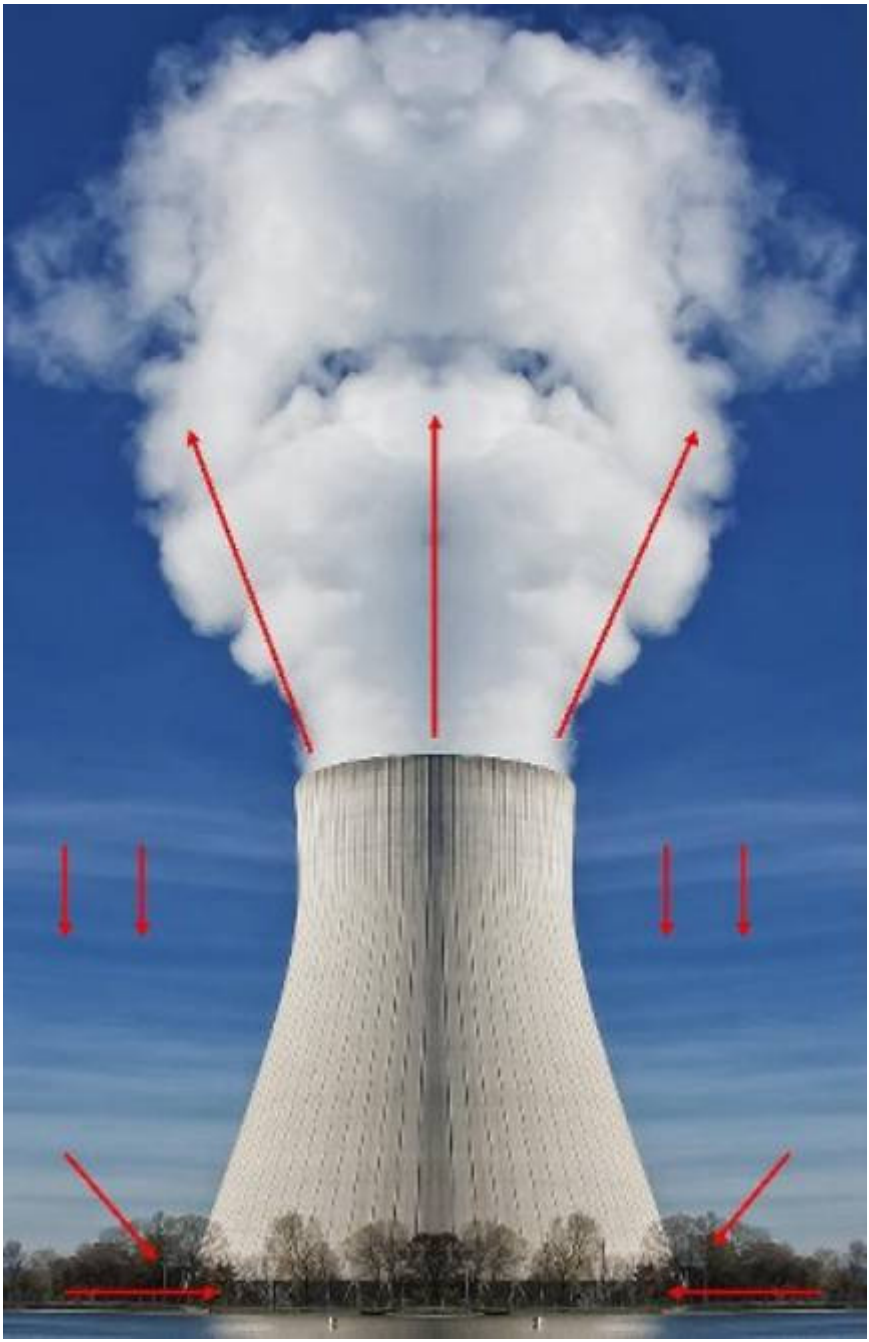
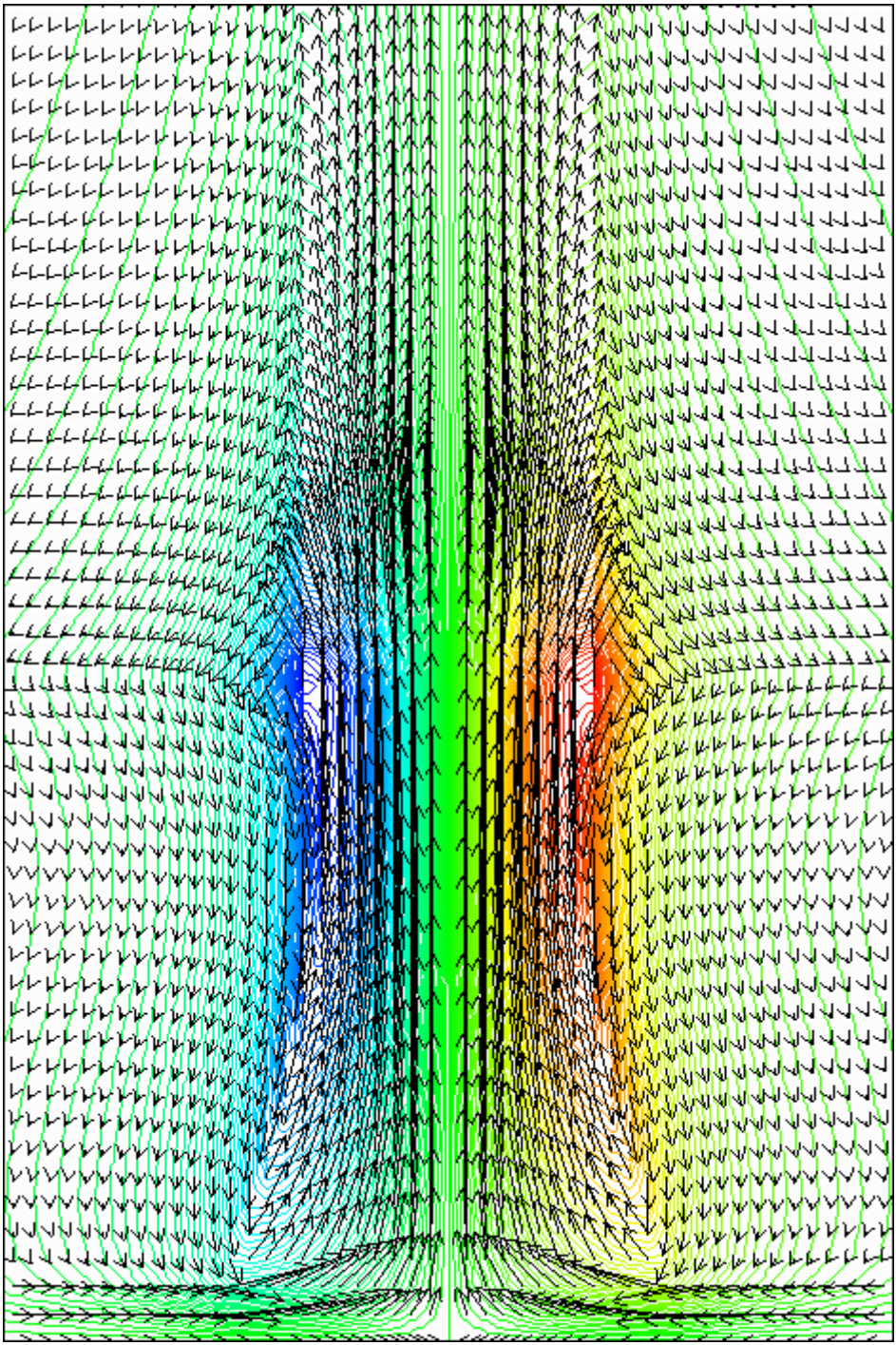
Complex Variables

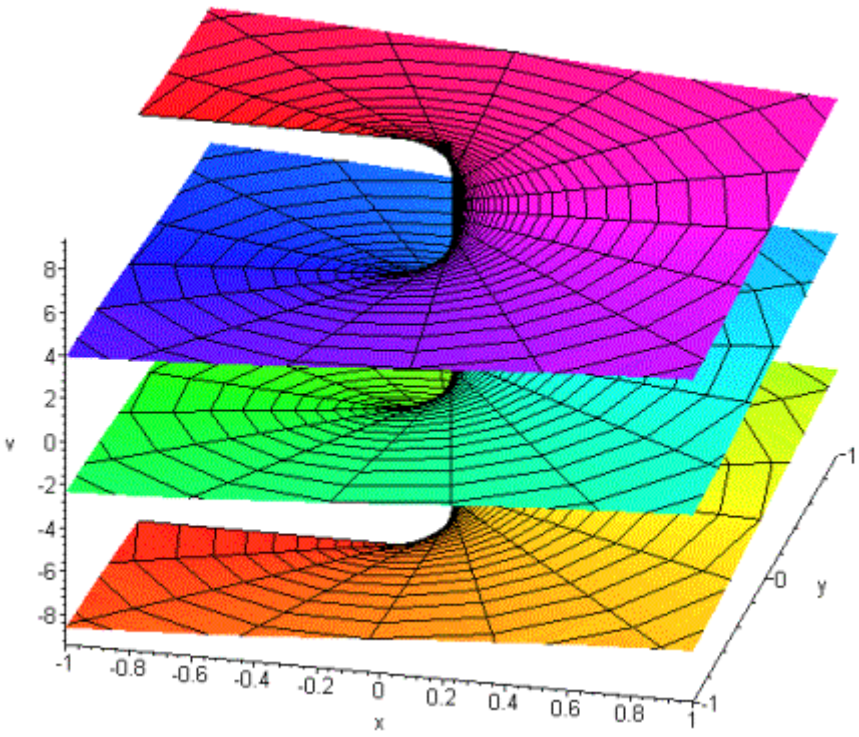
Practical Applications

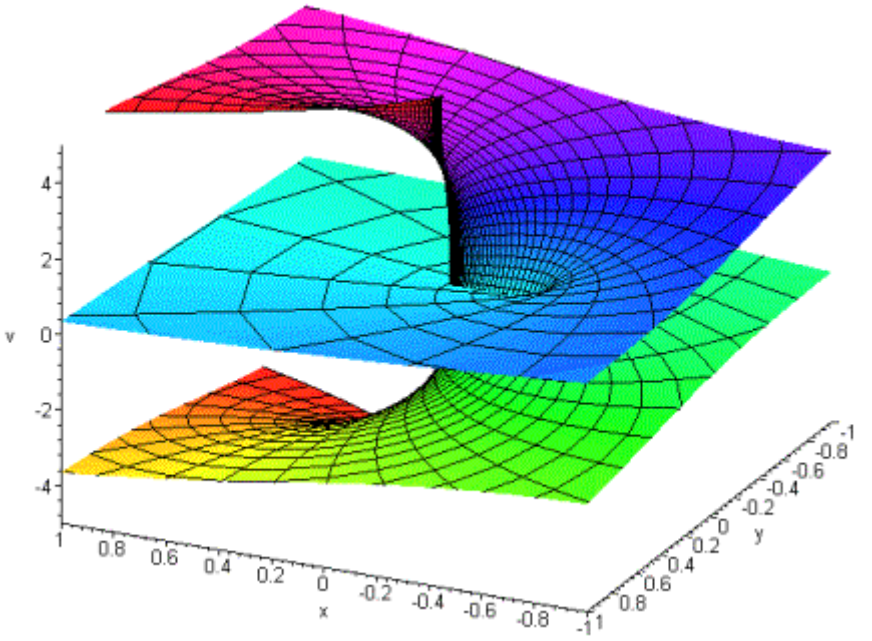
D. James Benton

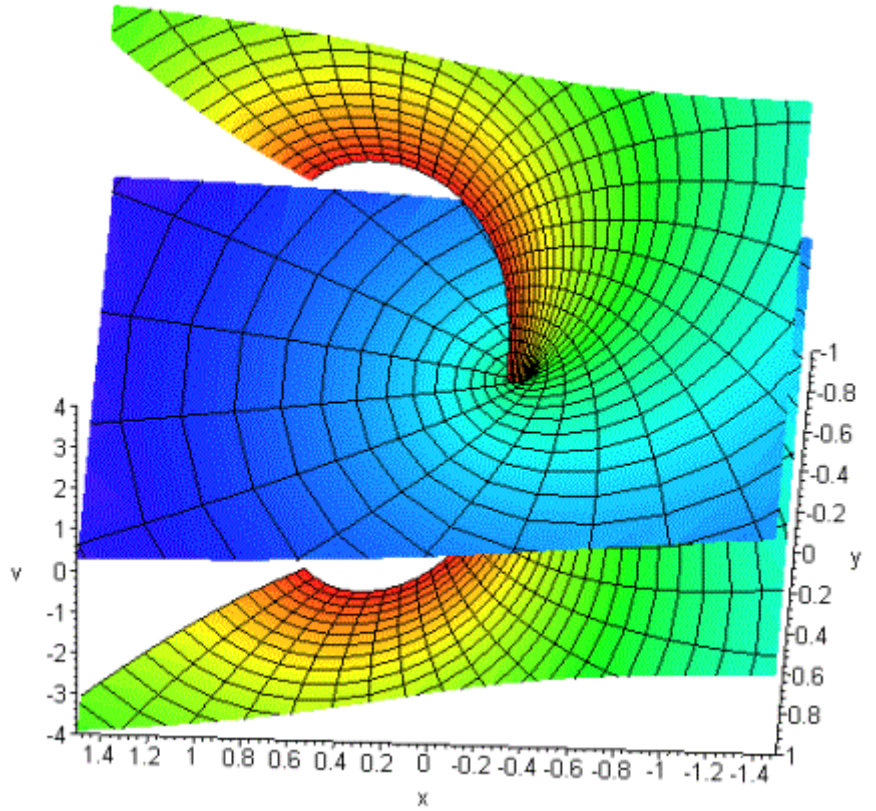
just the color figures for those who got the B&W version

