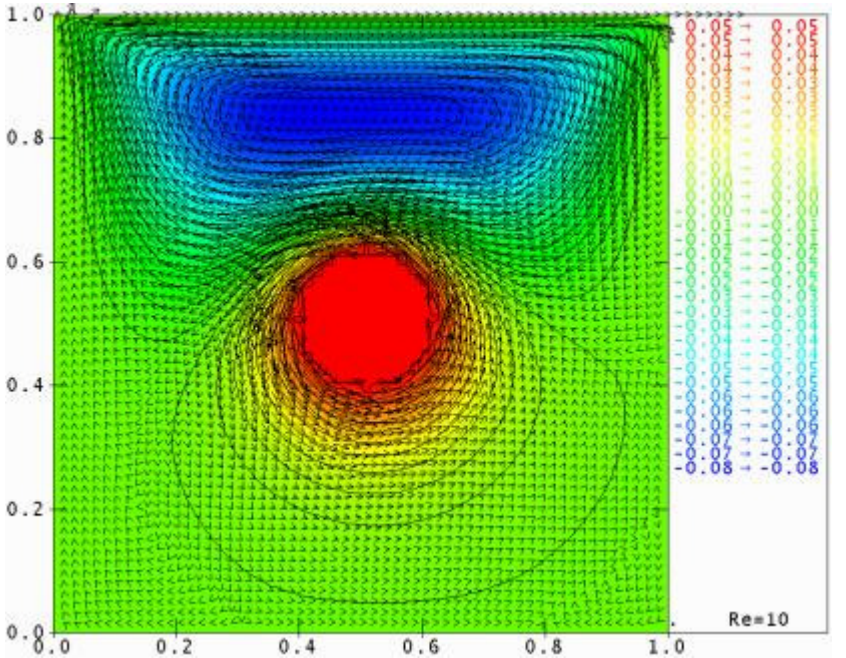

1.0
0.8
0.6
0.4
0.2
0.0
Y
0.0
0.2
0.4
0.6
0.8
1.0
X
-0.00 → 0.00
-0.00 → -0.00
-0.01 → -0.01
-0.01 → -0.01
-0.02 → -0.02
-0.03 → -0.03
-0.04 → -0.04
-0.05 → -0.05
-0.06 → -0.06
-0.07 → -0.07
-0.08 → -0.08
-0.09 → -0.09
-0.10 → -0.10
-0.10 → -0.10
-0.11 → -0.11
-0.11 → -0.11
-0.11 → -0.11
-0.12 → -0.12
Re=1000

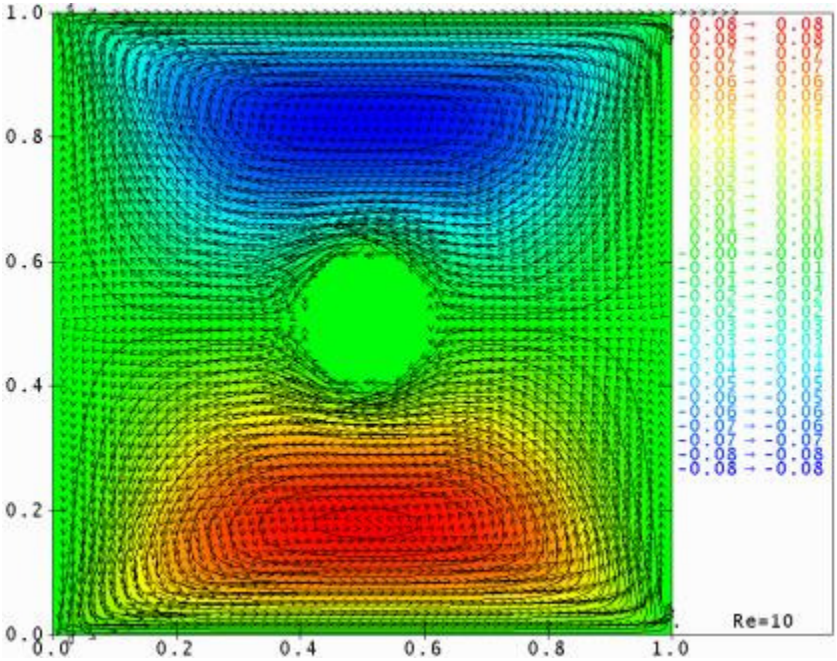

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
-0.00 → 0.00
-0.00 → -0.00
-0.01 → -0.01
-0.01 → -0.01
-0.02 → -0.02
-0.03 → -0.03
-0.04 → -0.04
-0.05 → -0.05
-0.06 → -0.06
-0.07 → -0.07
-0.08 → -0.08
-0.09 → -0.09
-0.10 → -0.10
-0.11 → -0.11
-0.12 → -0.12
Re=10

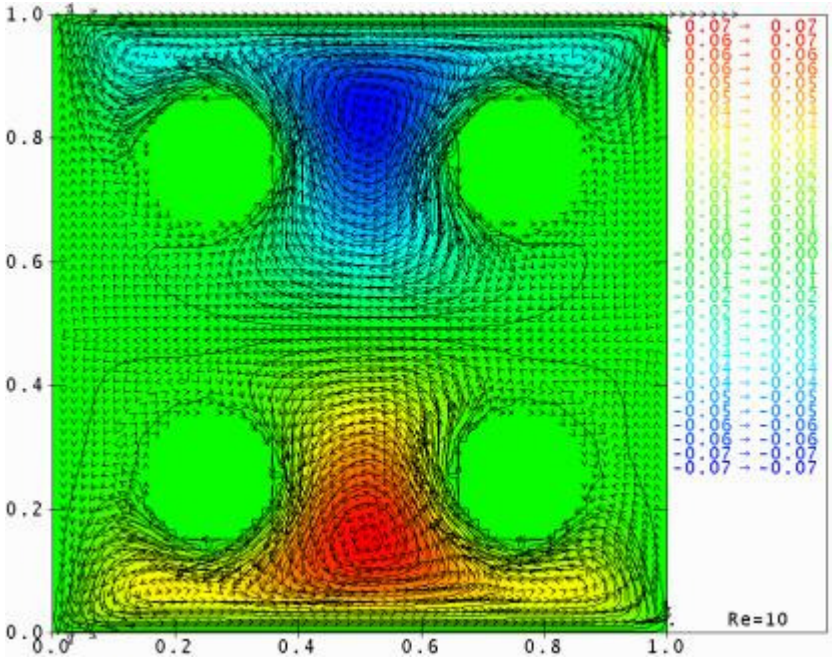

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
0.09 → 0.09
0.08 → 0.08
0.08 → 0.08
0.07 → 0.07
0.07 → 0.07
0.06 → 0.06
0.05 → 0.05
0.04 → 0.04
0.03 → 0.03
0.02 → 0.02
0.01 → 0.01
0.00 → 0.00
-0.01 → -0.01
-0.02 → -0.02
-0.03 → -0.03
-0.04 → -0.04
-0.05 → -0.05
-0.06 → -0.06
-0.07 → -0.07
-0.08 → -0.08
-0.09 → -0.09
Re=10

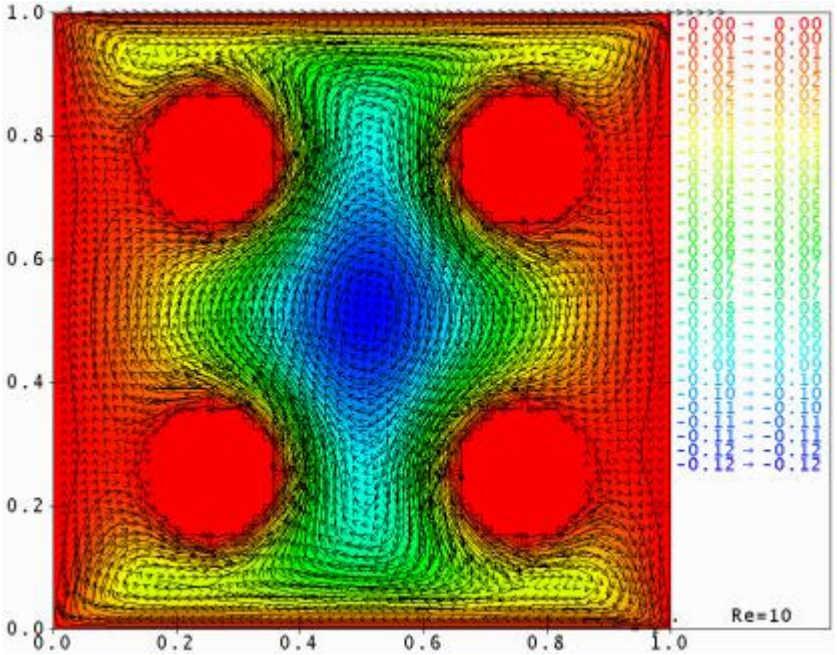

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
-0.00 → 0.00
-0.01 → -0.00
-0.01 → -0.01
-0.02 → -0.01
-0.02 → -0.02
-0.03 → -0.03
-0.04 → -0.04
-0.05 → -0.05
-0.06 → -0.06
-0.07 → -0.07
-0.08 → -0.08
-0.09 → -0.09
-0.10 → -0.10
-0.11 → -0.11
-0.12 → -0.12
-0.13 → -0.13
-0.14 → -0.14
-0.15 → -0.15
Re=10

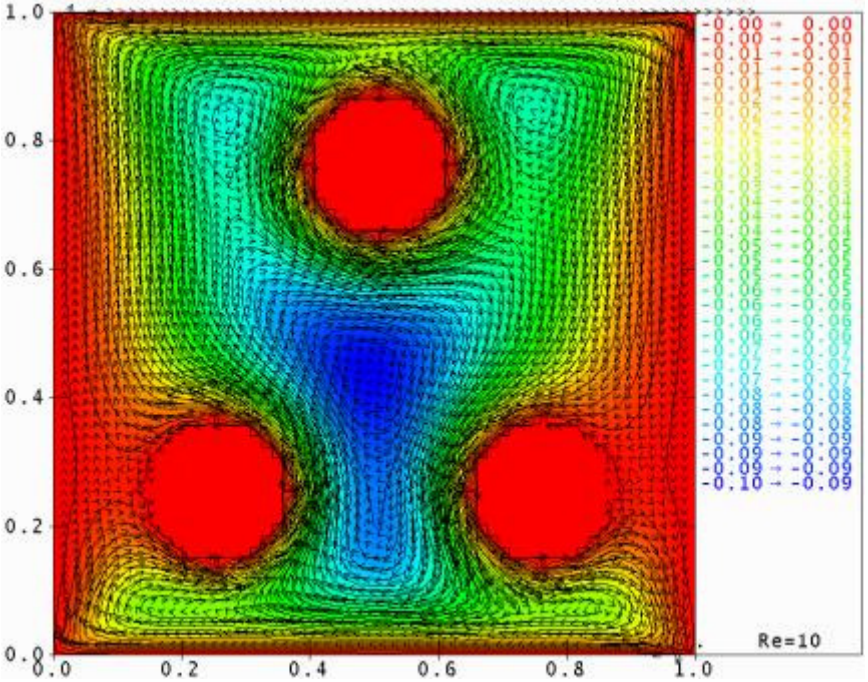

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
0.08 → 0.08
0.08 → 0.08
0.07 → 0.07
0.07 → 0.07
0.06 → 0.06
0.06 → 0.06
0.05 → 0.05
0.05 → 0.05
0.04 → 0.04
0.04 → 0.04
0.03 → 0.03
0.03 → 0.03
0.02 → 0.02
0.02 → 0.02
0.01 → 0.01
0.01 → 0.01
0.00 → 0.00
-0.00 → -0.00
-0.01 → -0.01
-0.01 → -0.01
-0.02 → -0.02
-0.02 → -0.02
-0.03 → -0.03
-0.03 → -0.03
-0.04 → -0.04
-0.04 → -0.04
-0.05 → -0.05
-0.05 → -0.05
-0.06 → -0.06
-0.06 → -0.06
-0.07 → -0.07
-0.07 → -0.07
-0.08 → -0.08
-0.08 → -0.08
Re=10