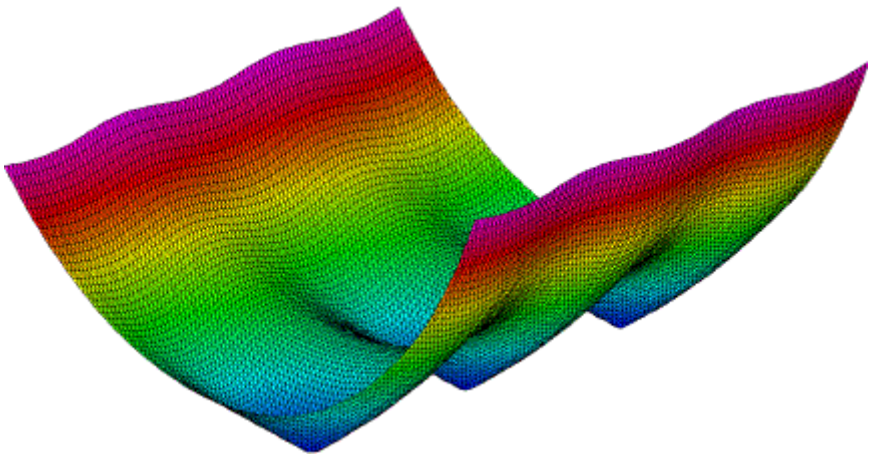


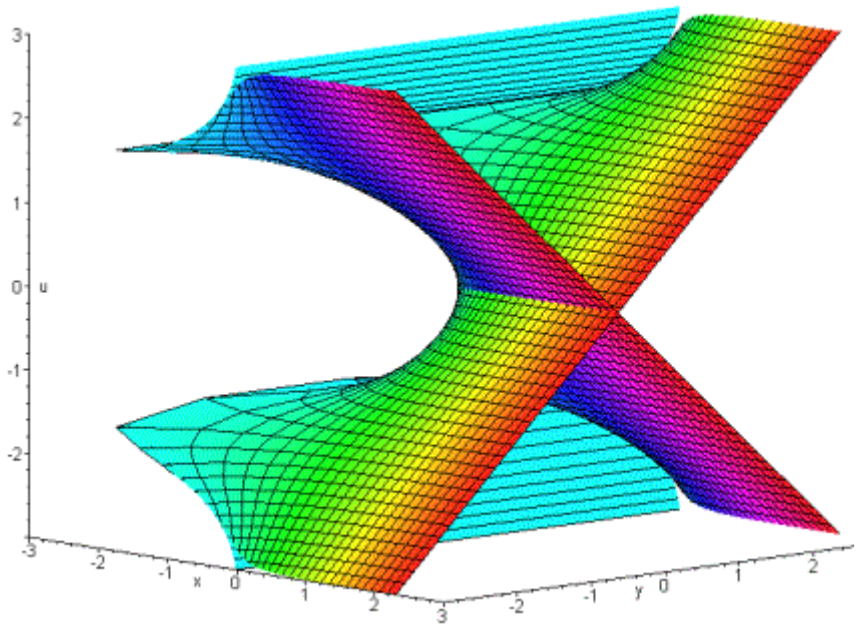


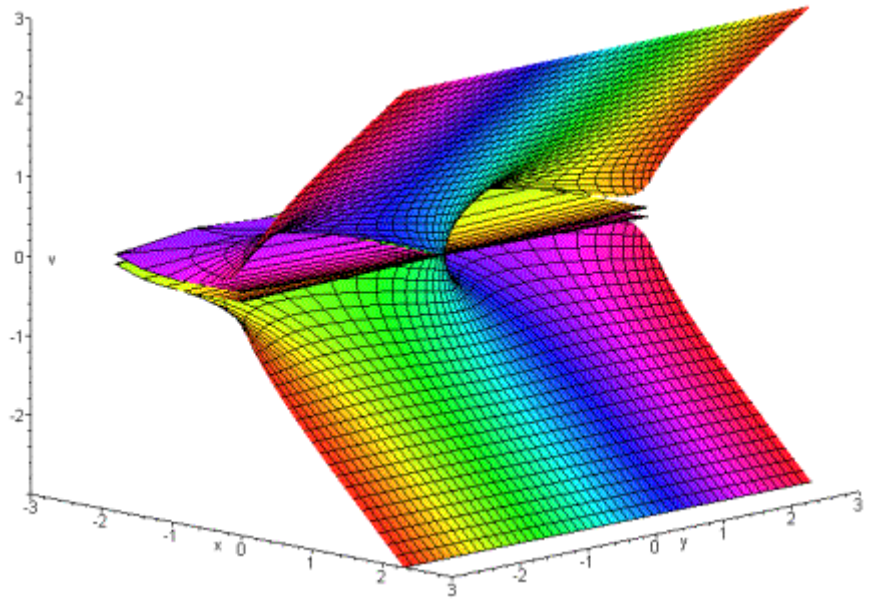

v
8
6
4
2
0
-2
-4
-6
-8
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x
-1
0
y
1

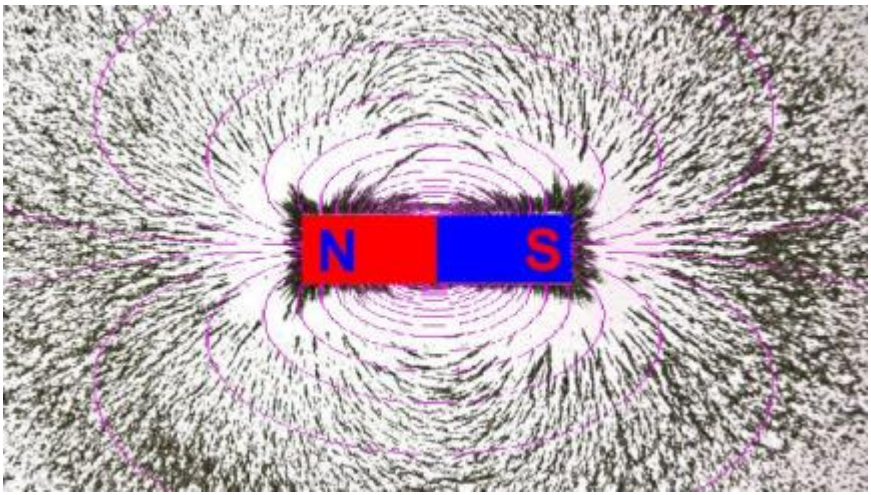

v
4
2
0
-2
-4
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
x
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
y

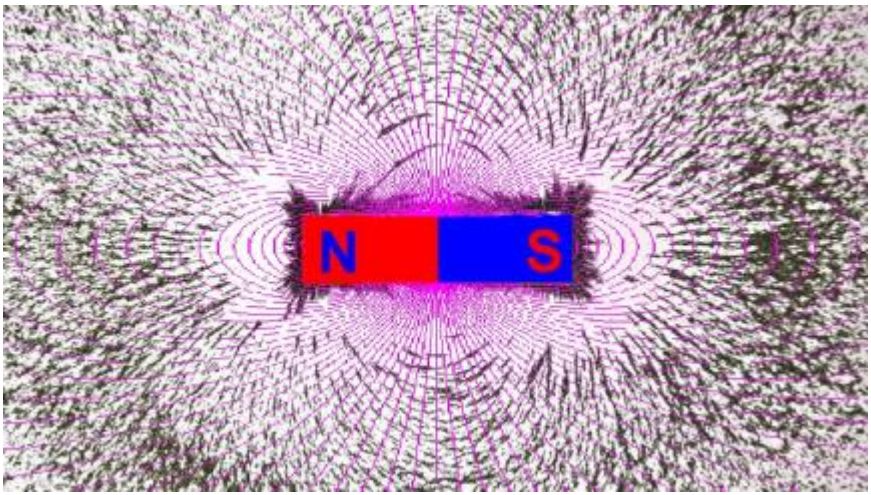

v
4
3
2
1
0
-1
-2
-3
-4
1.4 1.2 1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6 -0.8 -1 -1.2 -1.4
x
-1
-0.8
-0.6
-0.4
-0.2
0
y
0.2
0.4
0.6
0.8
1

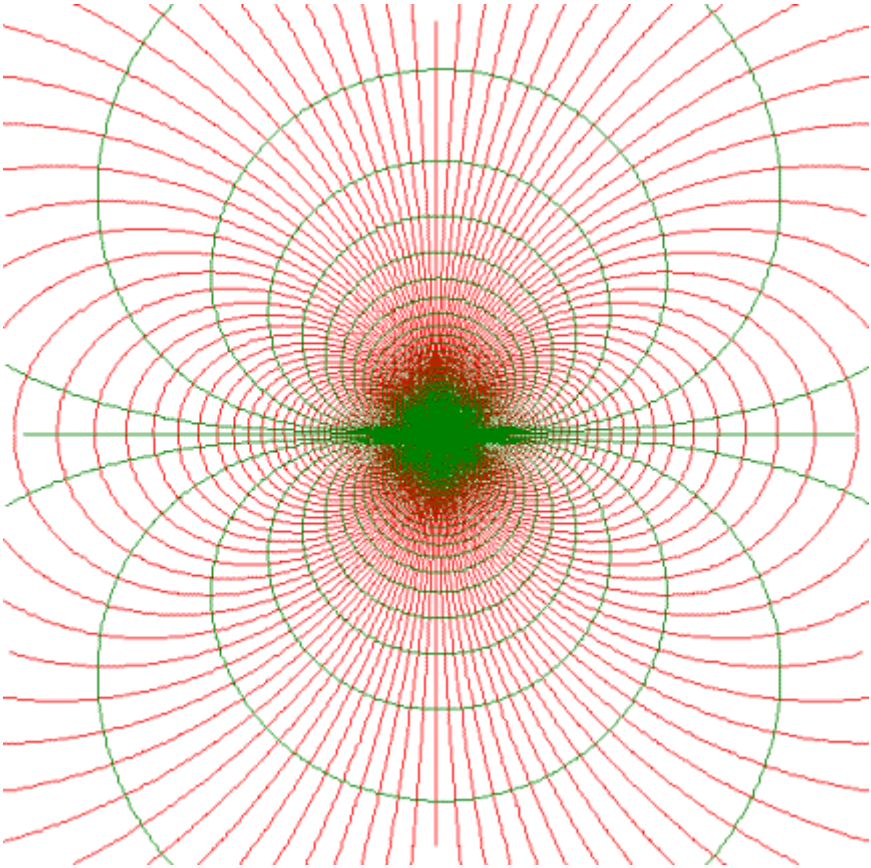


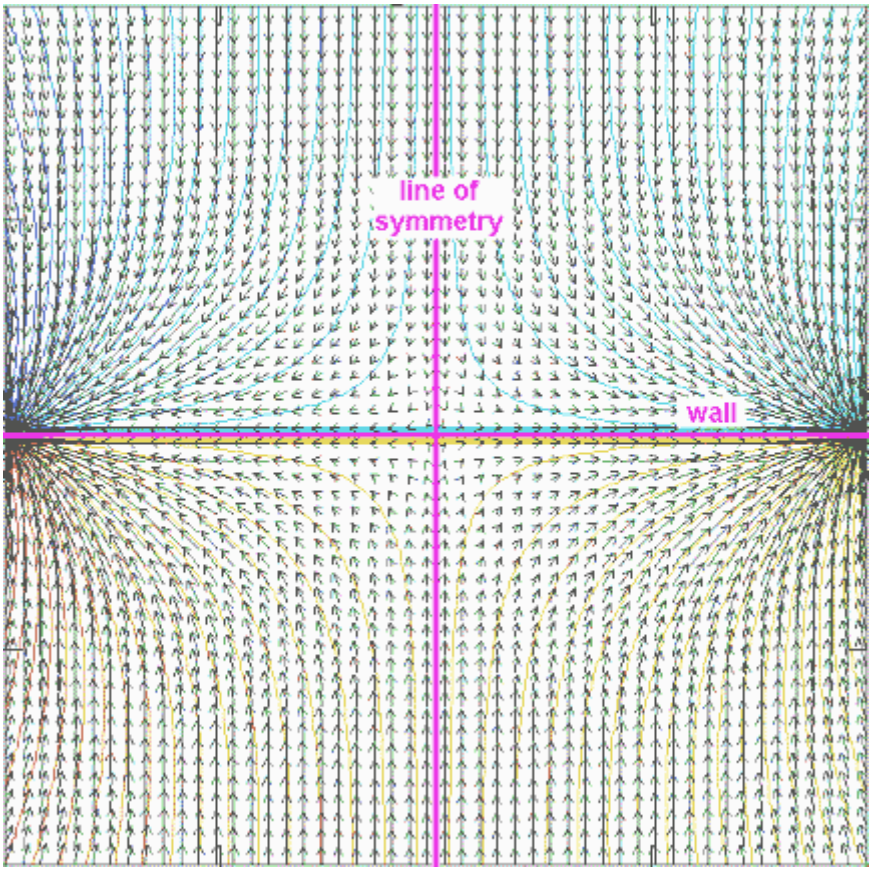

u
x
y

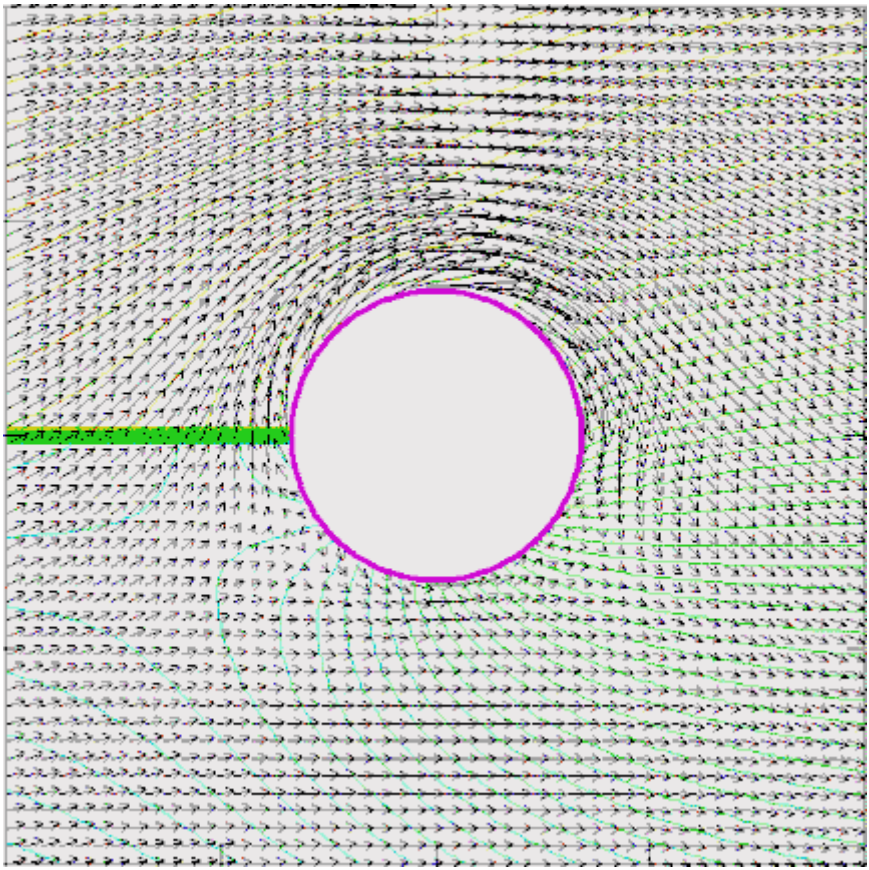

v
x
y

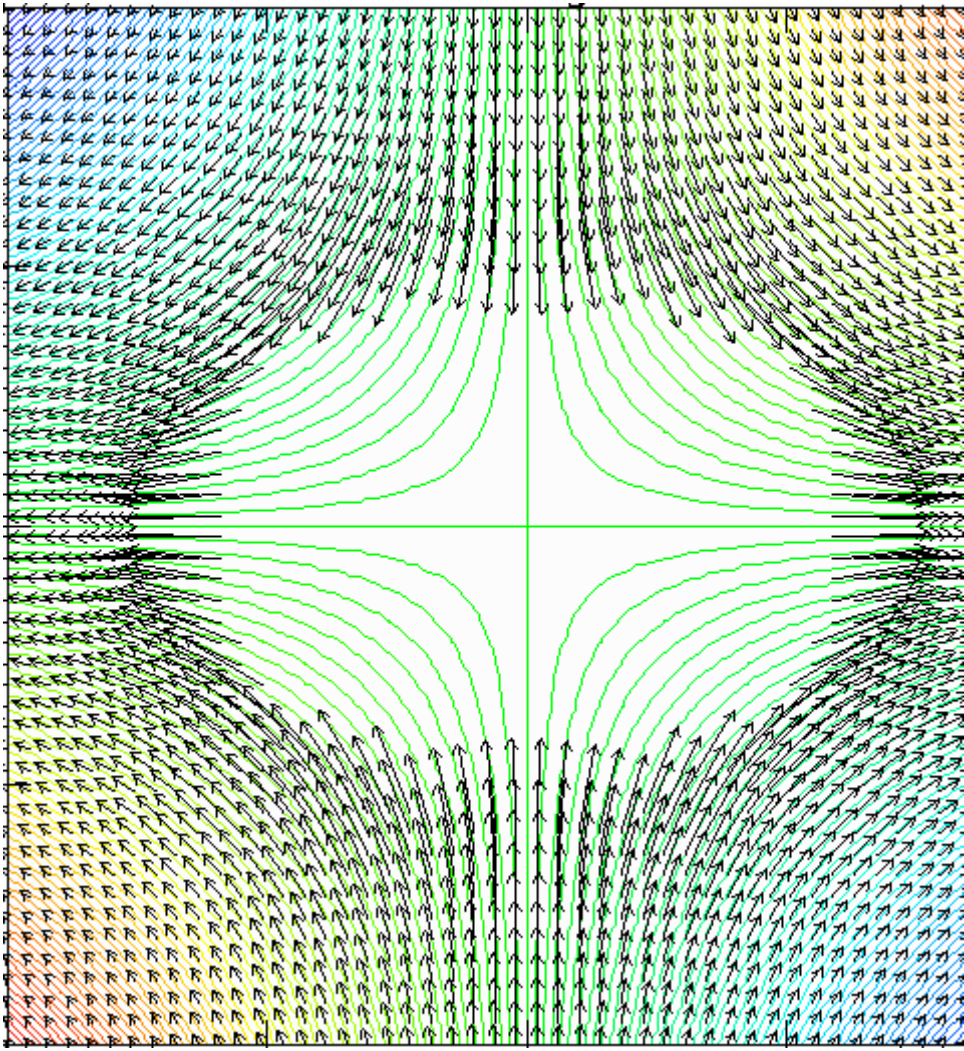

N
S

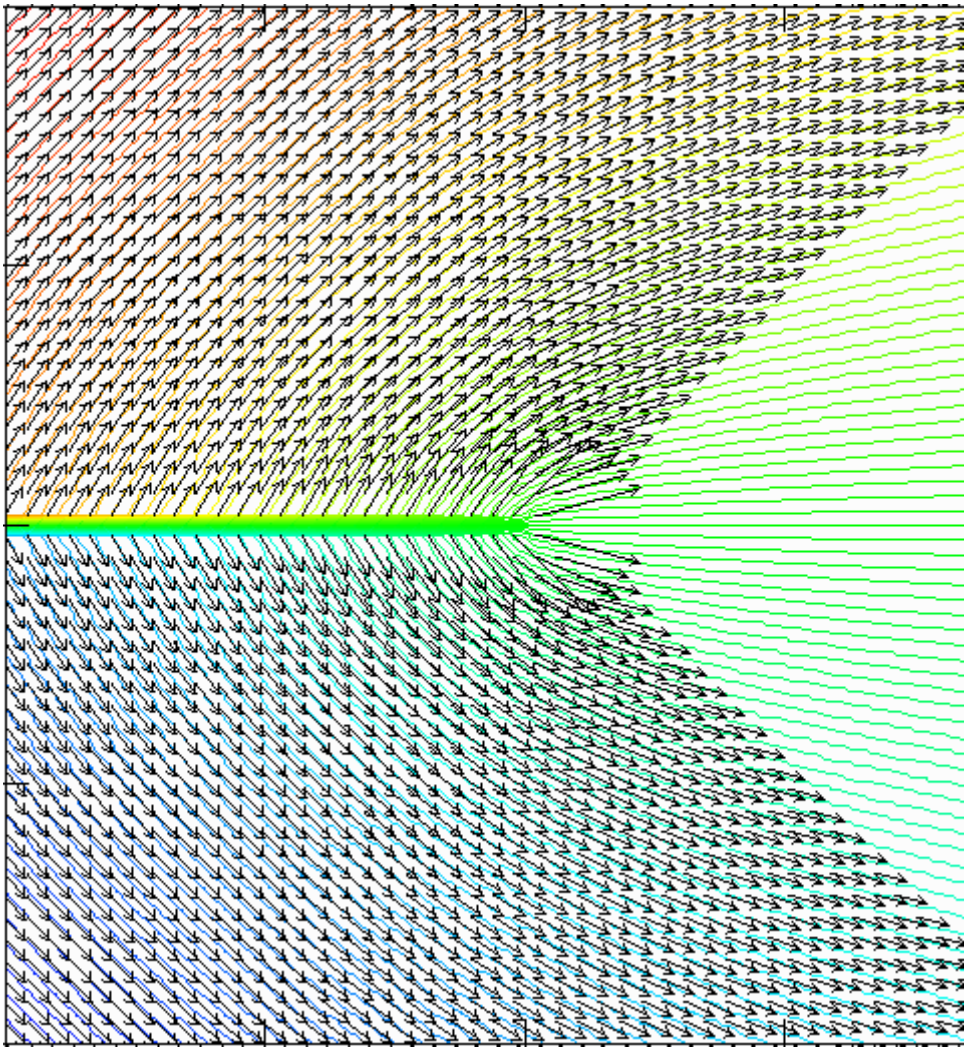

N
S

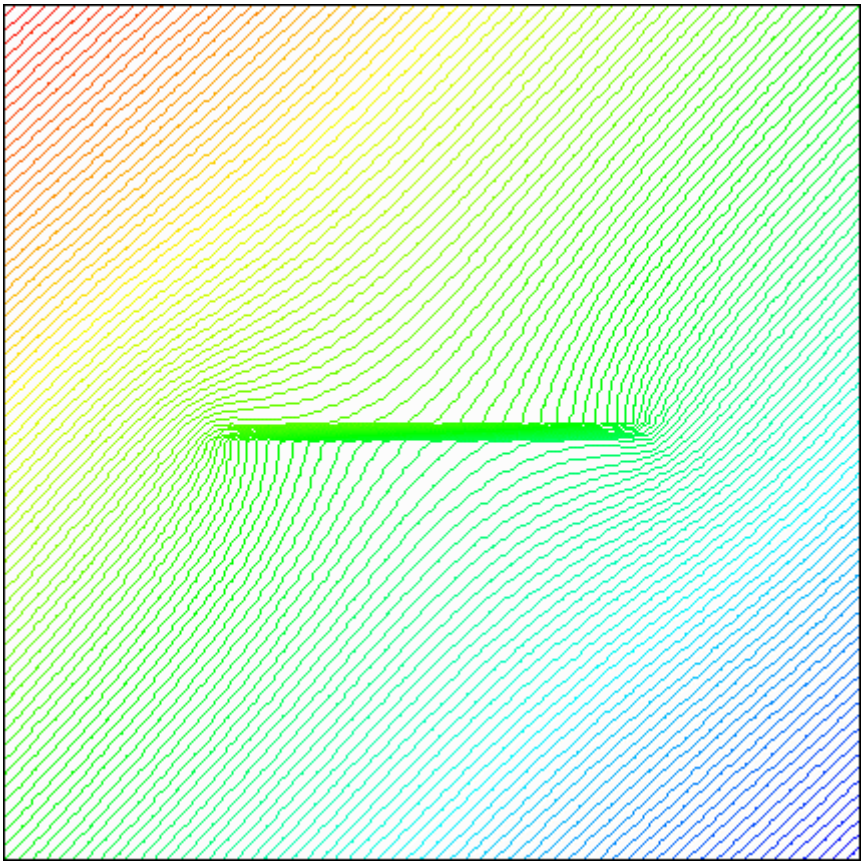


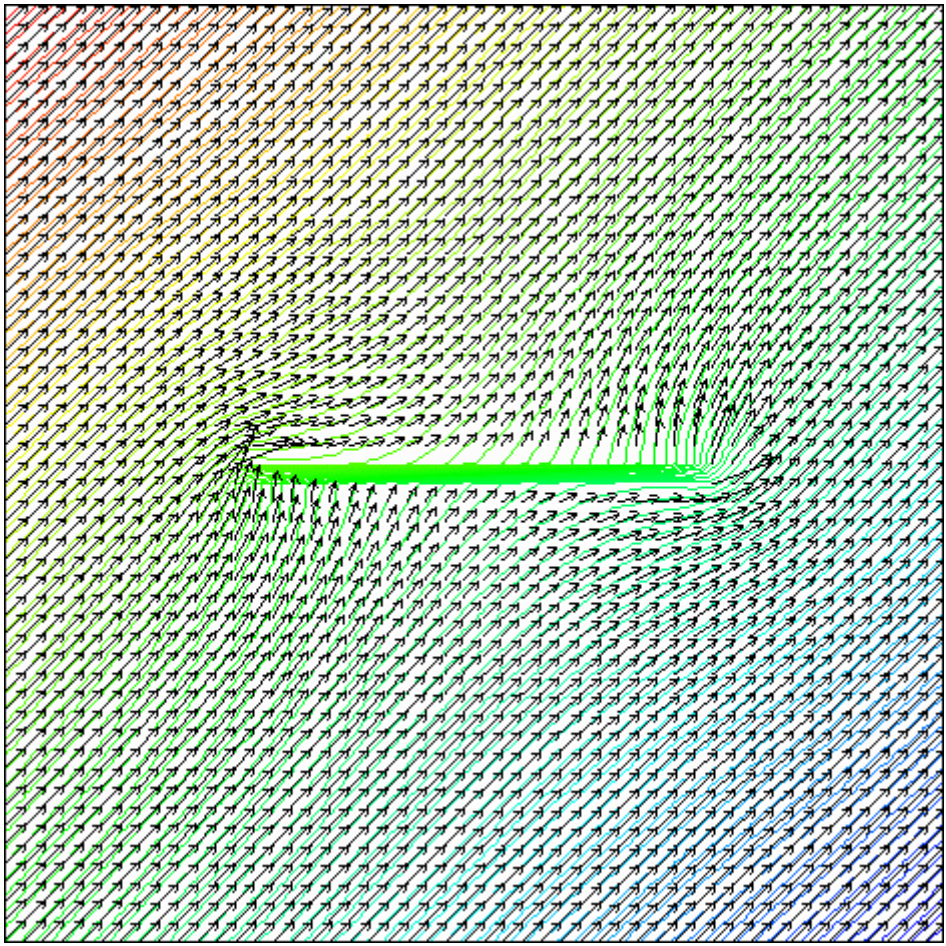

line of
symmetry
wall

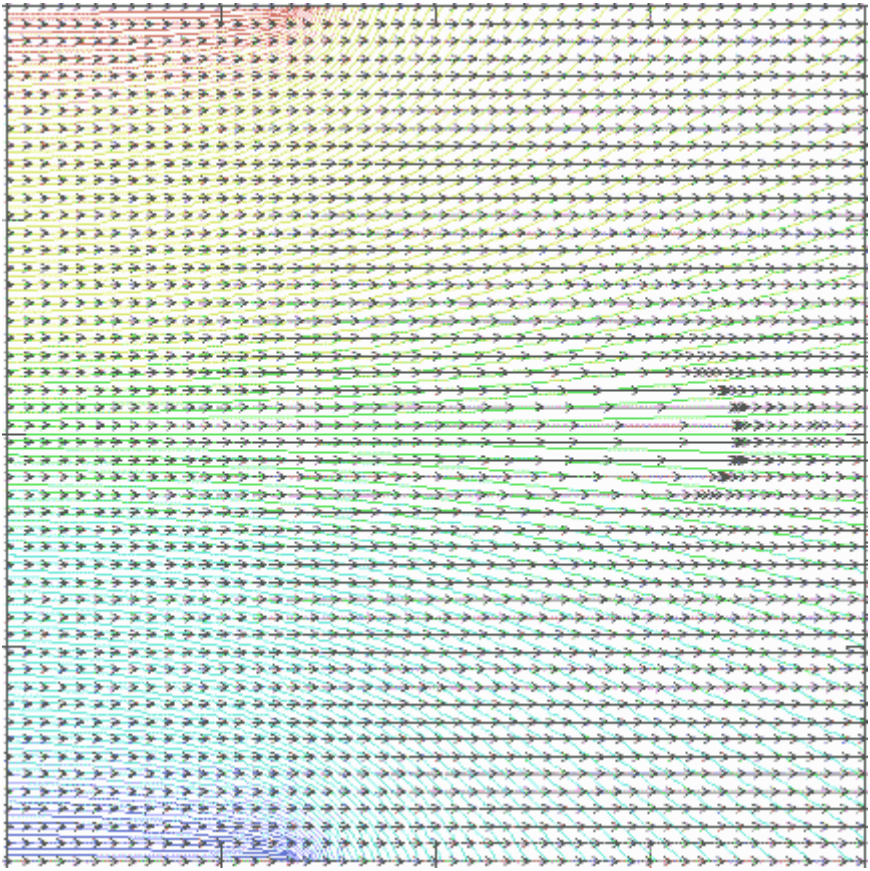


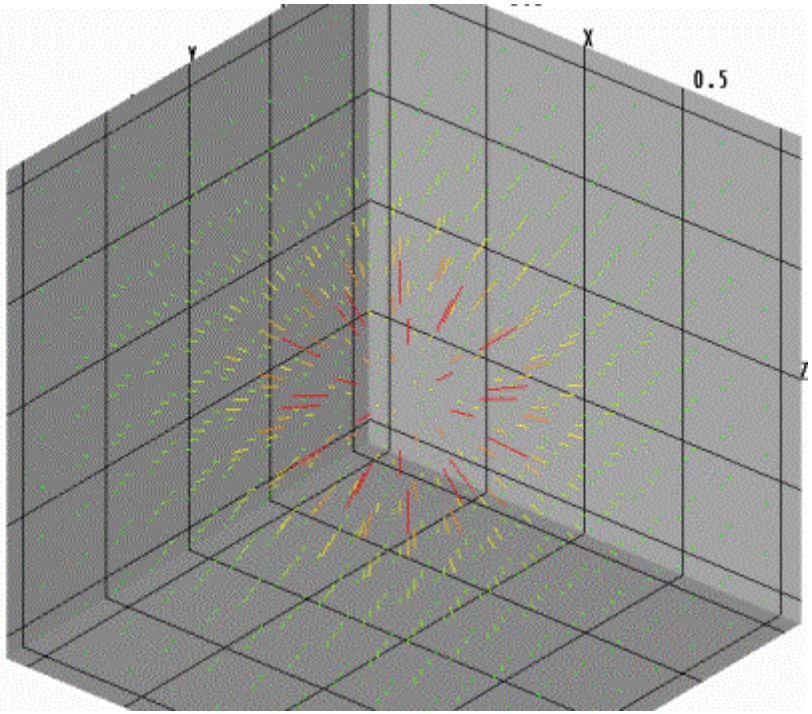


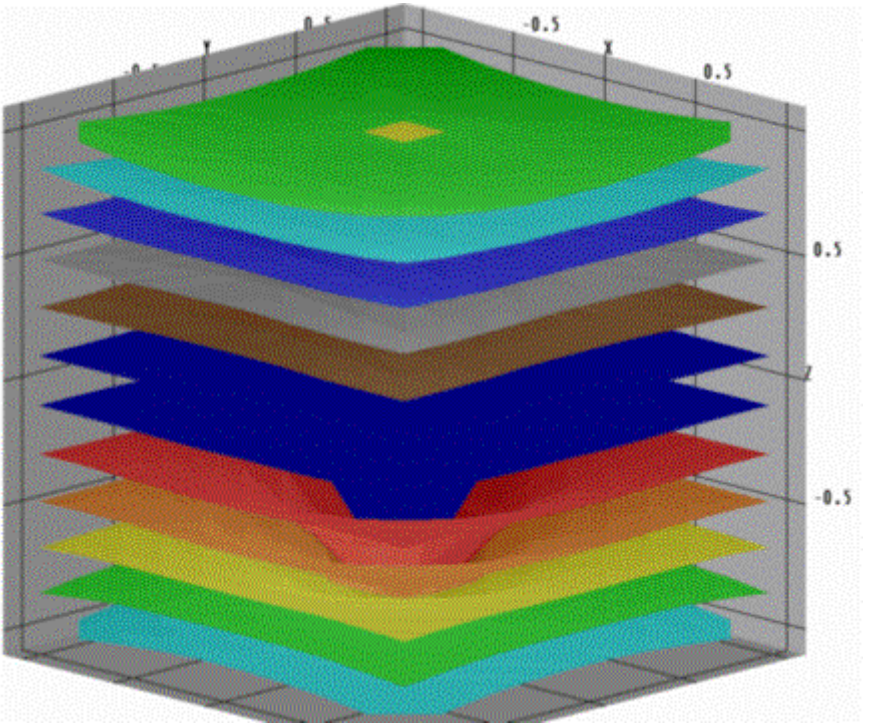


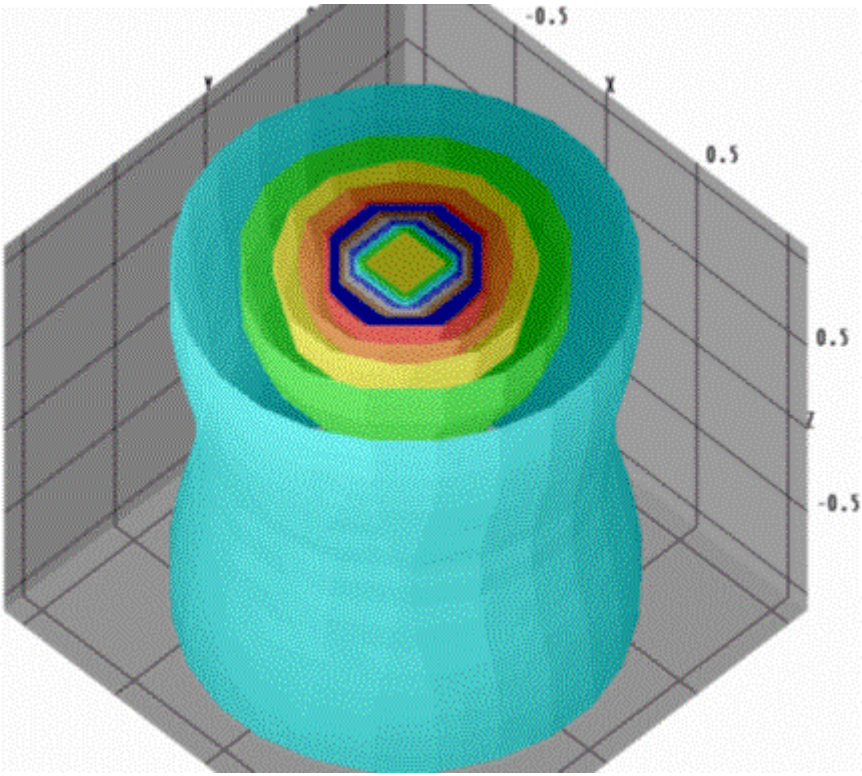