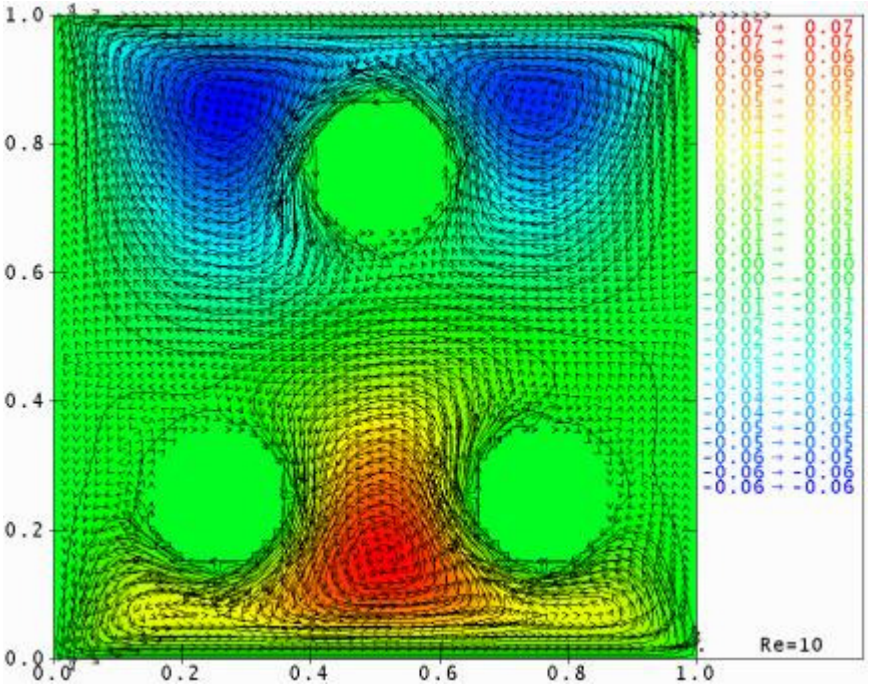

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
0.05 → 0.05
0.05 → 0.05
0.04 → 0.04
0.04 → 0.04
0.03 → 0.03
0.03 → 0.03
0.02 → 0.02
0.02 → 0.02
0.01 → 0.01
0.01 → 0.01
0.00 → 0.00
-0.00 → -0.00
-0.01 → -0.01
-0.01 → -0.01
-0.02 → -0.02
-0.02 → -0.02
-0.03 → -0.03
-0.03 → -0.03
-0.04 → -0.04
-0.04 → -0.04
-0.05 → -0.05
-0.05 → -0.05
-0.06 → -0.06
-0.06 → -0.06
-0.07 → -0.07
-0.07 → -0.07
-0.08 → -0.08
Re=10

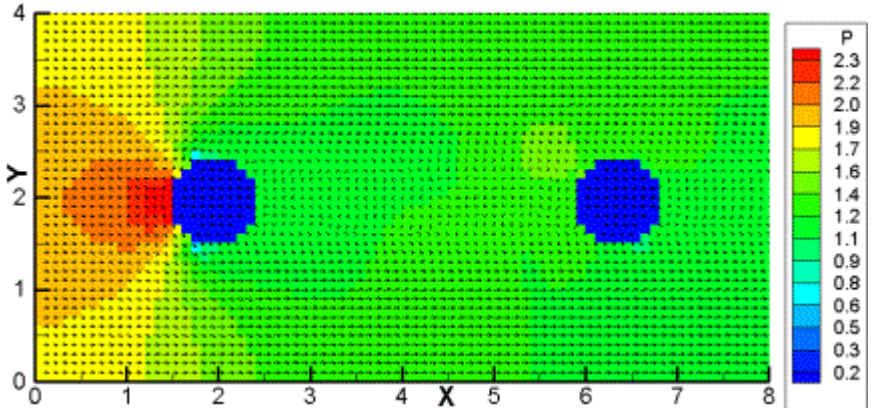

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
0.08 → 0.08
0.08 → 0.08
0.07 → 0.07
0.07 → 0.07
0.06 → 0.06
0.06 → 0.06
0.05 → 0.05
0.05 → 0.05
0.04 → 0.04
0.04 → 0.04
0.03 → 0.03
0.03 → 0.03
0.02 → 0.02
0.02 → 0.02
0.01 → 0.01
0.01 → 0.01
0.00 → 0.00
-0.01 → -0.01
-0.01 → -0.01
-0.02 → -0.02
-0.02 → -0.02
-0.03 → -0.03
-0.03 → -0.03
-0.04 → -0.04
-0.04 → -0.04
-0.05 → -0.05
-0.05 → -0.05
-0.06 → -0.06
-0.06 → -0.06
-0.07 → -0.07
-0.07 → -0.07
-0.08 → -0.08
-0.08 → -0.08
Re=10

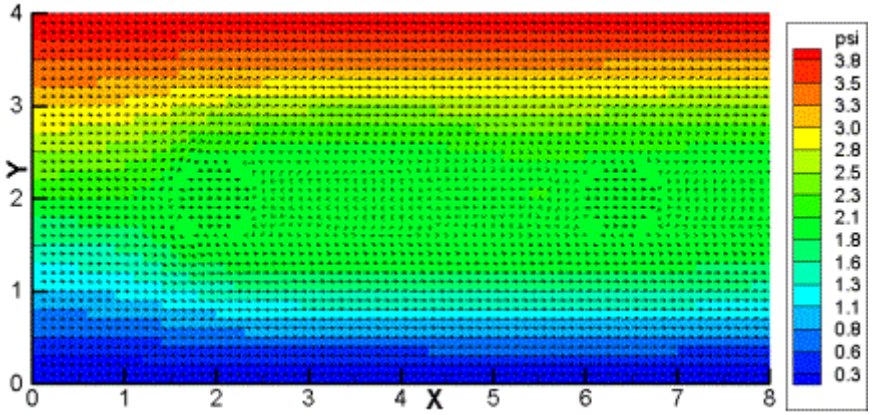

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
0.07 → 0.07
0.06 → 0.07
0.06 → 0.06
0.06 → 0.06
0.05 → 0.05
-0.07 → -0.07
-0.07 → -0.07
Re=10

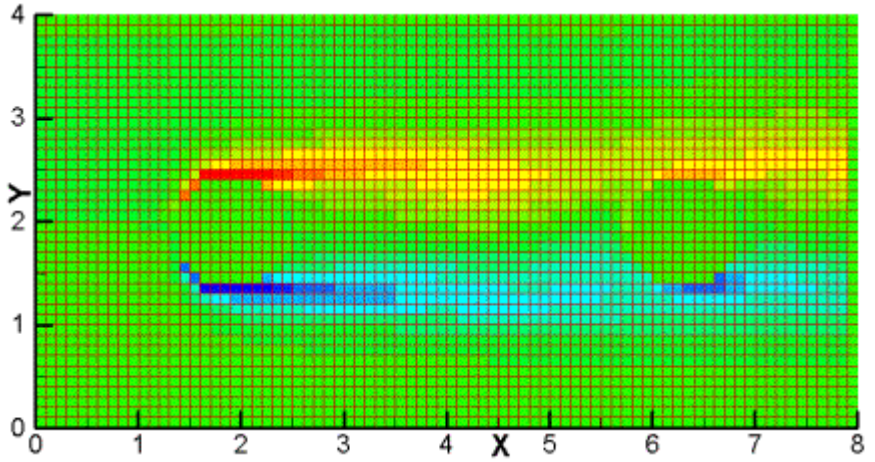

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Re=10

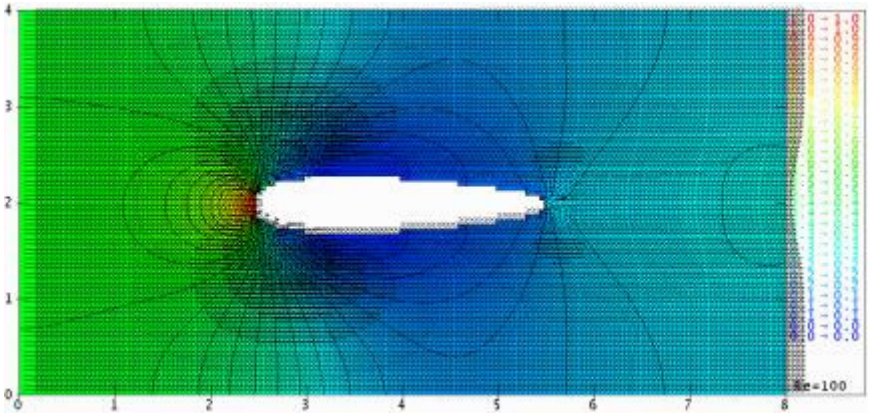

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Re=10

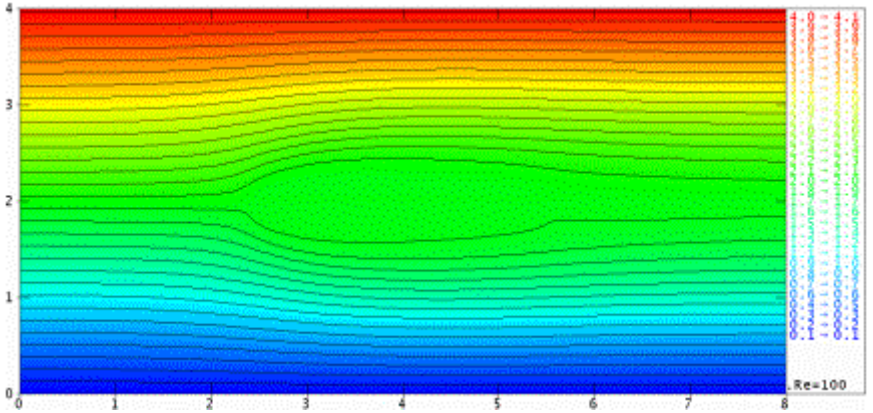

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Re=10

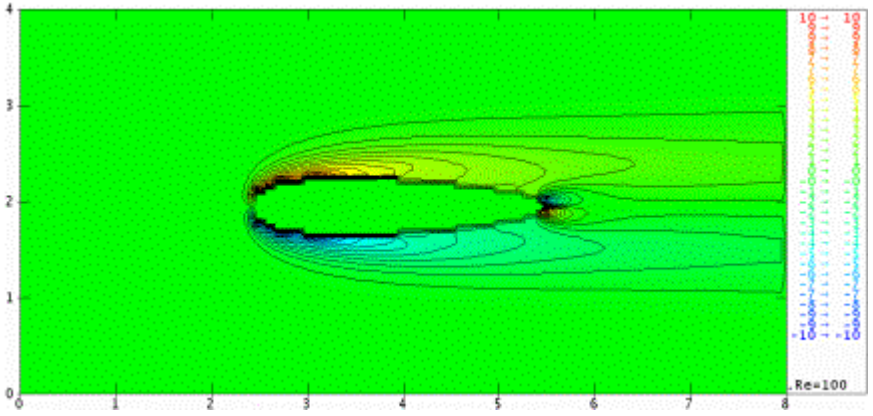

4
3
2
1
0
Y
0
1
2
3
4
X
5
6
7
8
P
2.3
2.2
2.0
1.9
1.7
1.6
1.4
1.2
1.1
0.9
0.8
0.6
0.5
0.3
0.2

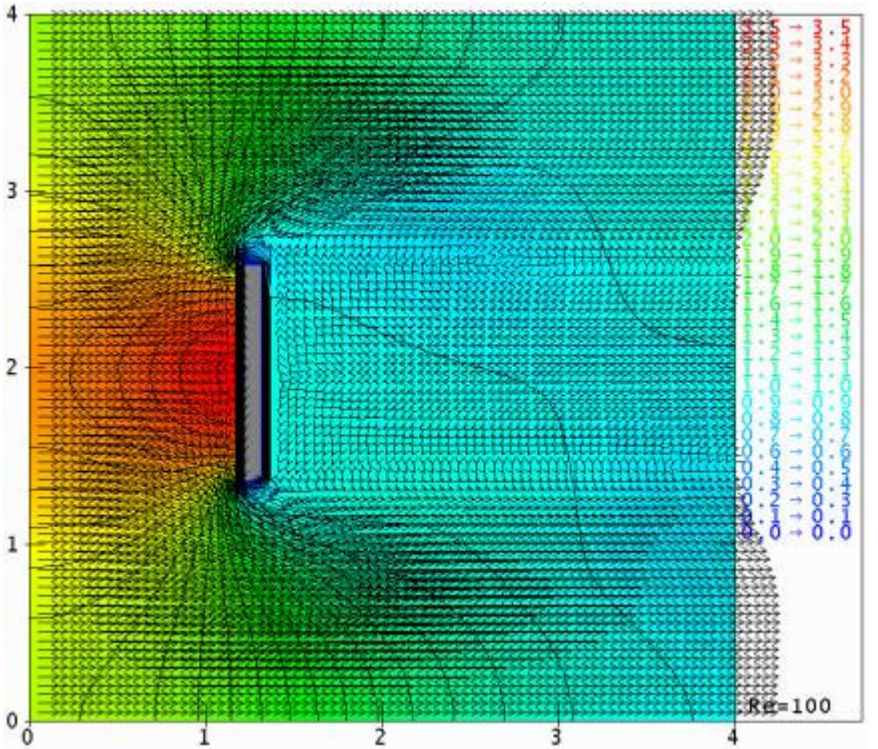

psi
3.8
3.5
3.3
3.0
2.8
2.5
2.3
2.1
1.8
1.6
1.3
1.1
0.8
0.6
0.3
Y
X
0
1
2
3
4
5
6
7
8