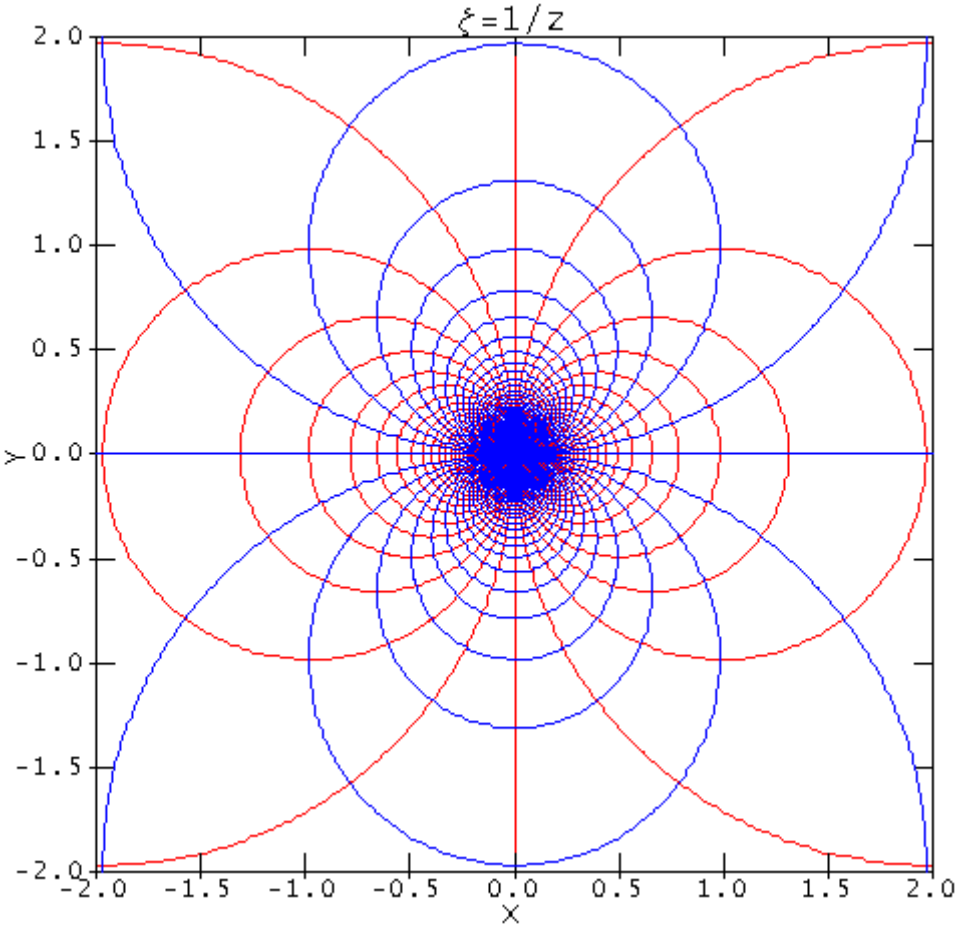


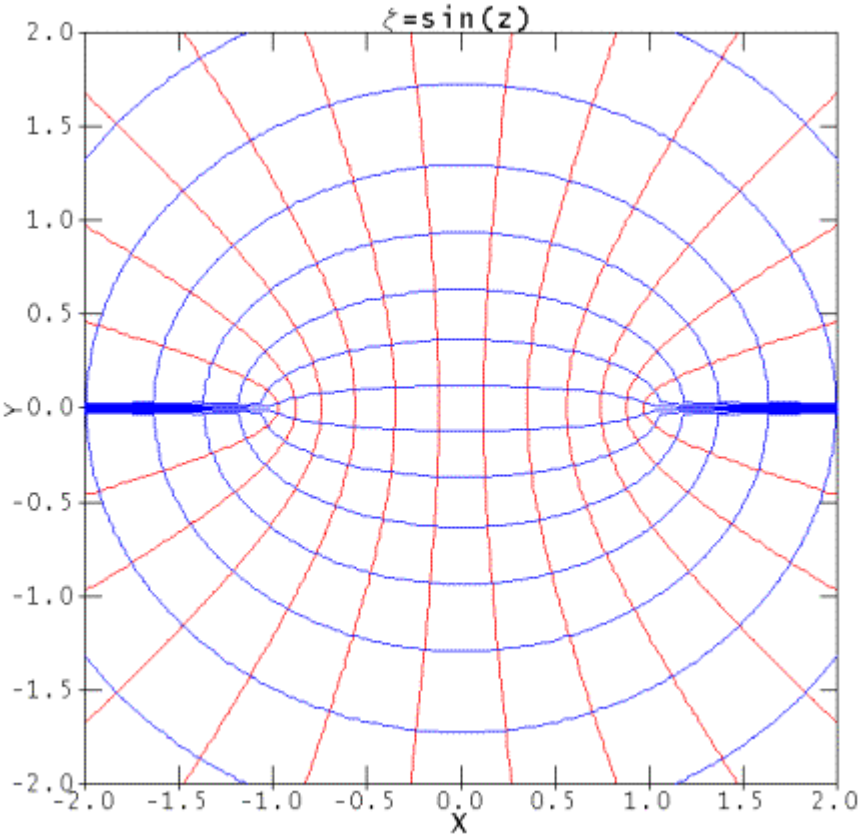


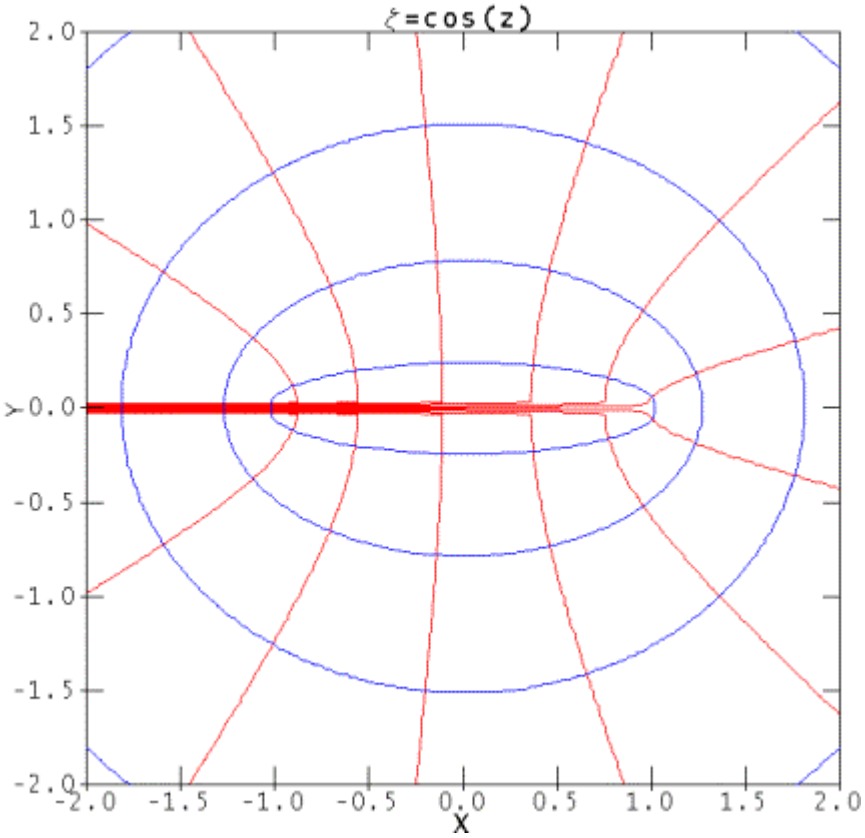

Y
X
0.5
Z

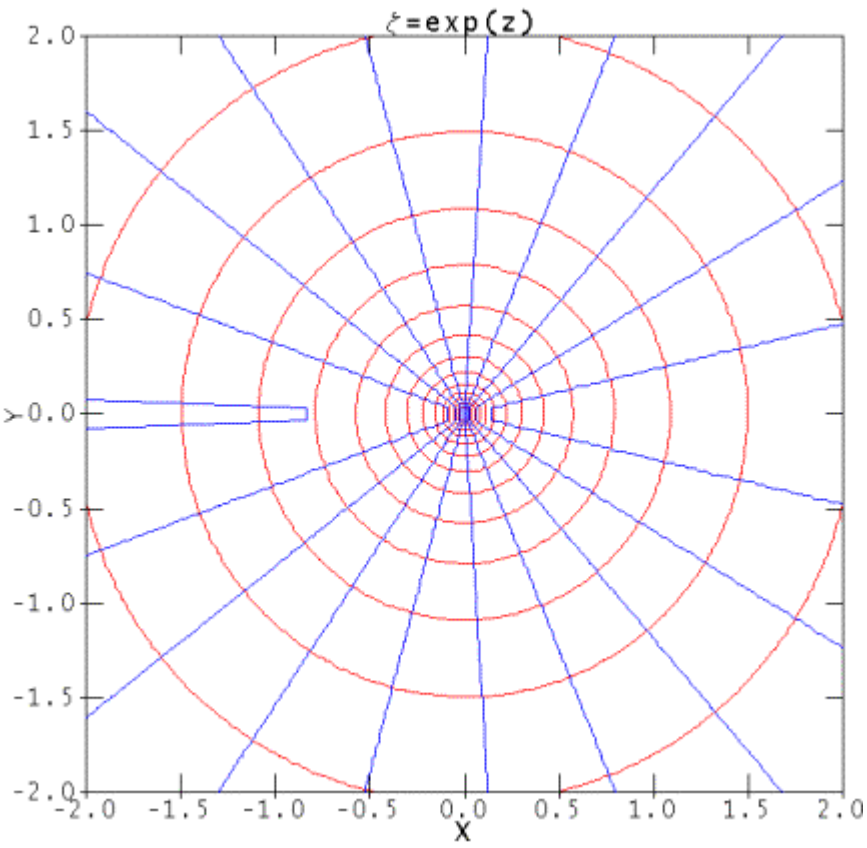

-0.5
X
0.5
0.5
Z
-0.5

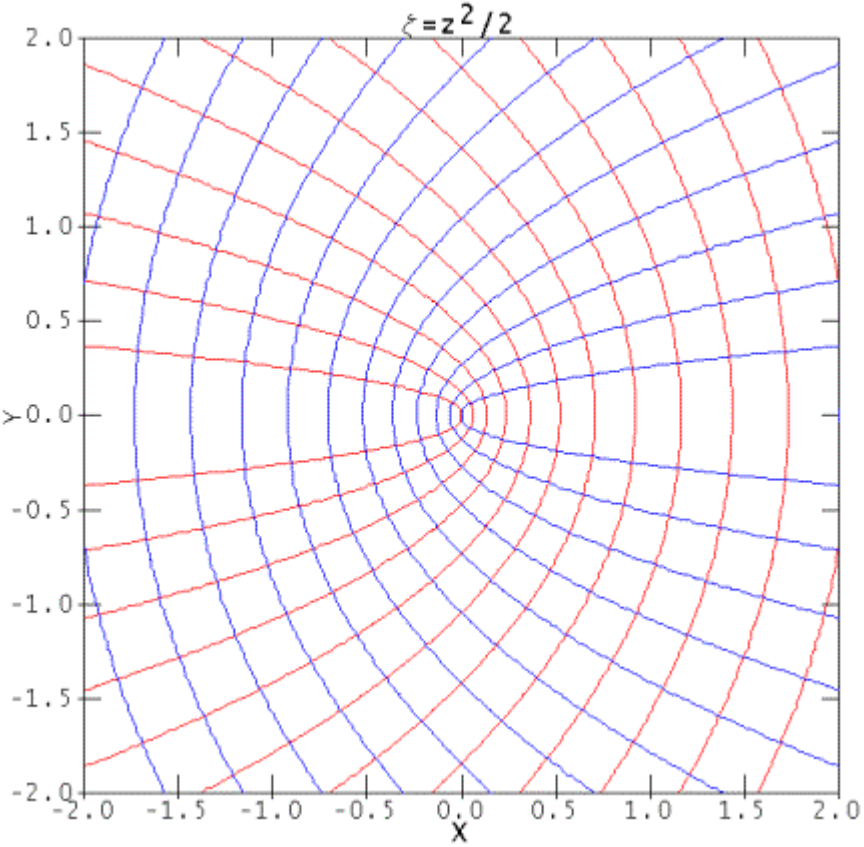

-0.5
X
0.5
0.5
Z
-0.5

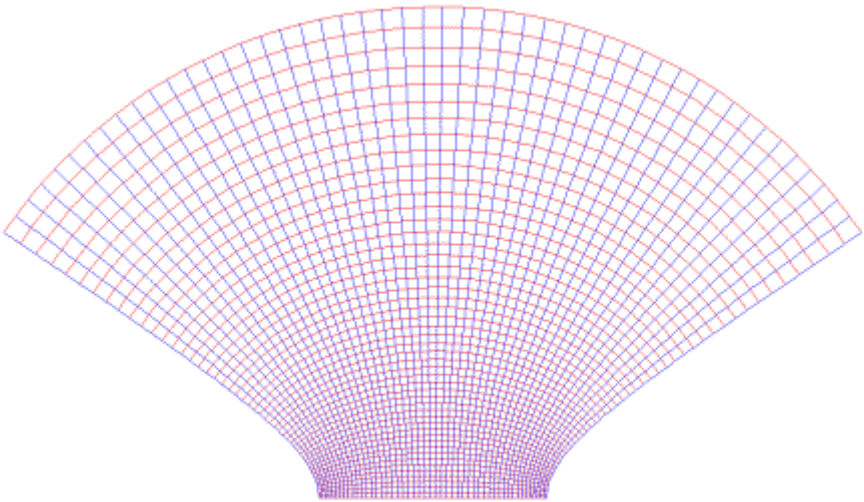

ζ=1/z
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
x

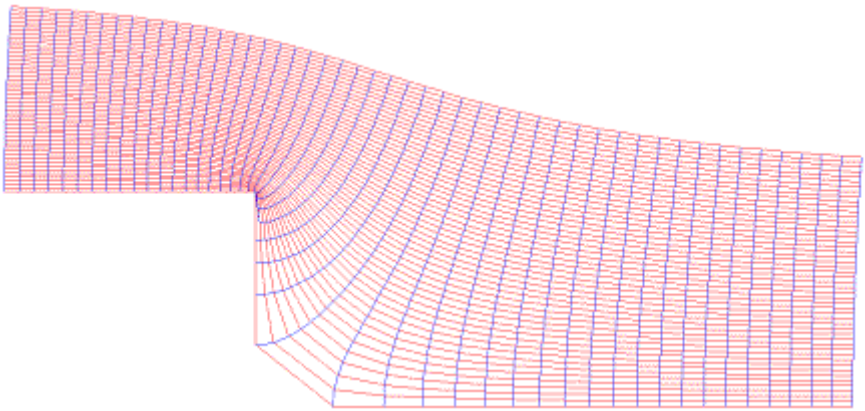

ζ=sin(z)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
X

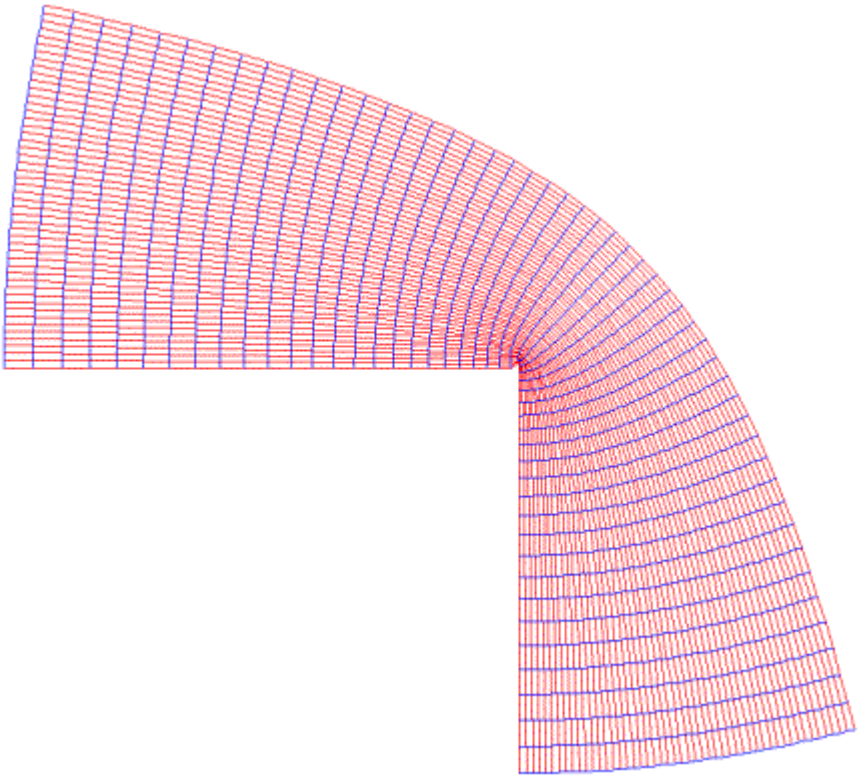

ζ=cos(z)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
x

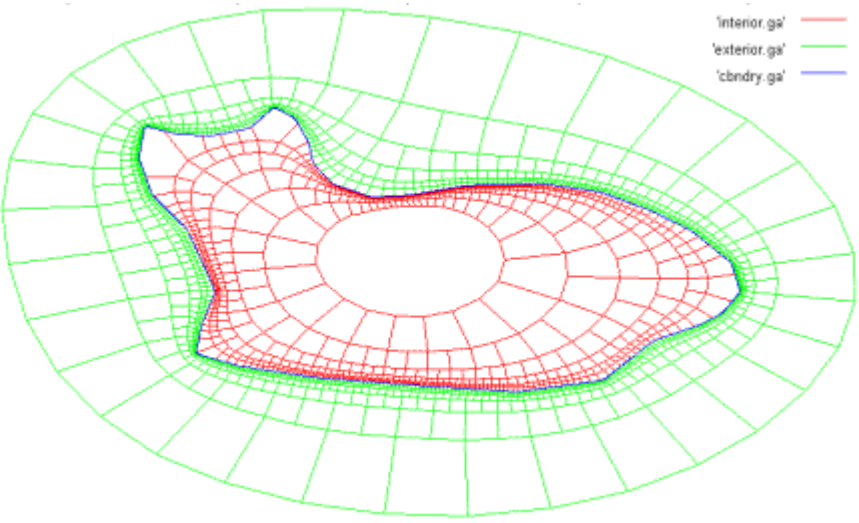

ζ=exp(z)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
X

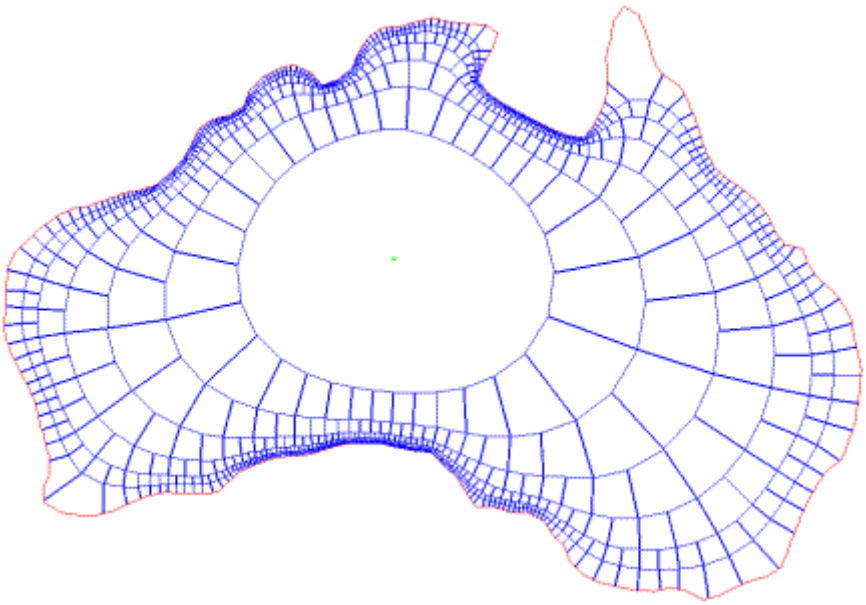

ζ=z²/2
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
X

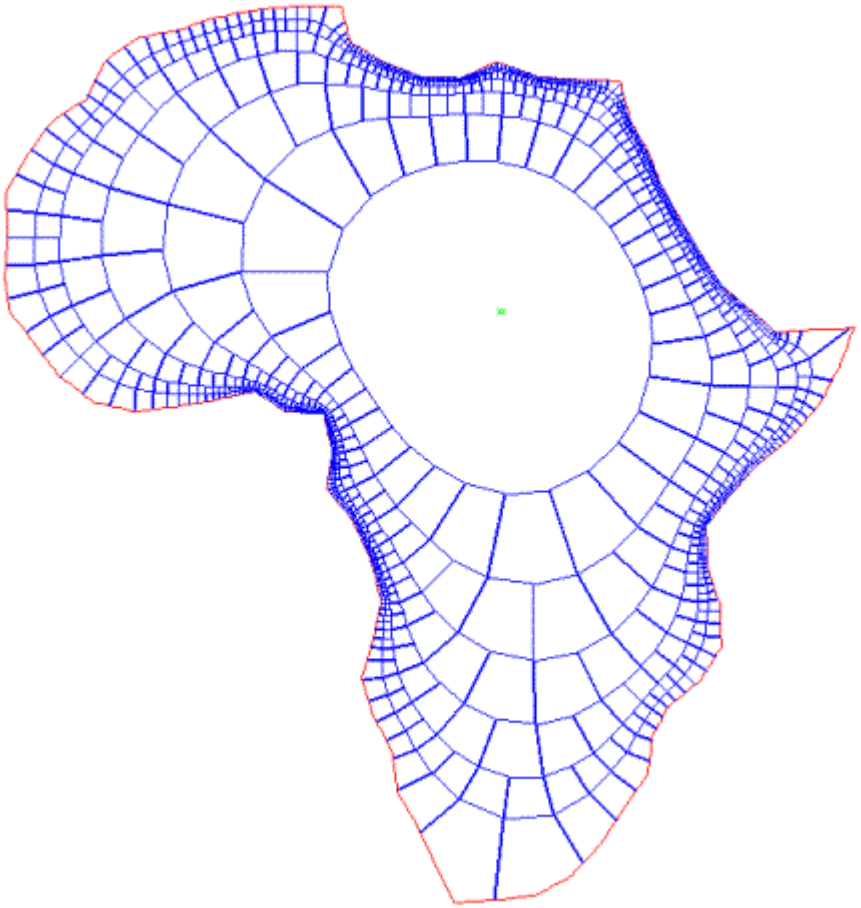


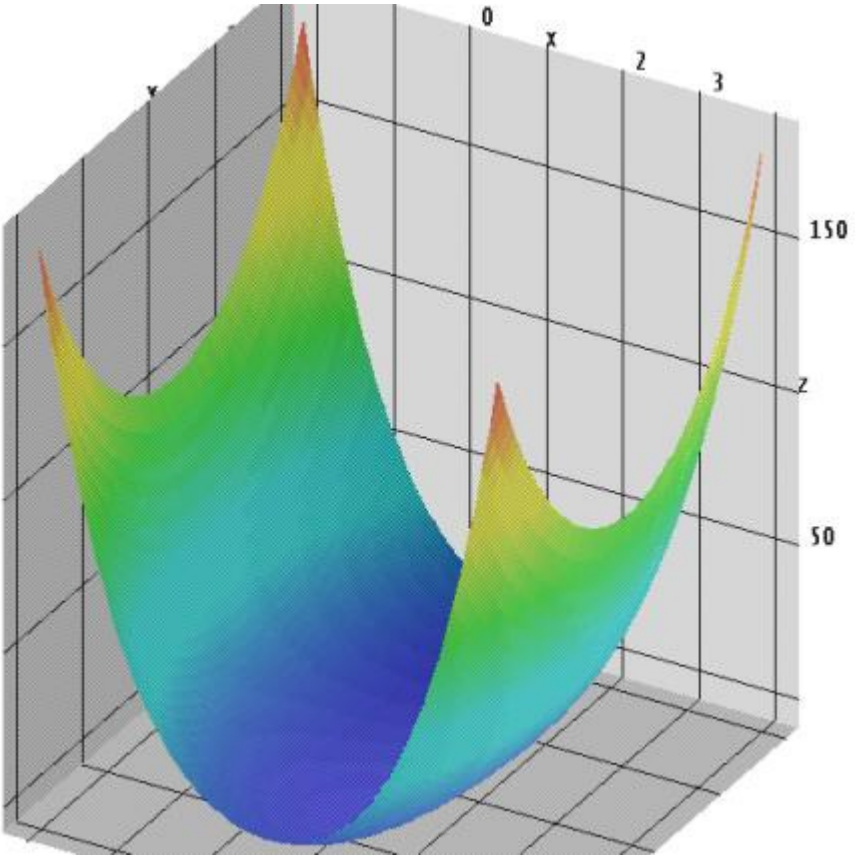


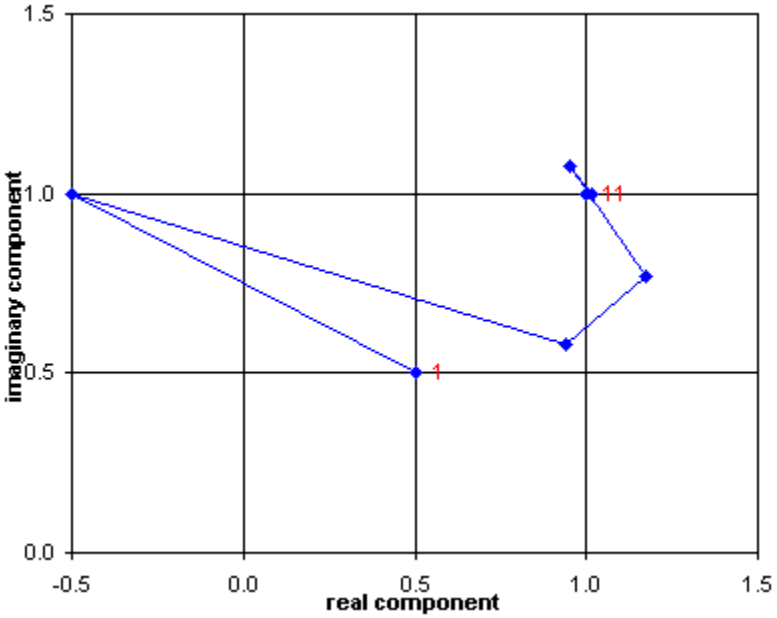


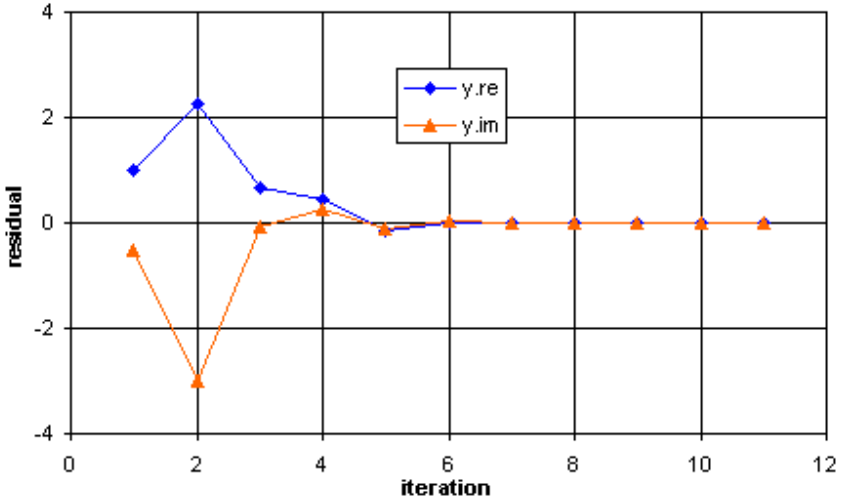

'interior.ga'
'exterior.ga'
'cbndry.ga'