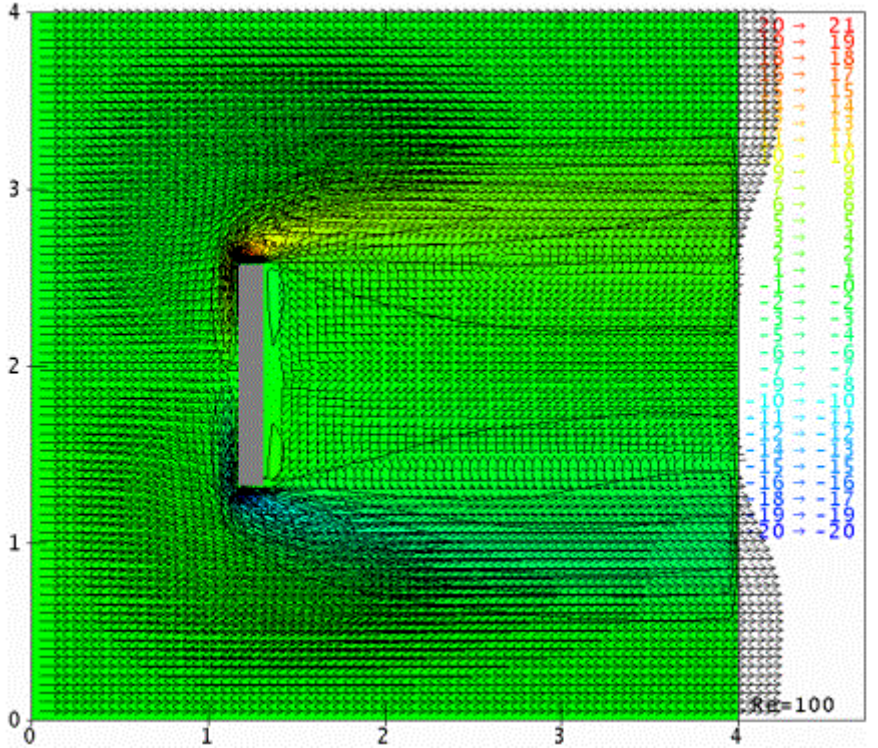

Y
X
0
1
2
3
4
5
6
7
8

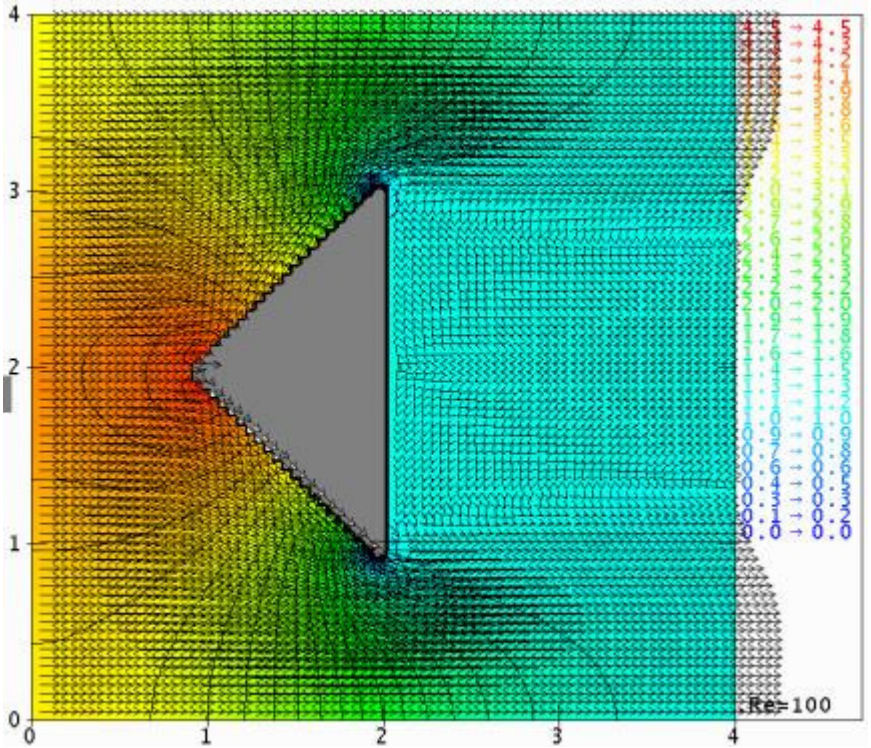

Re=100

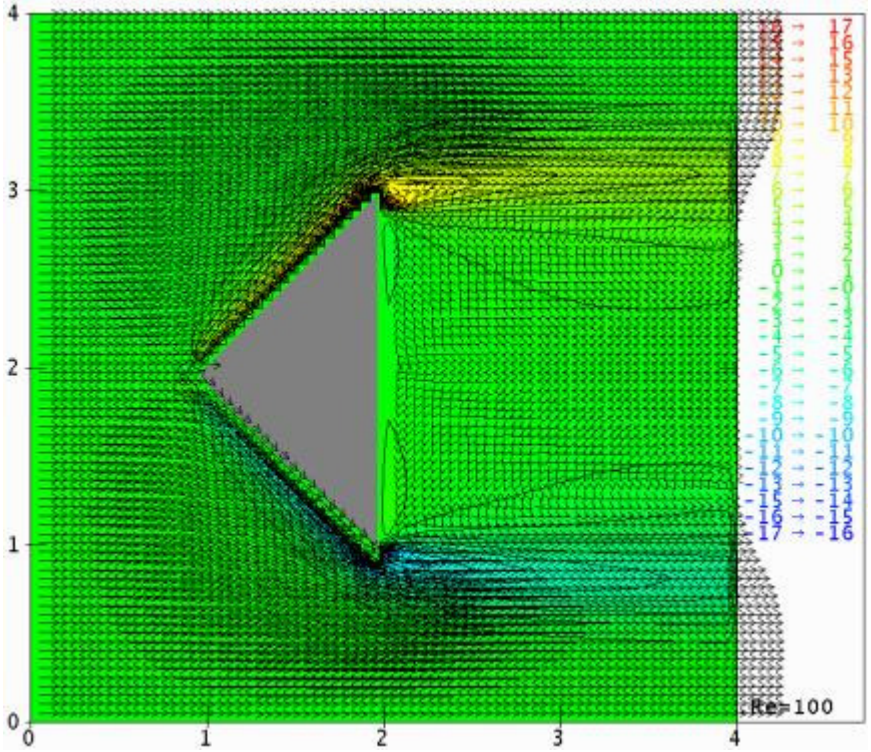

Re=100

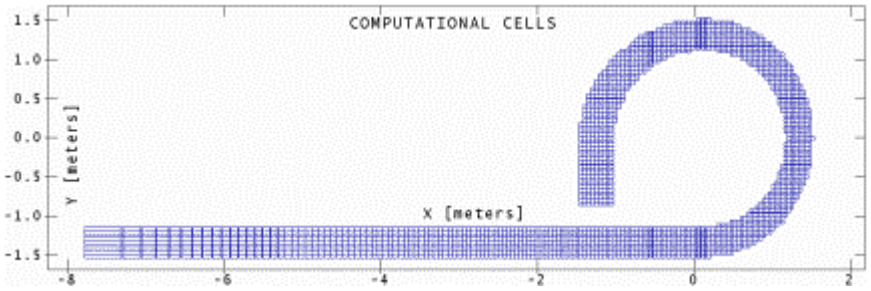

Re=100

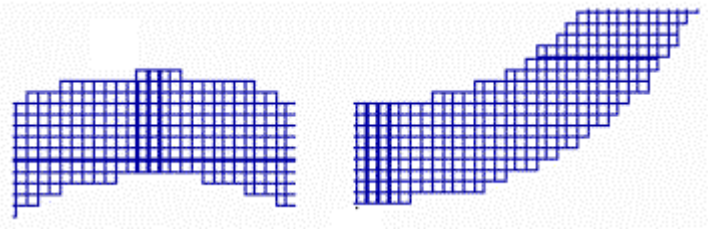

4
3
2
1
0
0
1
2
3
4
Re=100

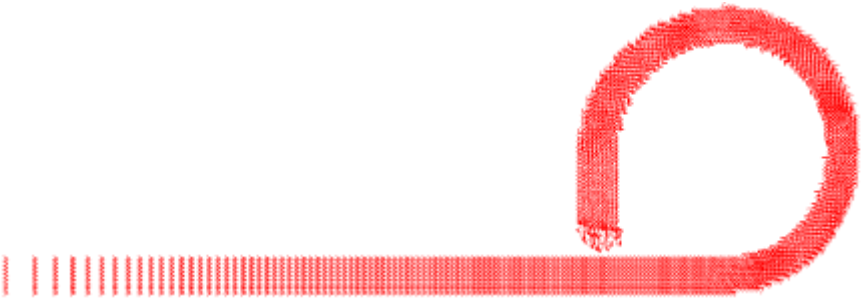

4
3
2
1
0
0
1
2
3
4
Re=100

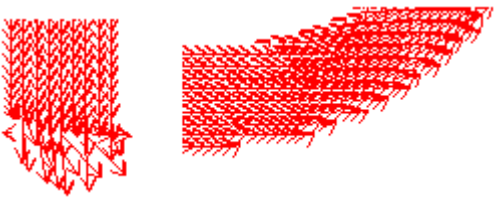

4
3
2
1
0
0
1
2
3
4
Re=100

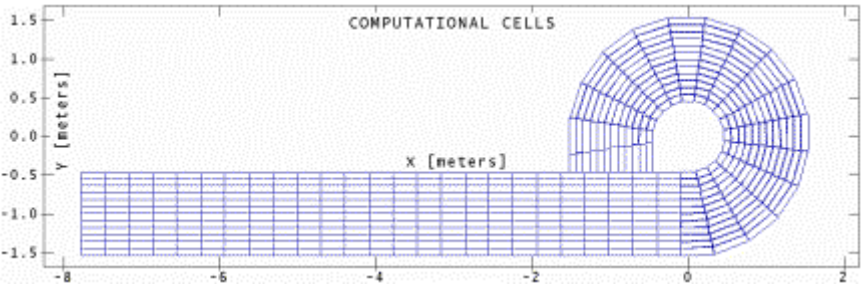

Re=100

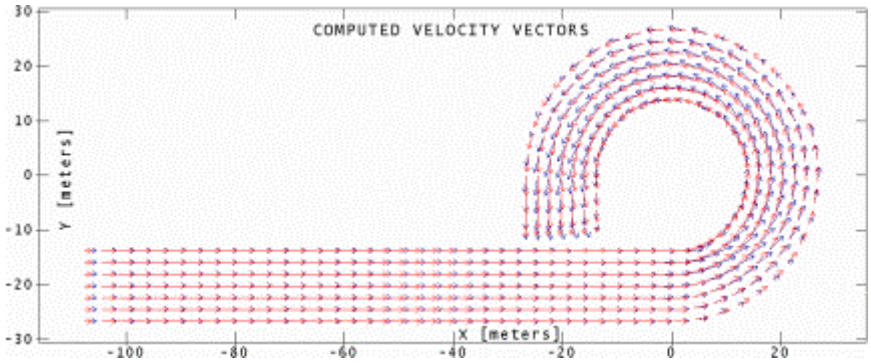

COMPUTATIONAL CELLS
Y [meters]
X [meters]