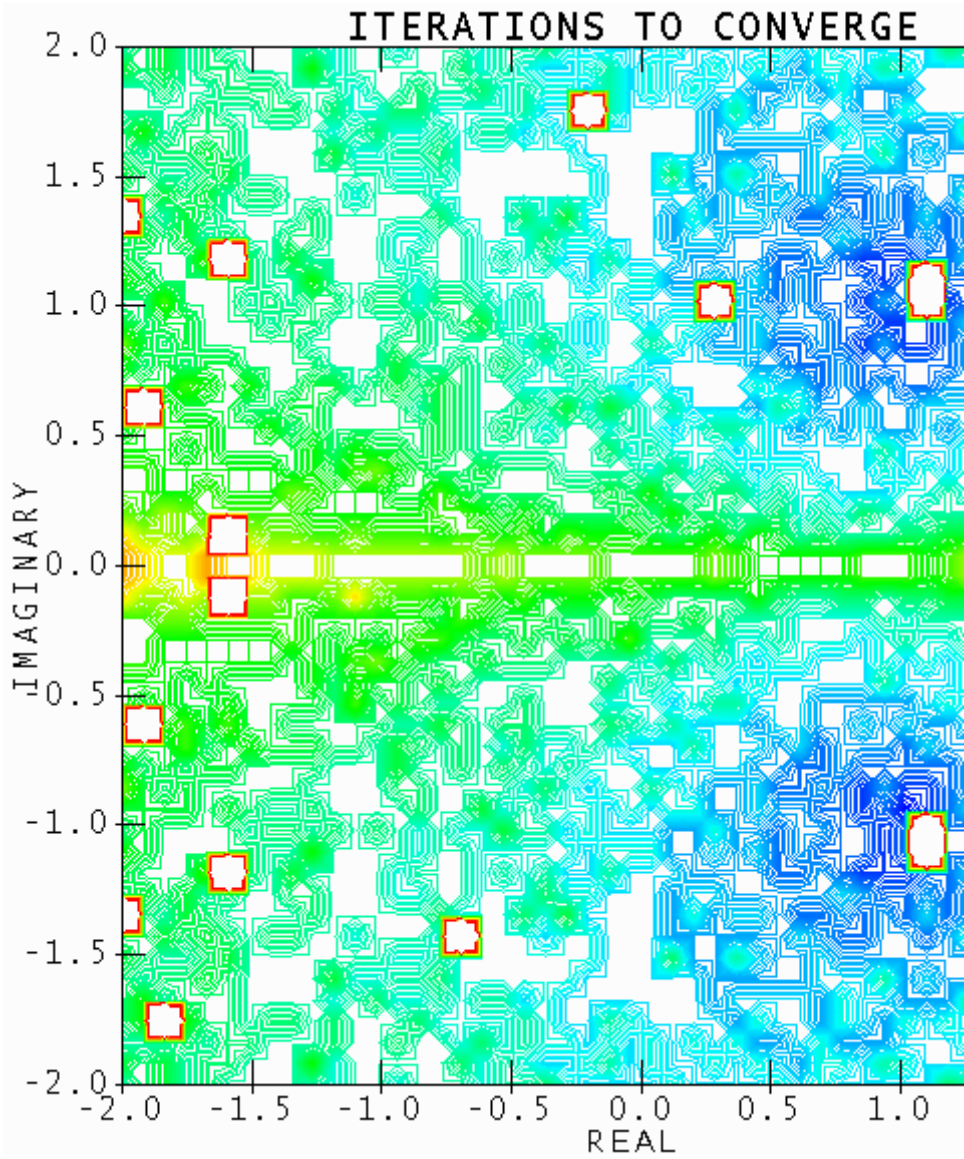


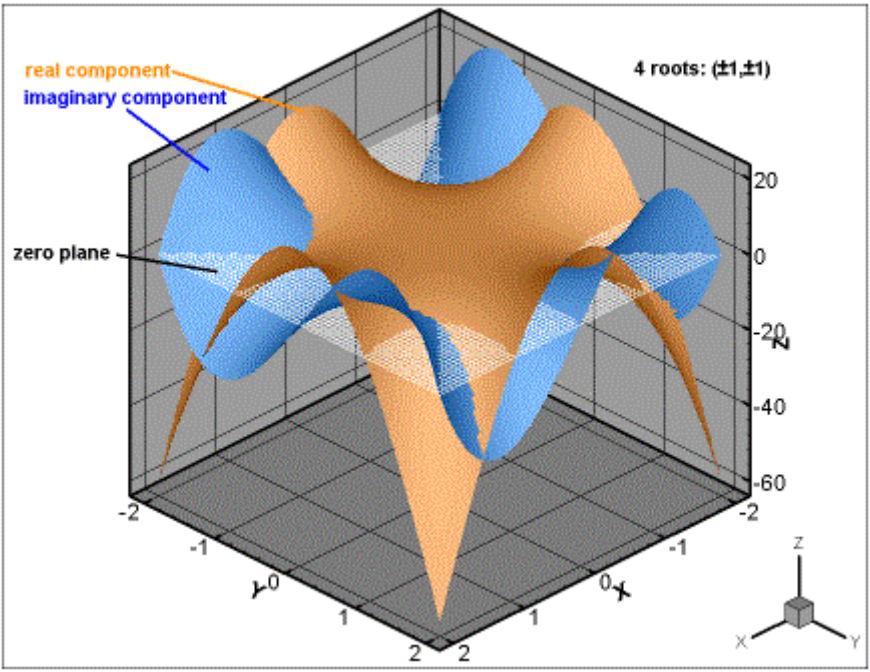


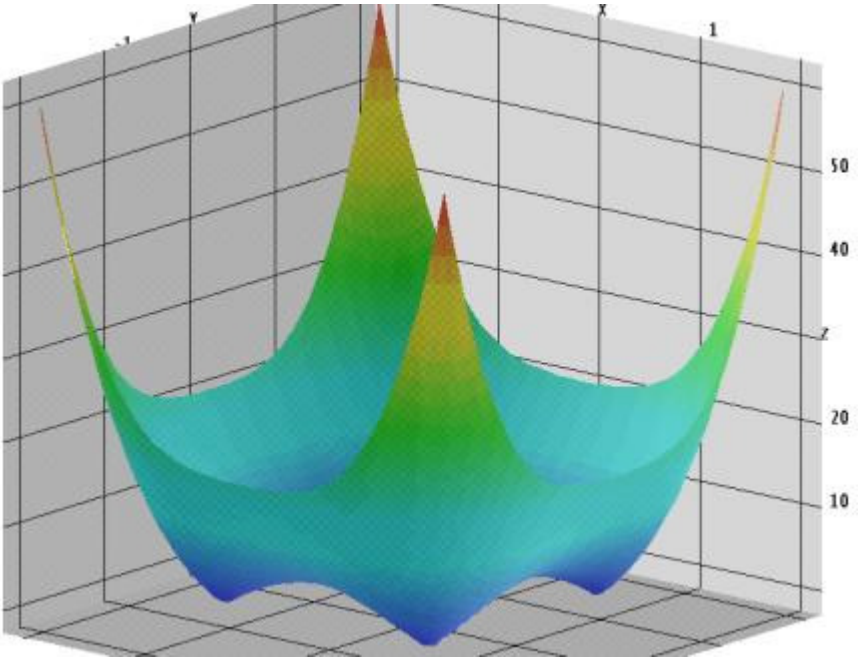

0
x
2
3
150
z
50


1.5
1.0
0.5
0.0
imaginary component
-0.5
0.0
0.5
1.0
1.5
real component
11
1

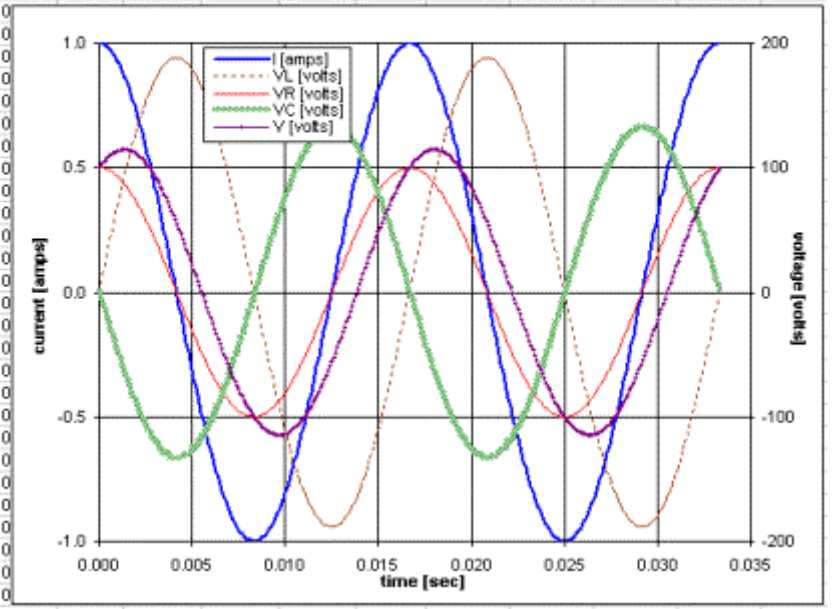

4
2
0
-2
-4
residual
0
2
4
6
8
10
12
iteration
y.re
y.im


ITERATIONS TO CONVERGE
2.0
1.5
1.0
0.5
0.0
0.5
-1.0
-1.5
-2.0
IMAGINARY
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
REAL

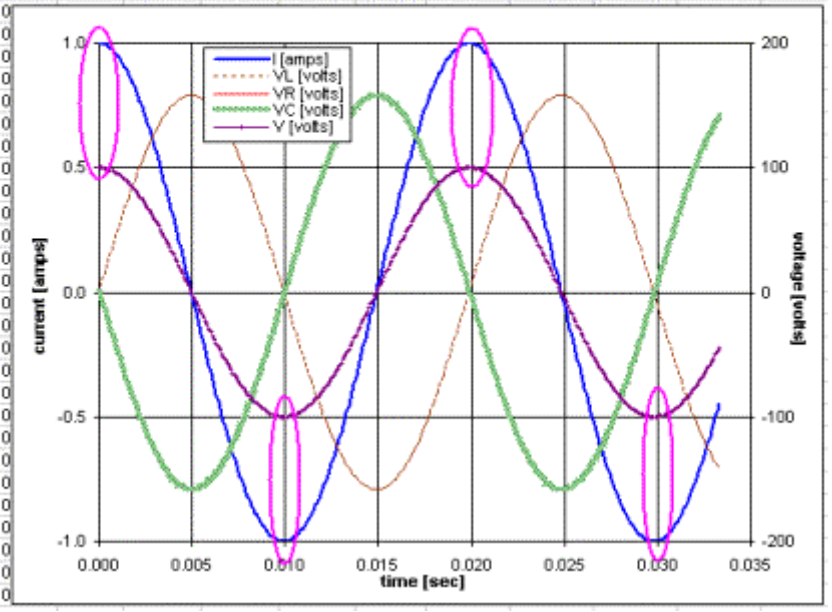

real component
imaginary component
zero plane
4 roots: (±1,±1)
20
0
-20
-40
-60
N
-2
-1
0
1
2
Y
X
Z

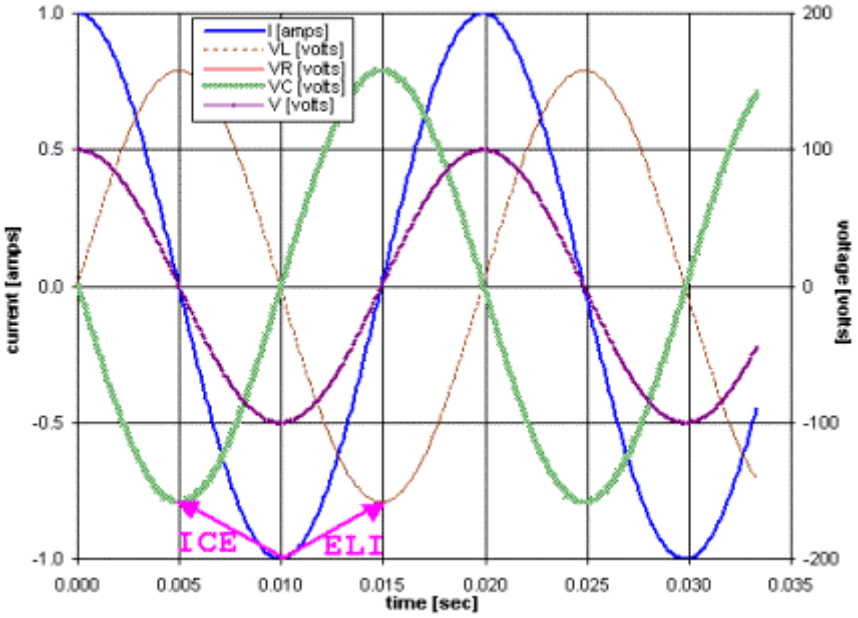

Y
X
1
50
40
Z
20
10

	A	B	C	D	E	F	G	H	I	J	K
1	series R-L-C circuit						377	ω [rad/sec]			
2	t [sec]	I [amps]	V_R [volts]	V_C [volts]	V_L [volts]	V [volts]	100	R [ohms]			
3	0.0000	1.000	100.0	0.0	0.0	100.0	0.00002	C [farad]			
4	0.0001	0.999	99.9	-5.0	7.1	102.0	0.5	L [henry]			
5	0.0002	0.997	99.7	-10.0	14.2	103.9	0.559	(ωL-1/ωC)/R			