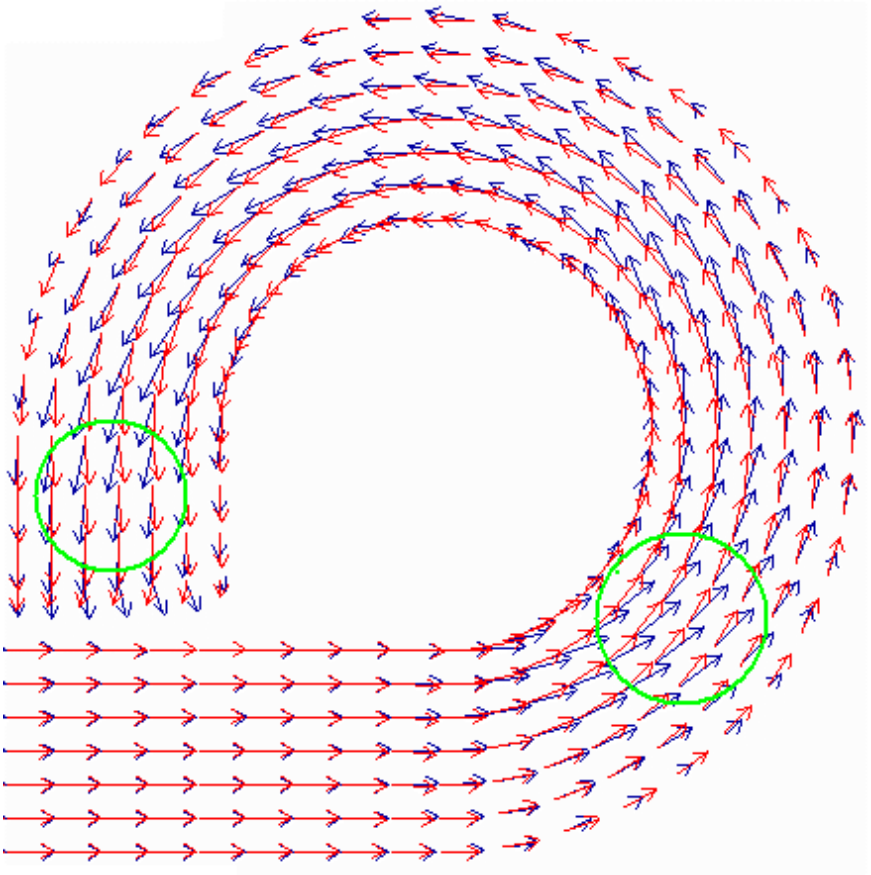


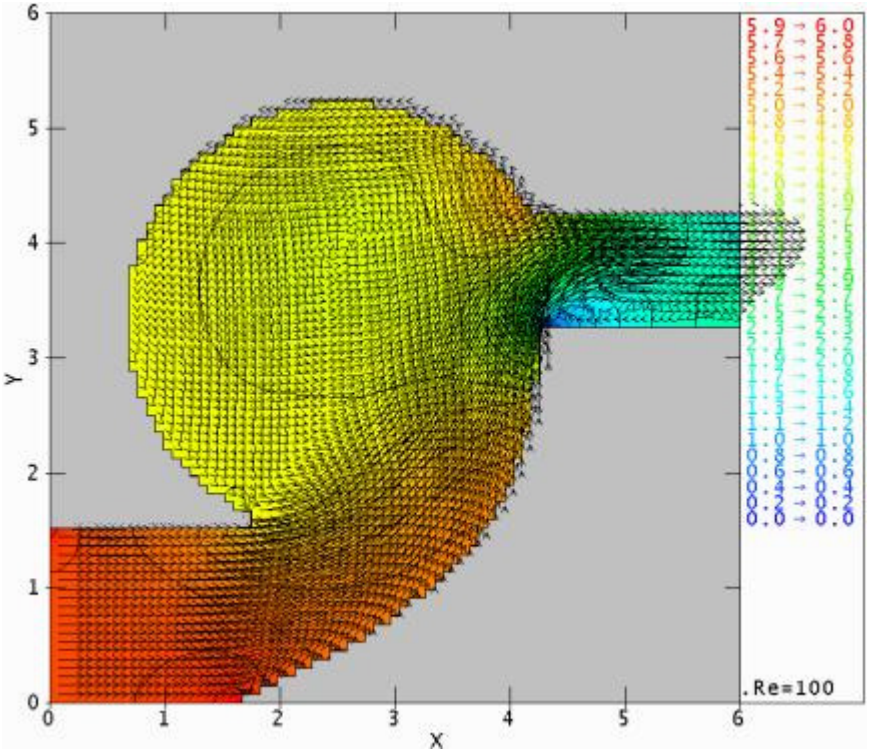


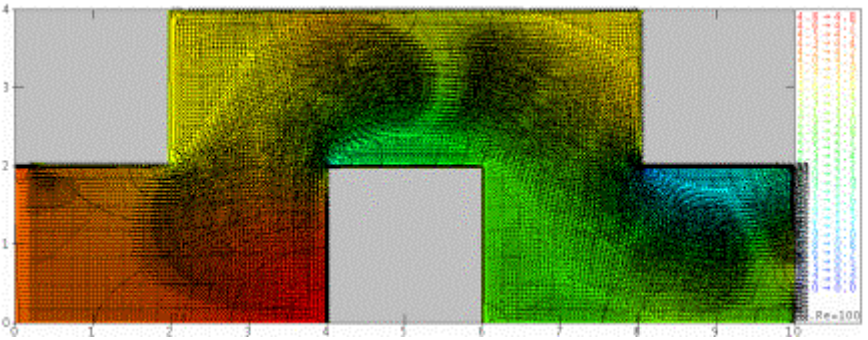


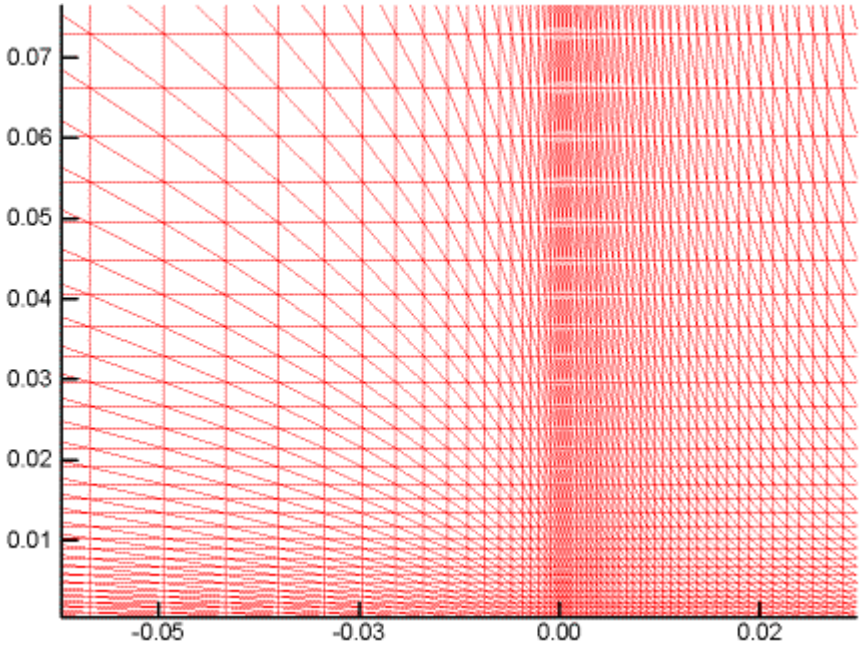

COMPUTATIONAL CELLS
Y [meters]
X [meters]
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-8
-6
-4
-2
0
2

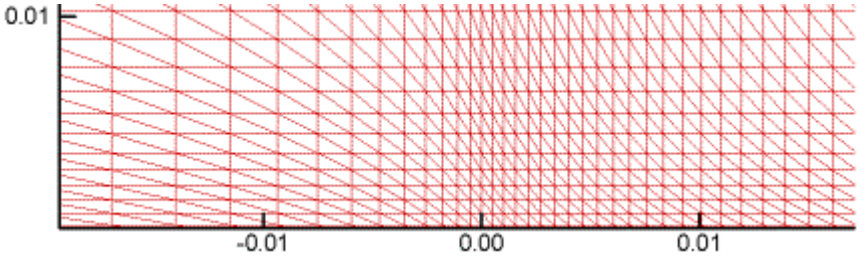

COMPUTED VELOCITY VECTORS
Y [meters]
X [meters]
30
20
10
0
-10
-20
-30
-100
-80
-60
-40
-20
0
20

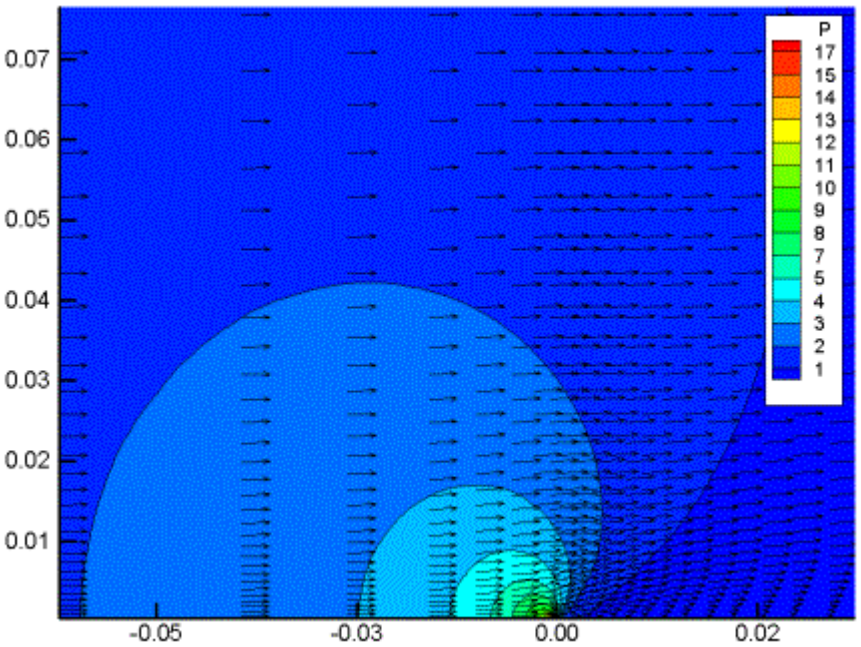


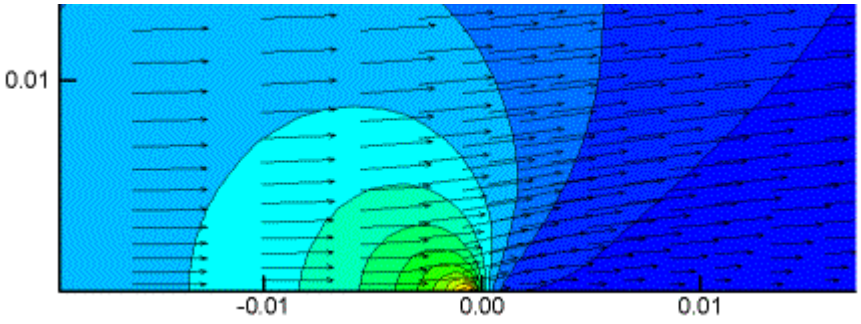

Y
X
.Re=100

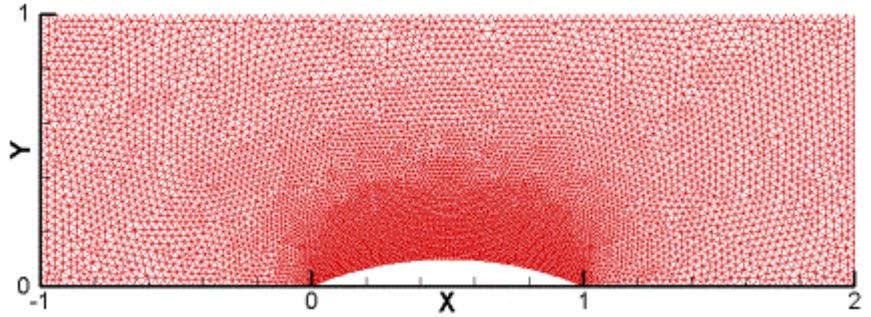

0
1
2
3
4
5
6
7
8
9
10
Re=100

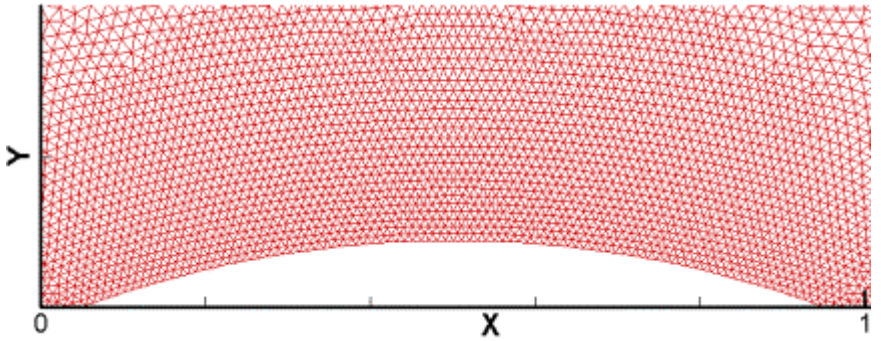

0.07
0.06
0.05
0.04
0.03
0.02
0.01
-0.05
-0.03
0.00
0.02

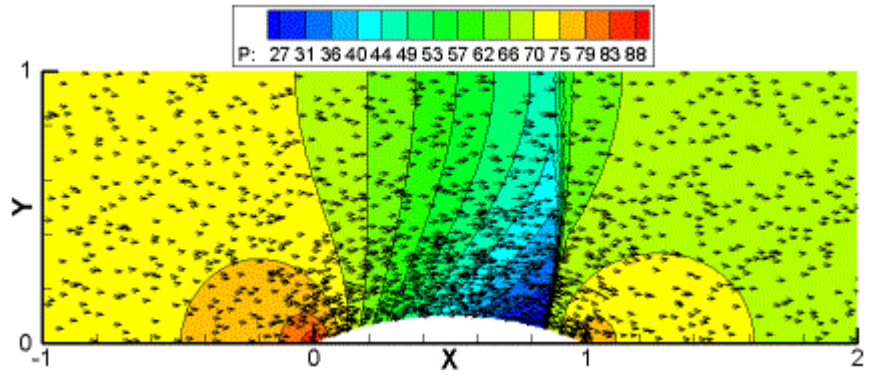

0.01
-0.01
0.00
0.01

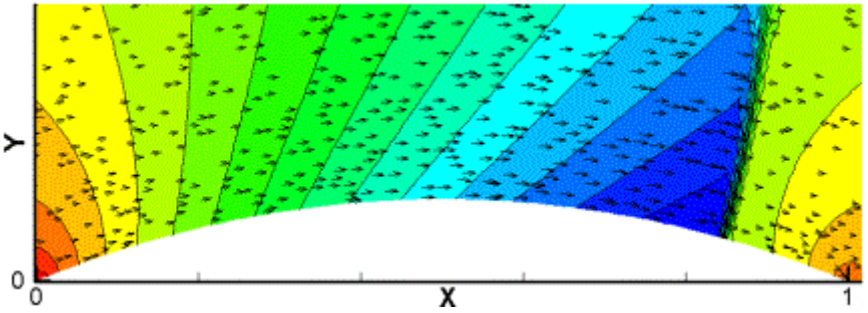

P
17
15
14
13
12
11
10
9
8
7
5
4
3
2
1
0.07
0.06
0.05
0.04
0.03
0.02
0.01
-0.05
-0.03
0.00
0.02

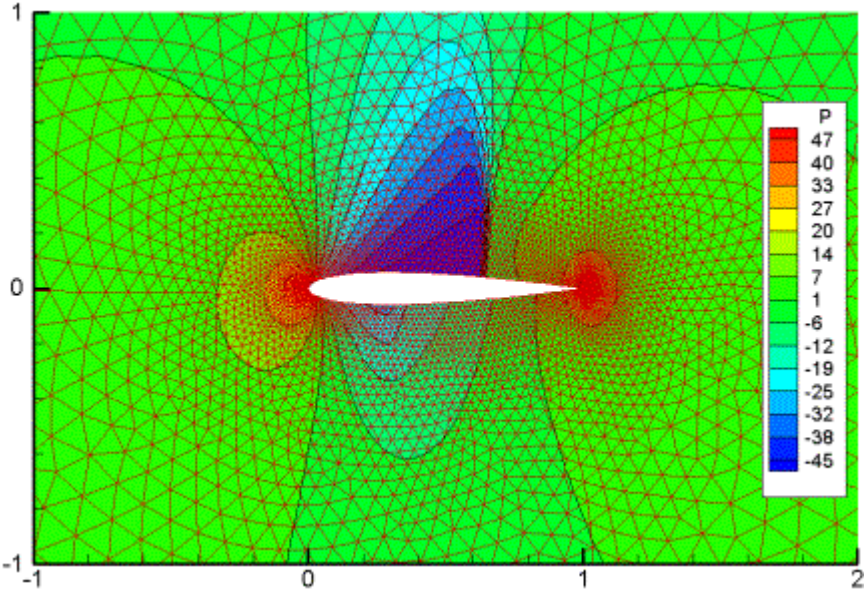

0.01
-0.01
0.00
0.01

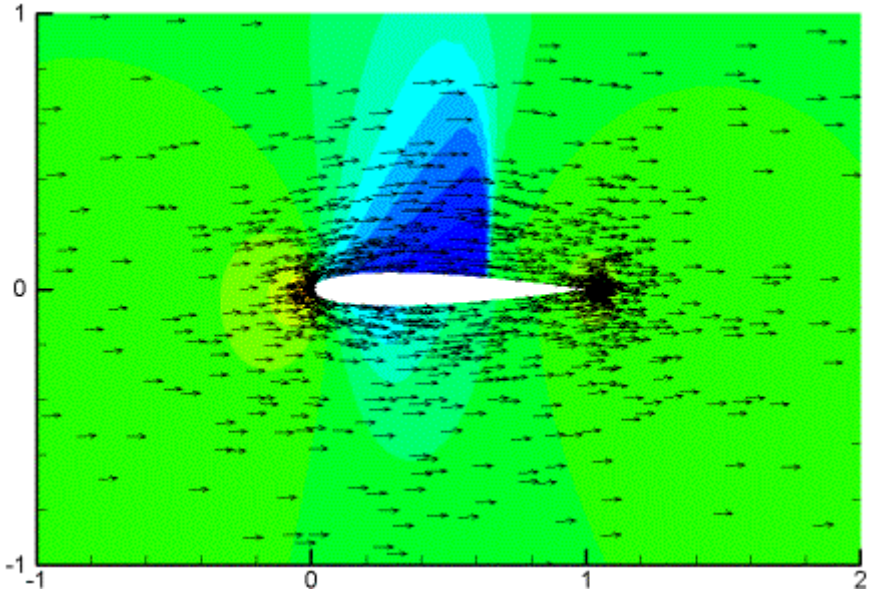

1
Y
0
-1
0
X
1
2