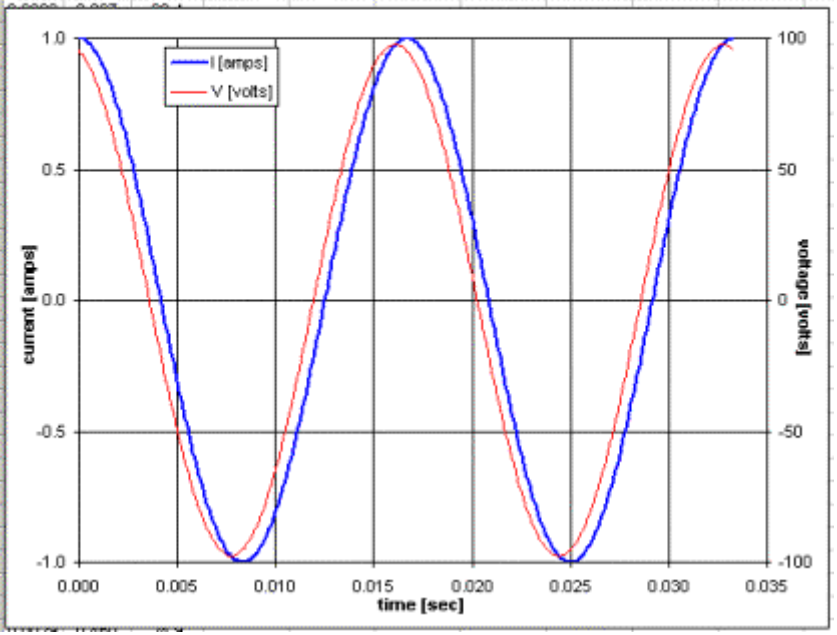


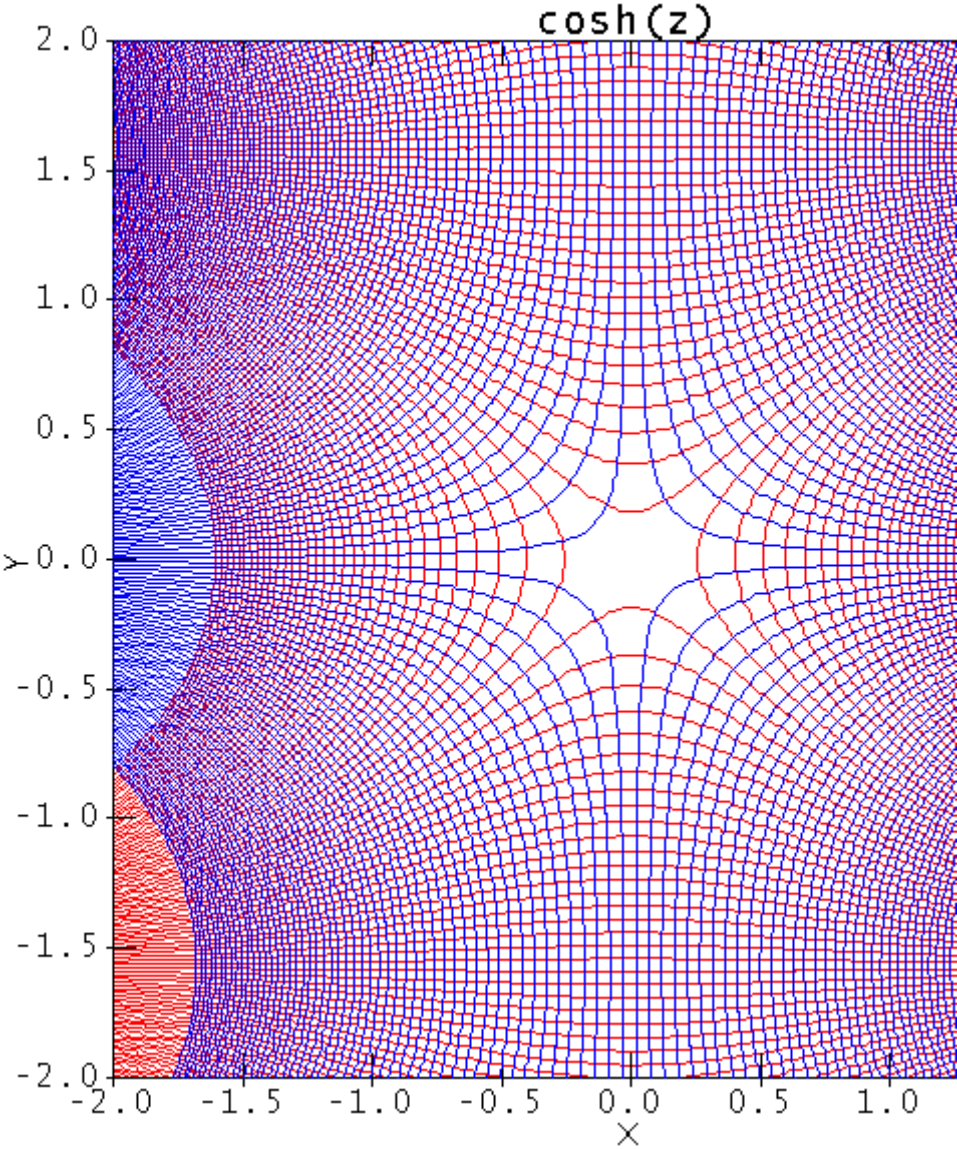
	A	B	C	D	E	F	G	H	I	J	K
1	series R-L-C circuit						316.228	ω [rad/sec]			
2	t [sec]	I [amps]	V_a [volts]	V_c [volts]	V_L [volts]	V [volts]	100	R [ohms]			
3	0.0000	1.000	100.0	0.0	0.0	100.0	0.00002	C [farad]			
4	0.0001	1.000	100.0	-5.0	5.0	100.0	0.5	L [henry]			
5	0.0002	0.998	99.8	-10.0	10.0	99.8	0.000	(ωL-1/ωC)/R			

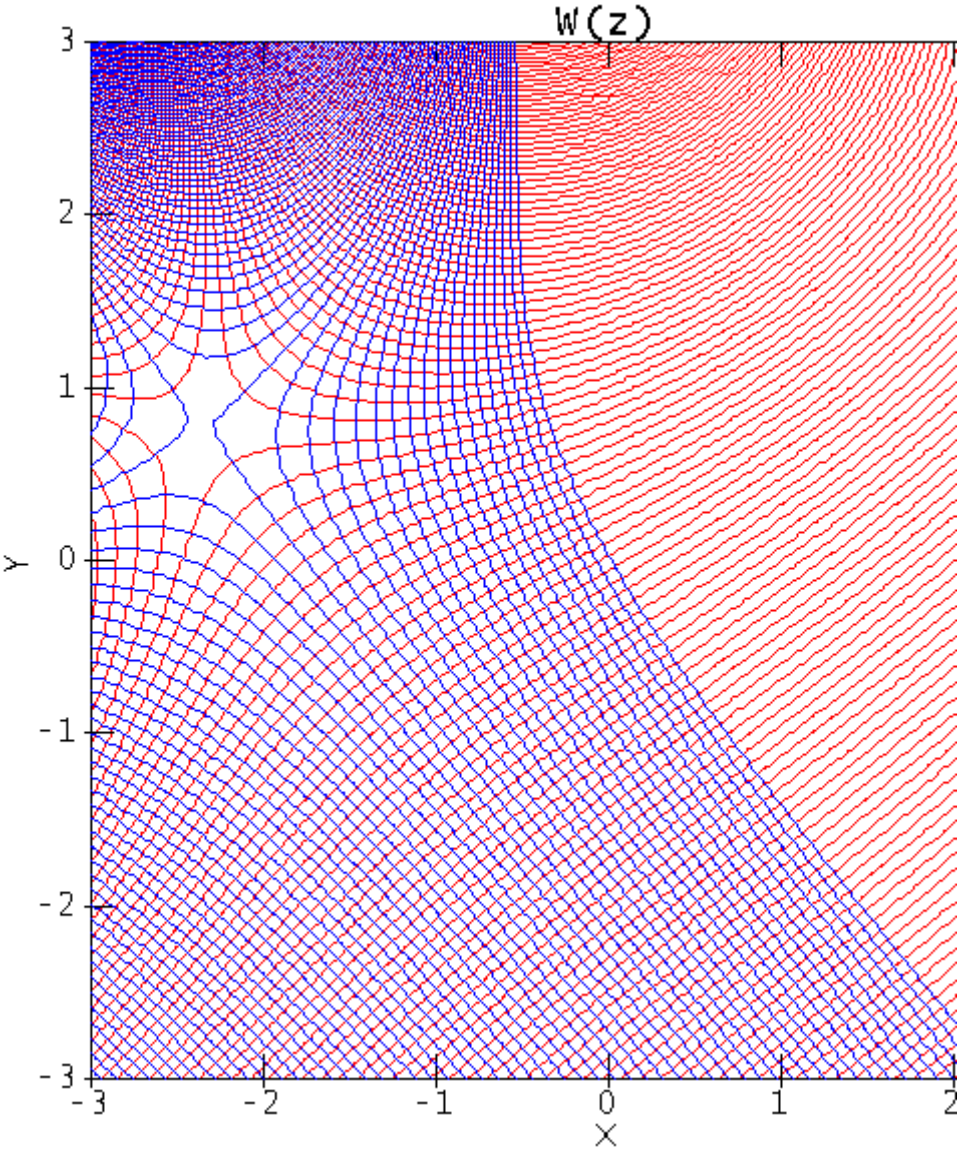


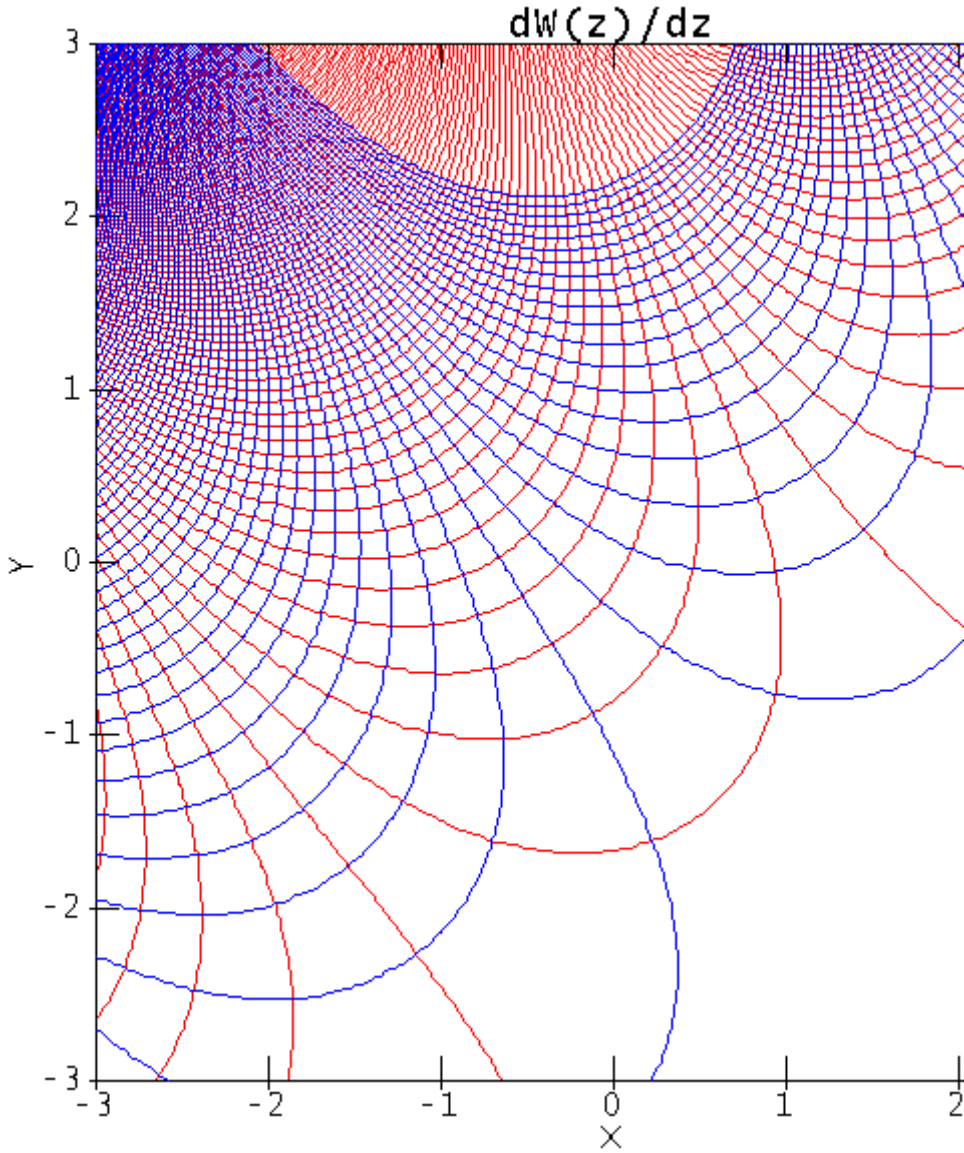

I [amps]
VL [volts]
VR [volts]
VC [volts]
V [volts]
current [amps]
voltage [volts]
time [sec]
1.0
0.5
0.0
-0.5
-1.0
200
100
0
-100
-200
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
ICE
ELI

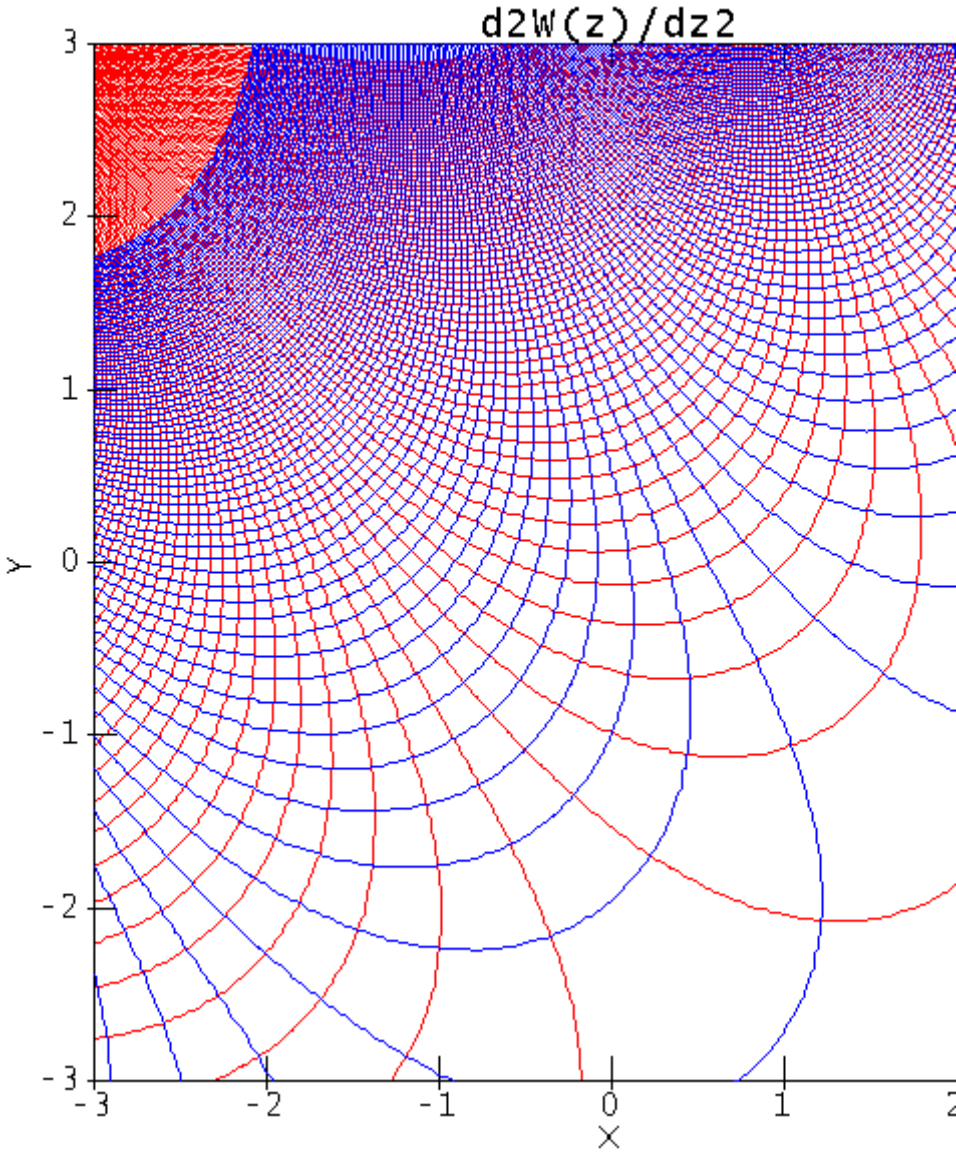
	A	B	C	D	E	F	G	H	I	J
1	parallel R-L-C circuit			377	ω [rad/sec]					
2	t [sec]	I [amps]	V [volts]	100	R [ohms]					
3	0.0000	1.000	95.2	0.00002	C [farad]					
4	0.0001	0.999	94.4	0.5	L [henry]					