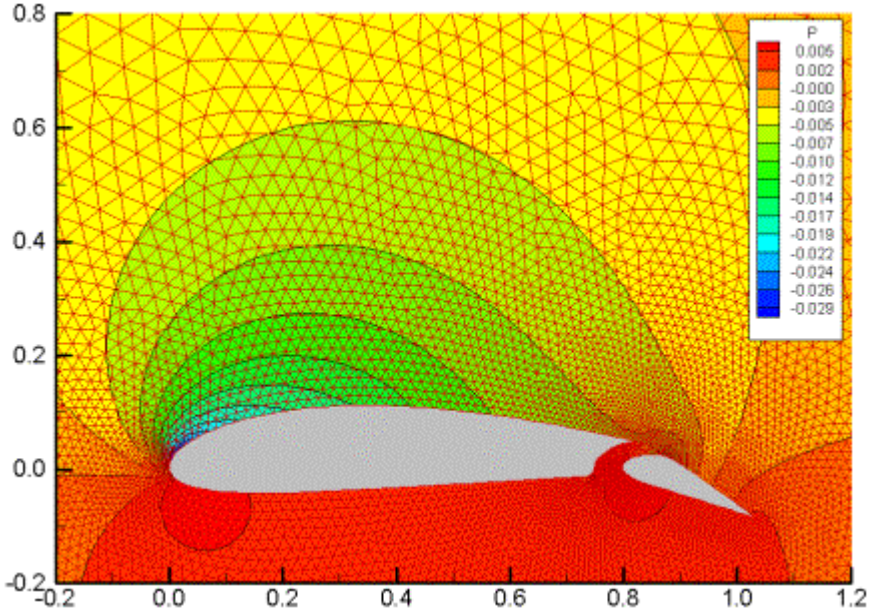

Y
0
X
1

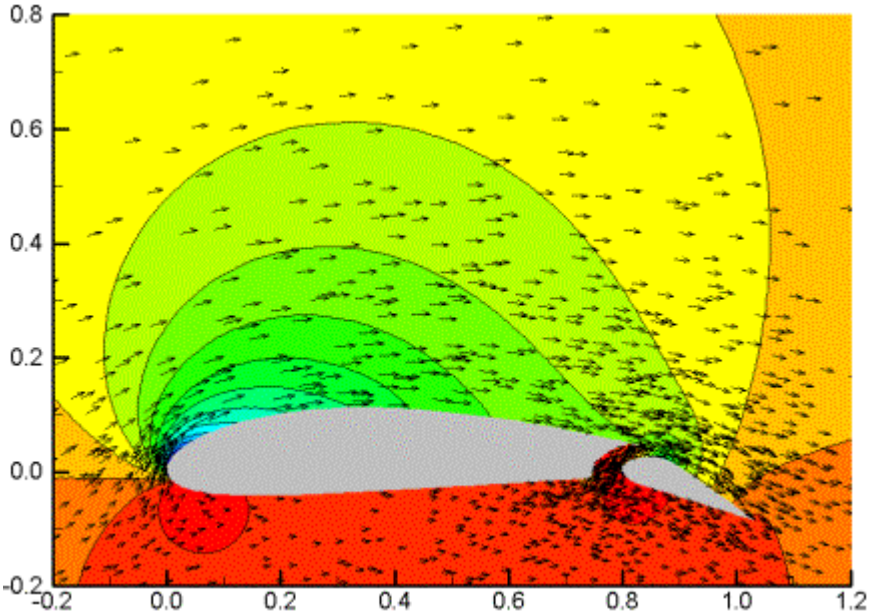

P: 27 31 36 40 44 49 53 57 62 66 70 75 79 83 88
1
Y
0
-1
0
X
1
2

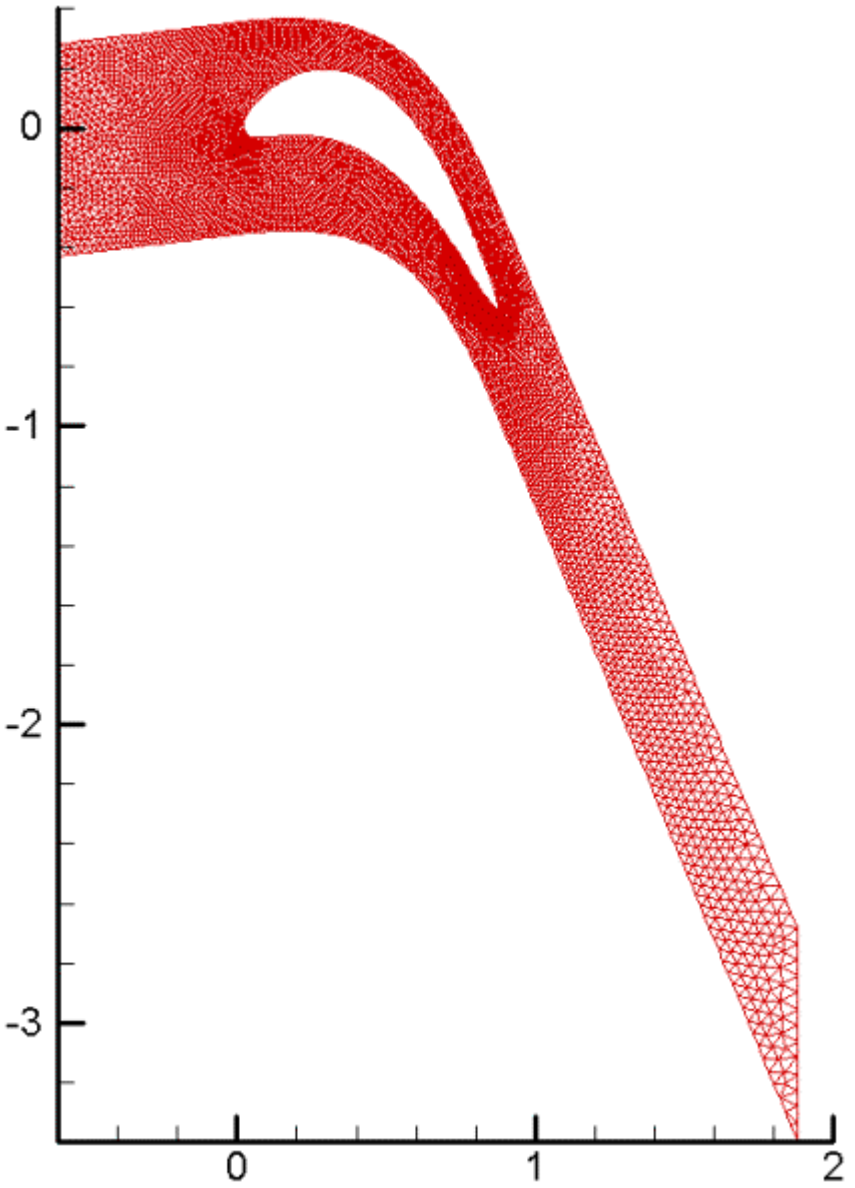

Y
0
0
X
1

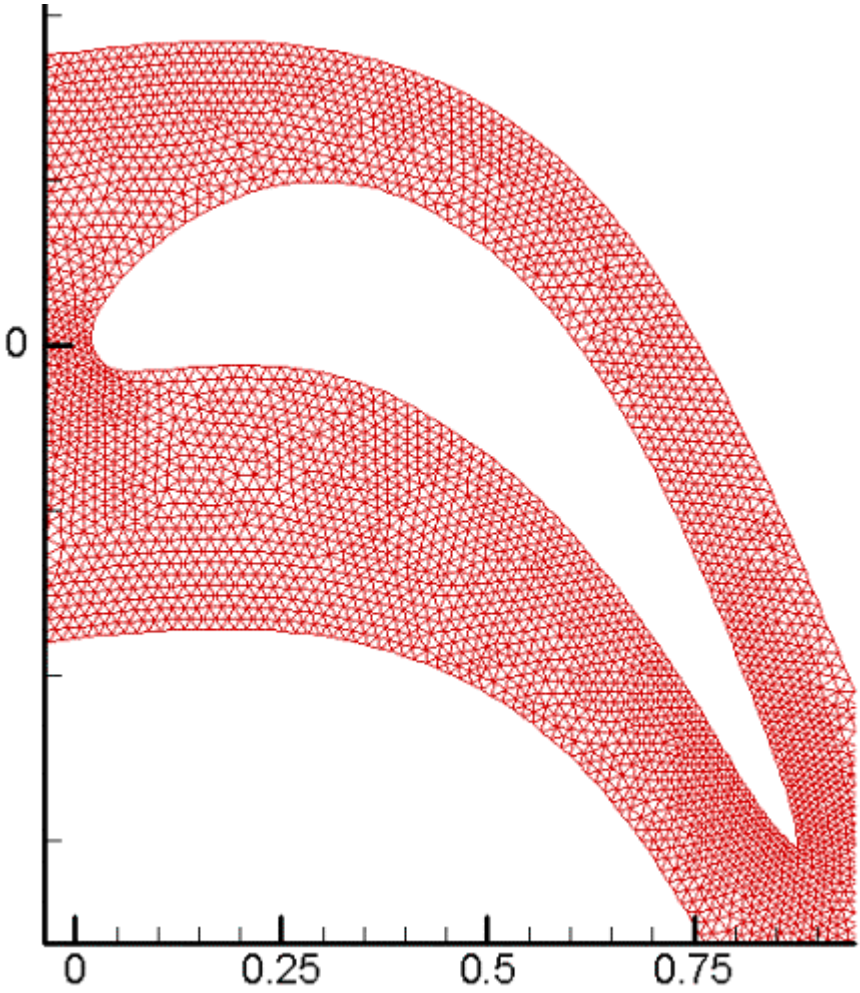

1
0
-1
-1
0
1
2
P
47
40
33
27
20
14
7
1
-6
-12
-19
-25
-32
-38
-45

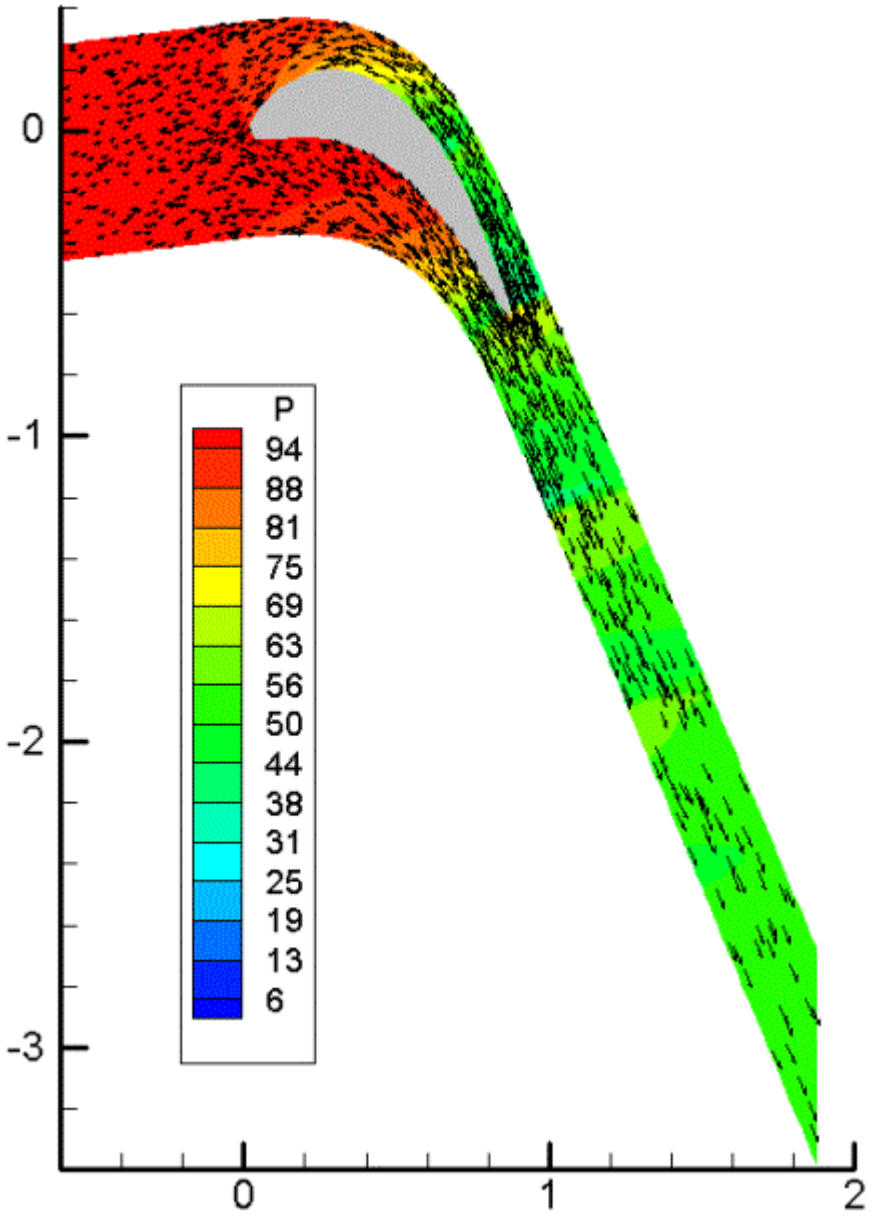

1
0
-1
-1
0
1
2

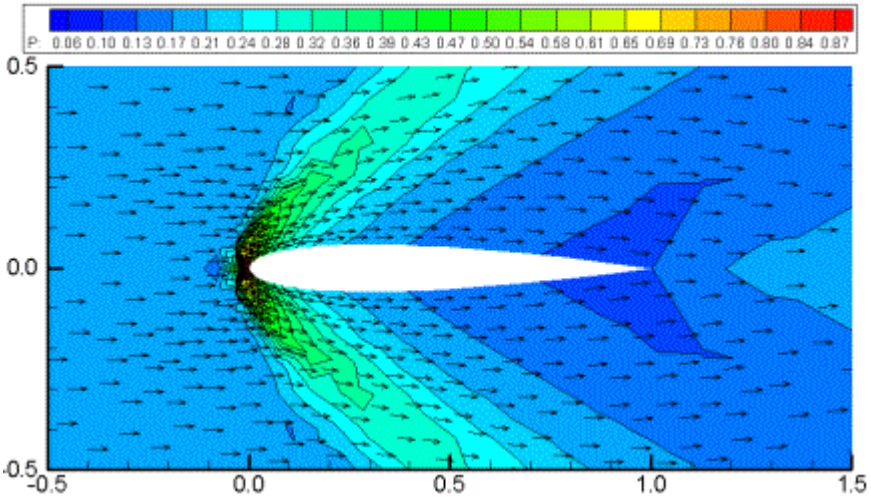

P
0.005
0.002
-0.000
-0.003
-0.005
-0.007
-0.010
-0.012
-0.014
-0.017
-0.019
-0.022
-0.024
-0.026
-0.029
0.8
0.6
0.4
0.2
0.0
-0.2
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2

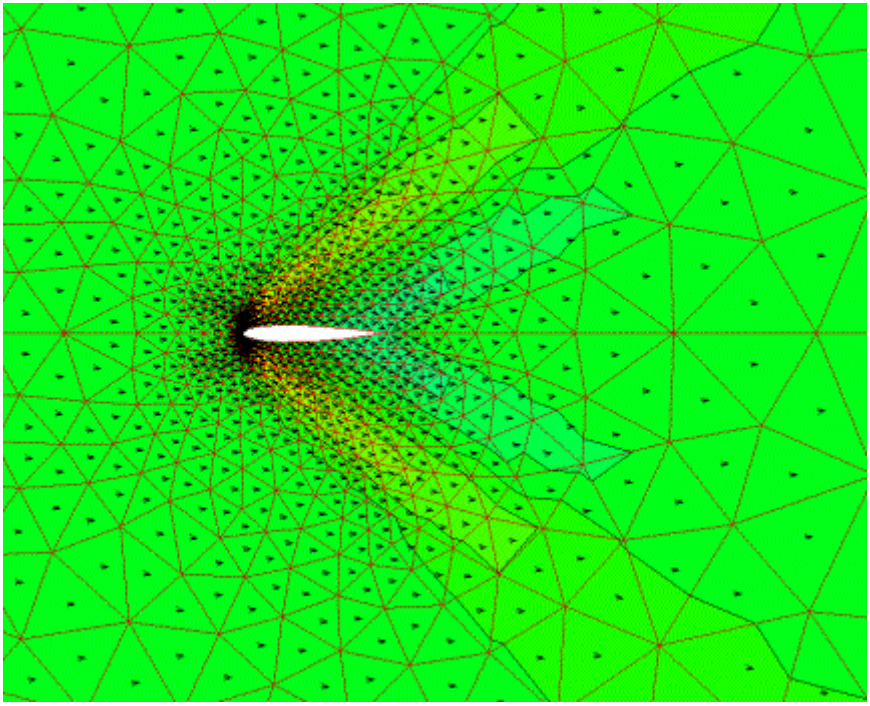

0.8
0.6
0.4
0.2
0.0
-0.2
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2

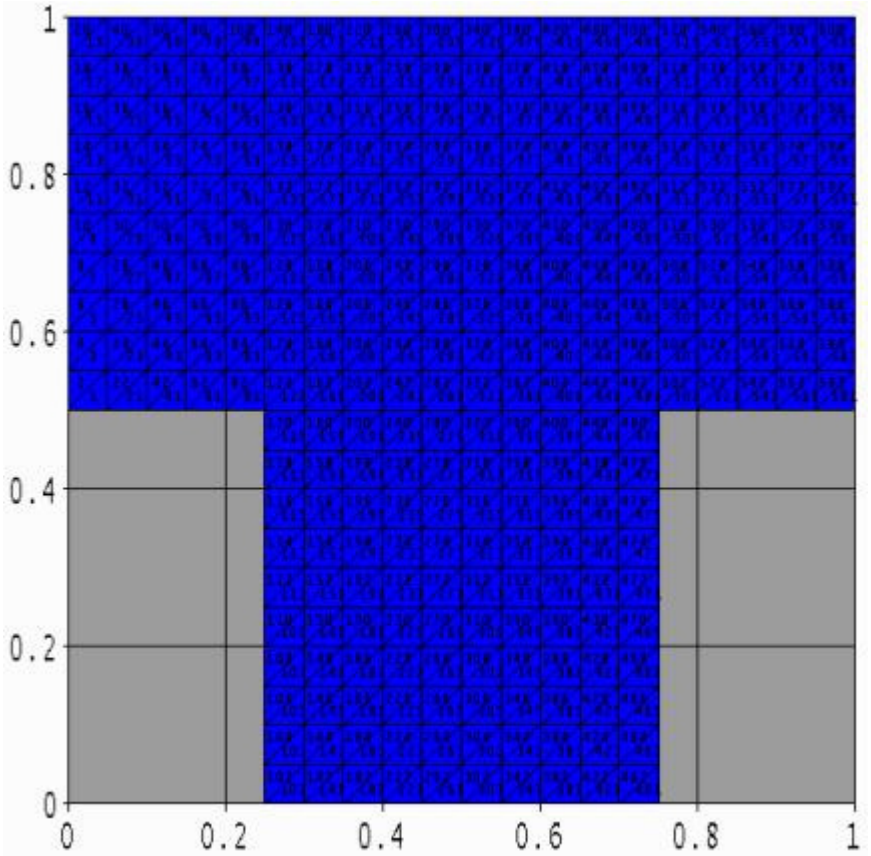

0
-1
-2
-3
0
1
2

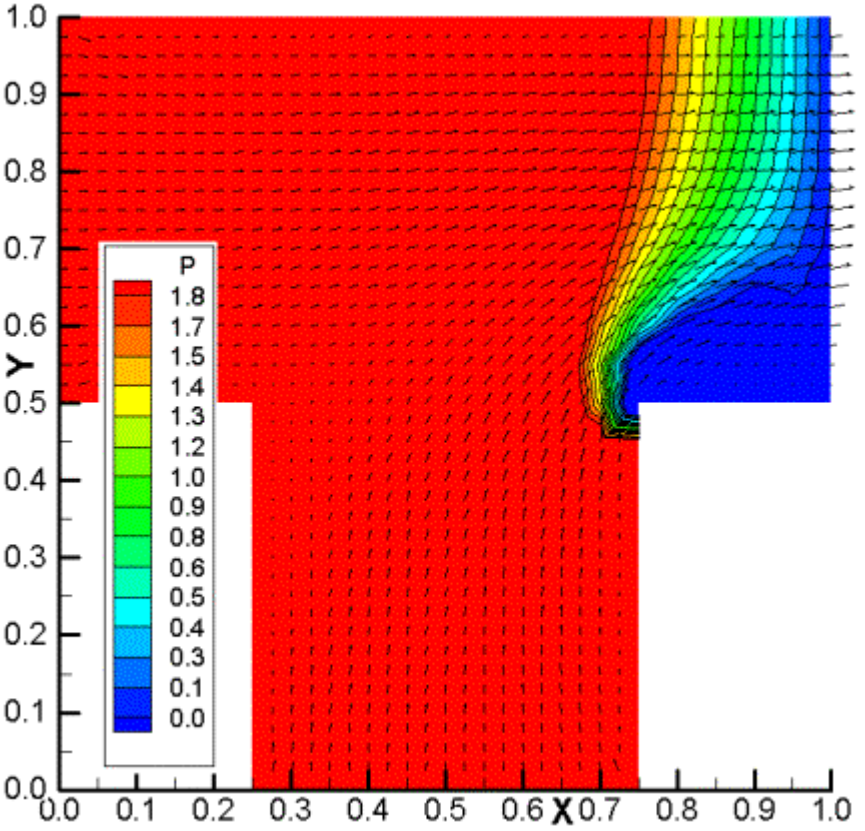

0
0
0.25
0.5
0.75

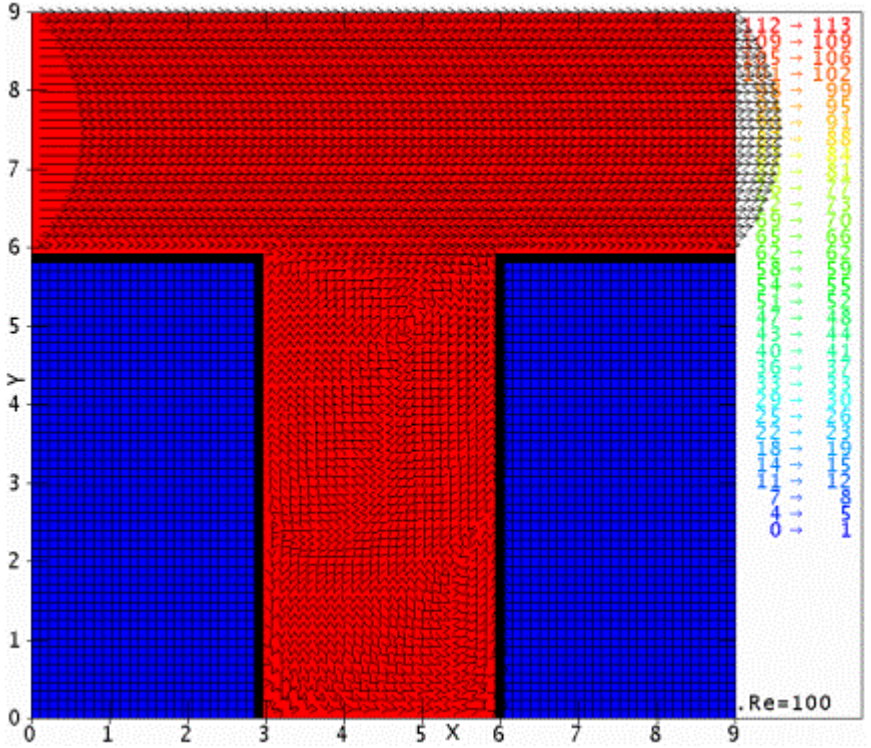

P
94
88
81
75
69
63
56
50
44
38
31
25
19
13
6
0
-1
-2
-3
0
1
2

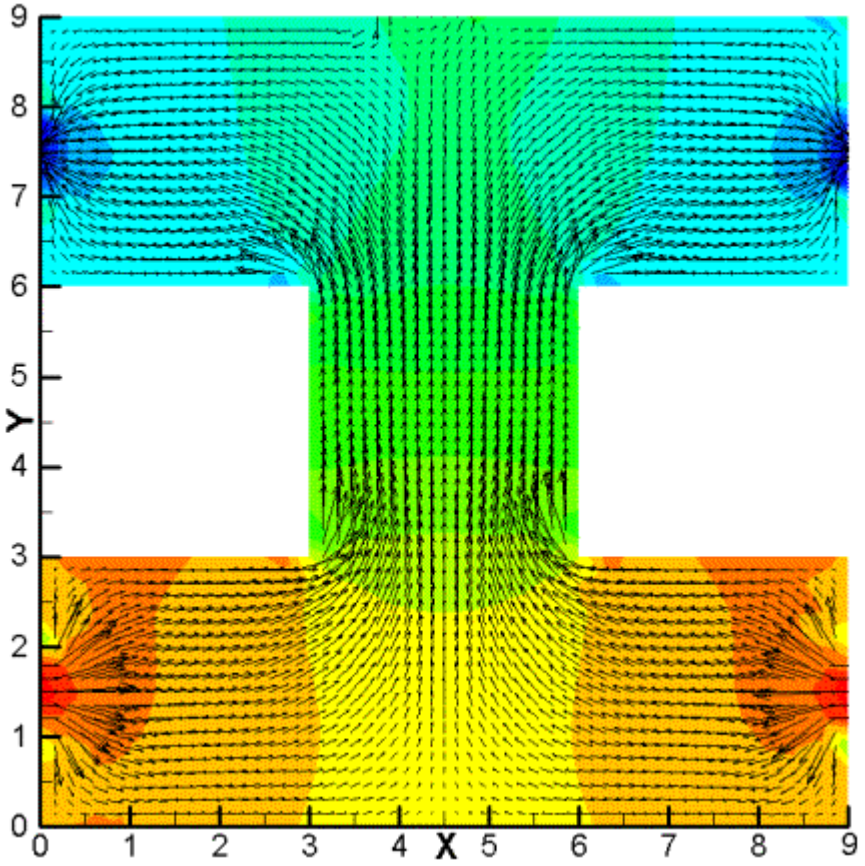

P: 0.06 0.10 0.13 0.17 0.21 0.24 0.28 0.32 0.36 0.39 0.43 0.47 0.50 0.54 0.58 0.61 0.65 0.69 0.73 0.76 0.80 0.84 0.87
0.5
0.0
-0.5
-0.5
0.0
0.5
1.0
1.5

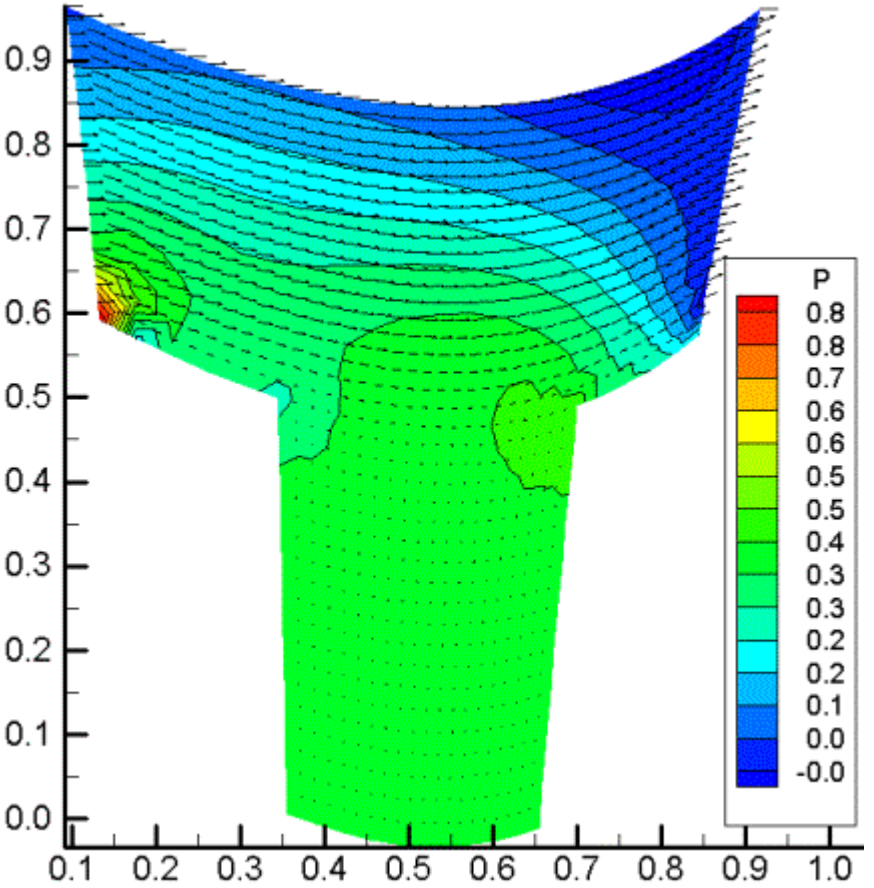


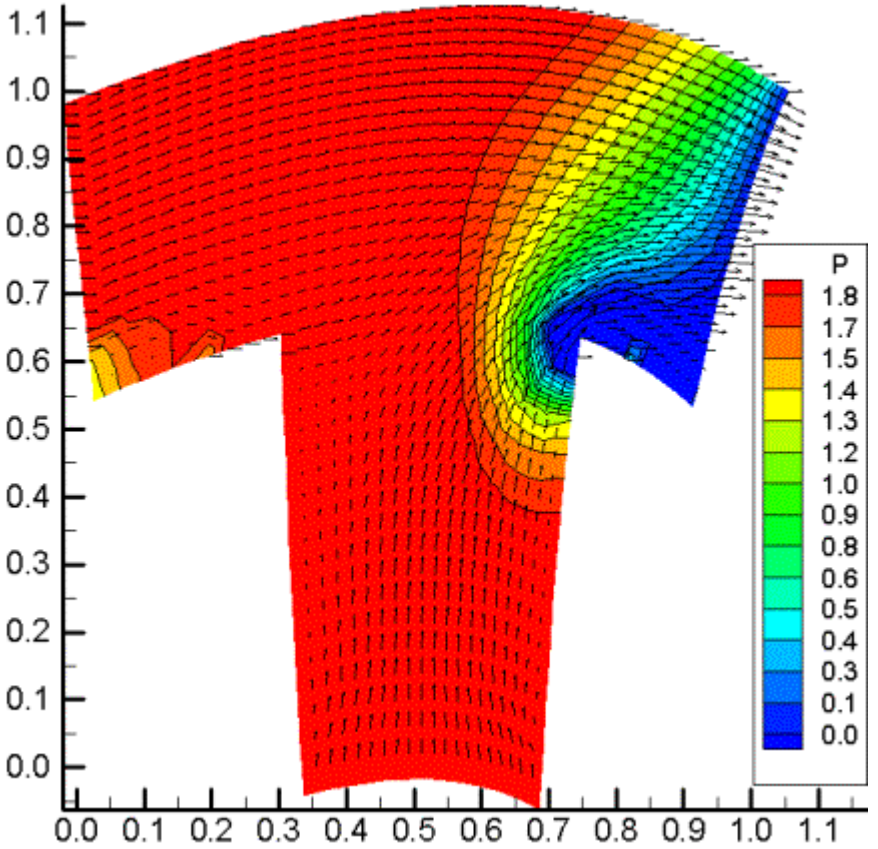

1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

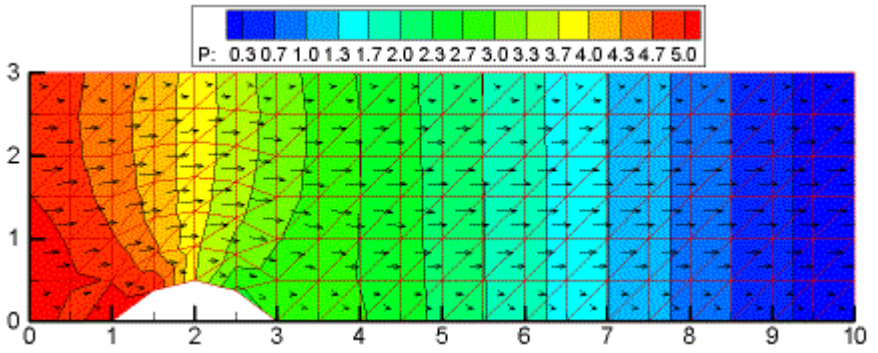

P
1.8
1.7
1.5
1.4
1.3
1.2
1.0
0.9
0.8
0.6
0.5
0.4
0.3
0.1
0.0
Y
X
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

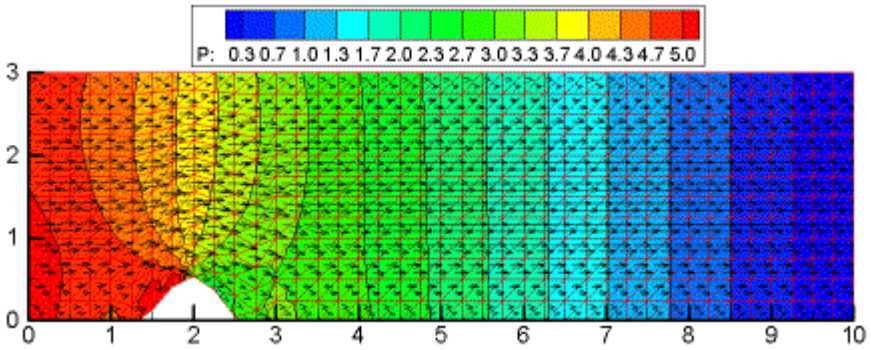

9
8
7
6
5
4
3
2
1
0
Y
0
1
2
3
4
5
X
6
7
8
9
.Re=100

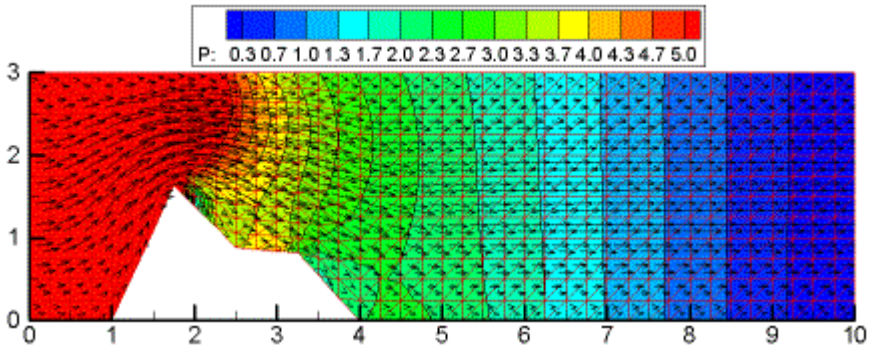

9
8
7
6
5
Y
4
3
2
1
0
0
1
2
3
4
X
5
6
7
8
9

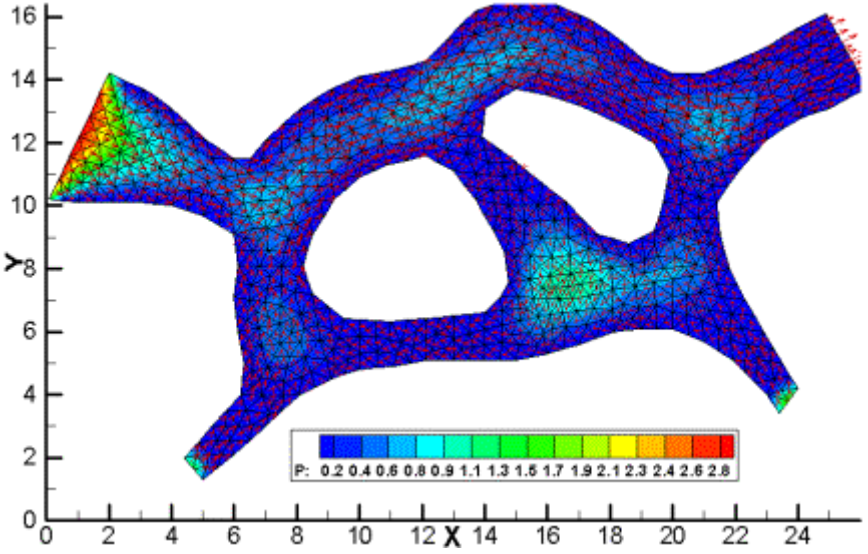

0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
P
0.8
0.8
0.7
0.6
0.6
0.5
0.5
0.4
0.3
0.3
0.2
0.2
0.1
0.0
-0.0

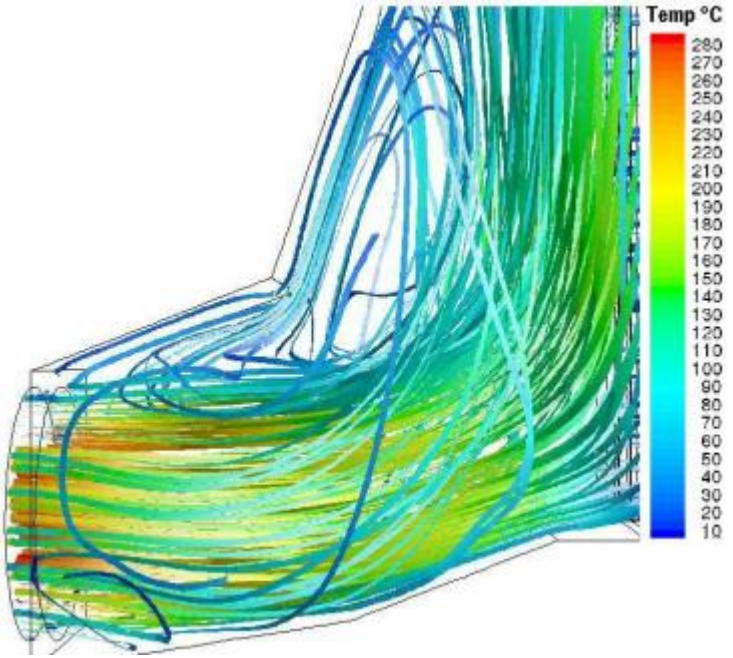

1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1
P
1.8
1.7
1.5
1.4
1.3
1.2
1.0
0.9
0.8
0.6
0.5
0.4
0.3
0.1
0.0

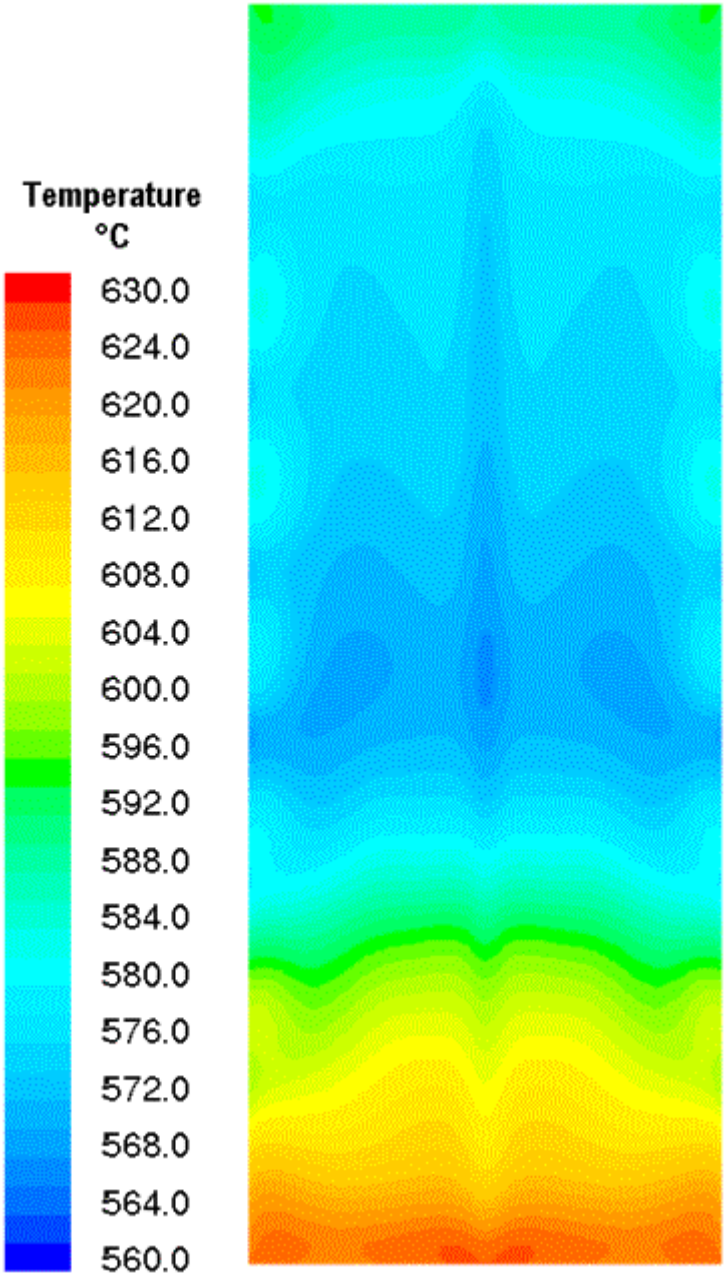

P: 0.3 0.7 1.0 1.3 1.7 2.0 2.3 2.7 3.0 3.3 3.7 4.0 4.3 4.7 5.0
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10

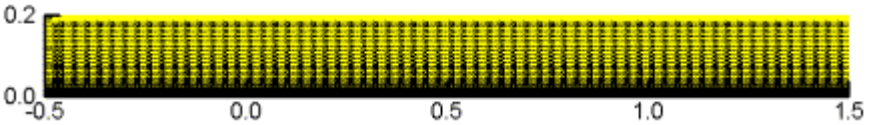

P: 0.3 0.7 1.0 1.3 1.7 2.0 2.3 2.7 3.0 3.3 3.7 4.0 4.3 4.7 5.0
0
1
2
3
0
1
2
3
4
5
6
7
8
9
10

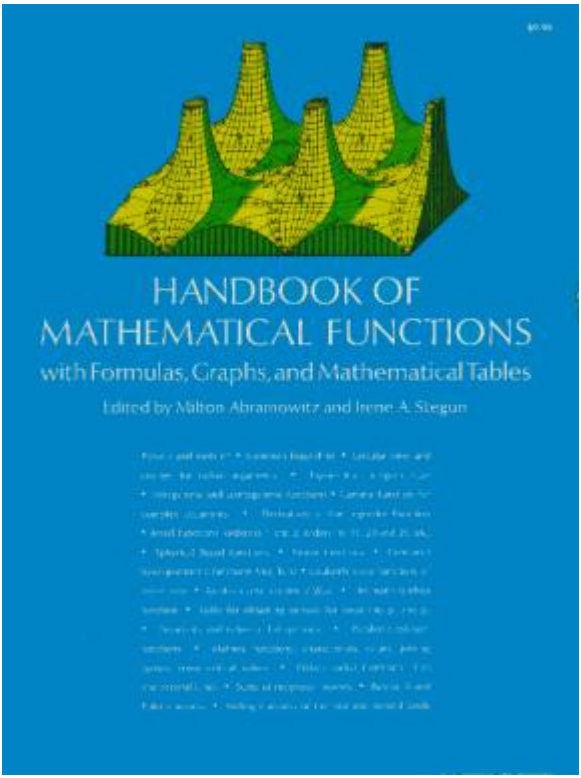

P: 0.3 0.7 1.0 1.3 1.7 2.0 2.3 2.7 3.0 3.3 3.7 4.0 4.3 4.7 5.0
0
1
2
3
0
1
2
3
4
5
6
7
8
9
10


Y
X
P: 0.2 0.4 0.6 0.8 0.9 1.1 1.3 1.5 1.7 1.9 2.1 2.3 2.4 2.6 2.8


Temp °C
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10


Temperature
°C
630.0
624.0
620.0
616.0
612.0
608.0
604.0
600.0
596.0
592.0
588.0
584.0
580.0
576.0
572.0
568.0
564.0
560.0


0.2
0.0
-0.5
0.0
0.5
1.0
1.5


HANDBOOK OF
MATHEMATICAL FUNCTIONS
with Formulas, Graphs, and Mathematical Tables
Edited by Milton Abramowitz and Irene A. Stegun