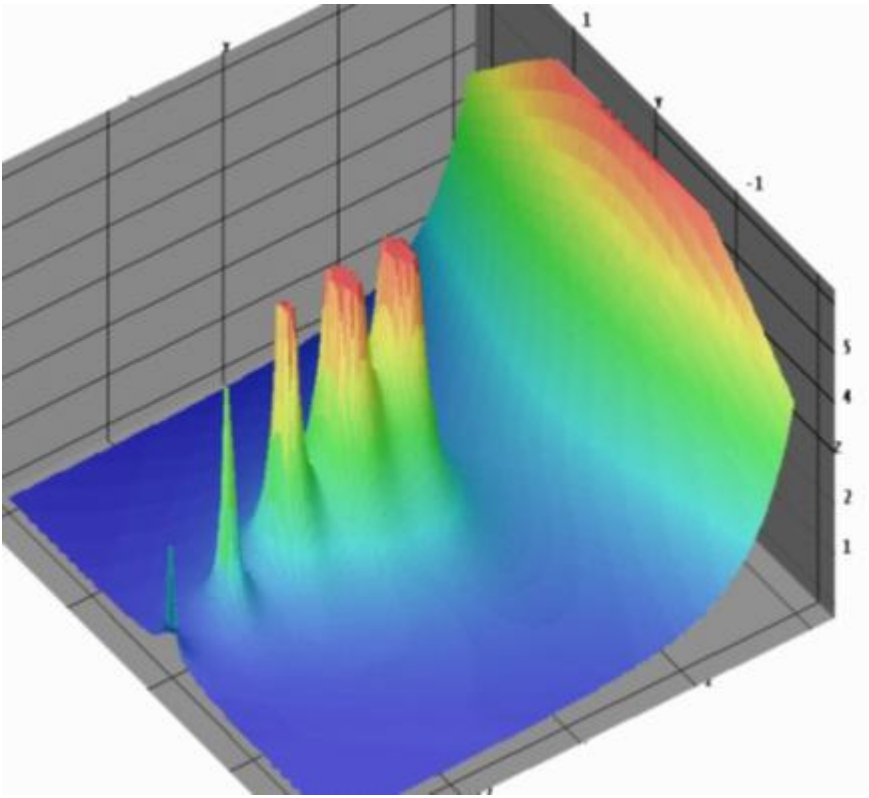


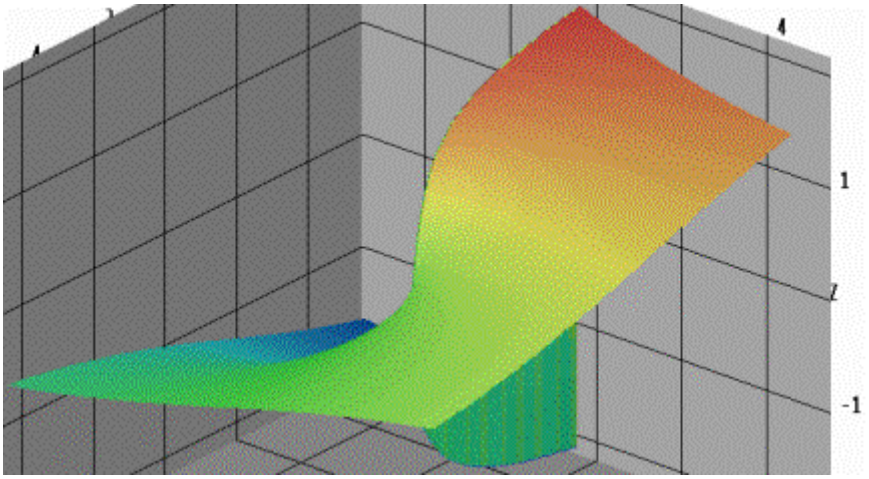

cosh(z)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
X

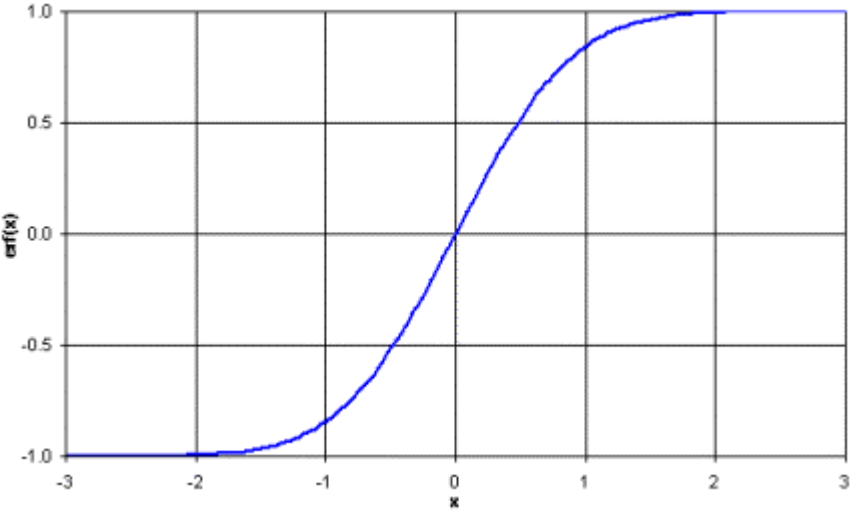

W(z)
3
2
1
0
-1
-2
-3
Y
-3
-2
-1
0
1
2
X

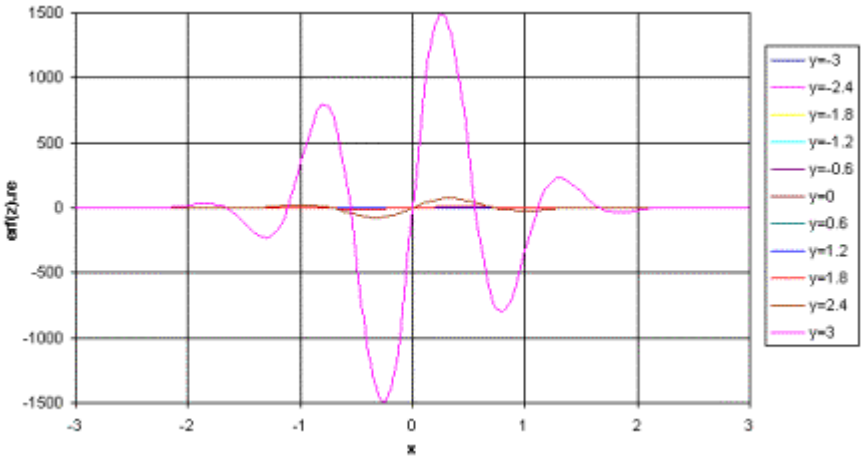

dW(z)/dz
3
2
1
0
-1
-2
-3
Y
-3
-2
-1
0
1
2
X

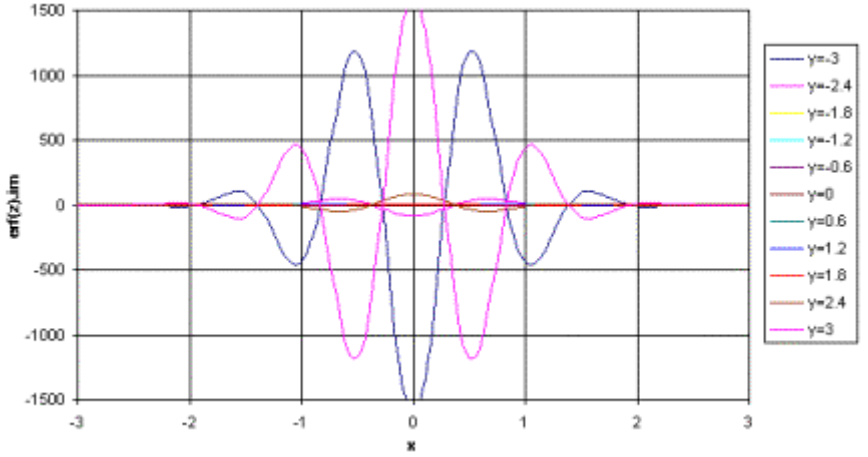

d2W(z)/dz2
3
2
1
0
-1
-2
-3
Y
-3
-2
-1
0
1
X

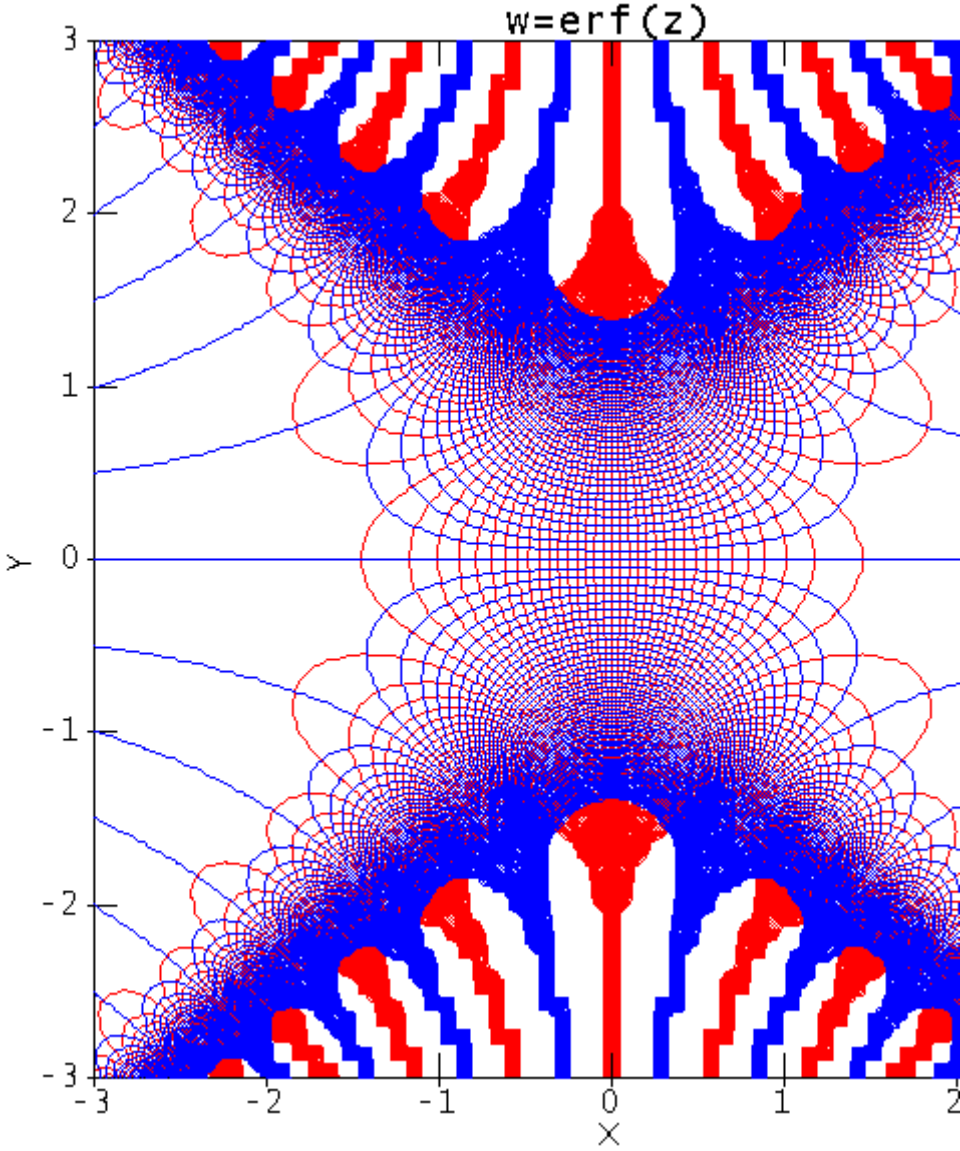

1
-1
5
4
2
1

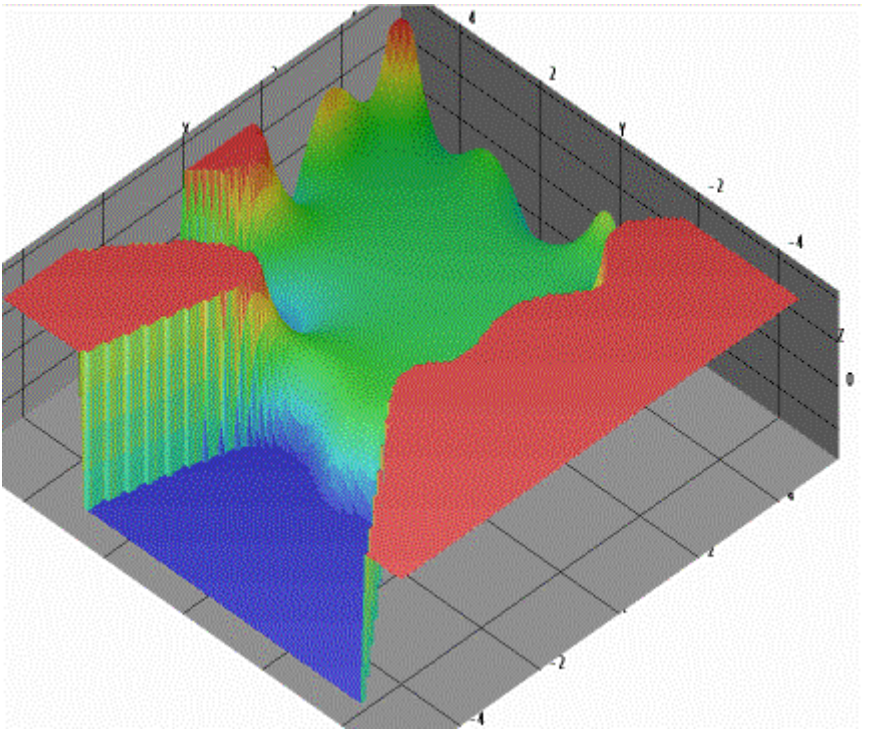

1
-1

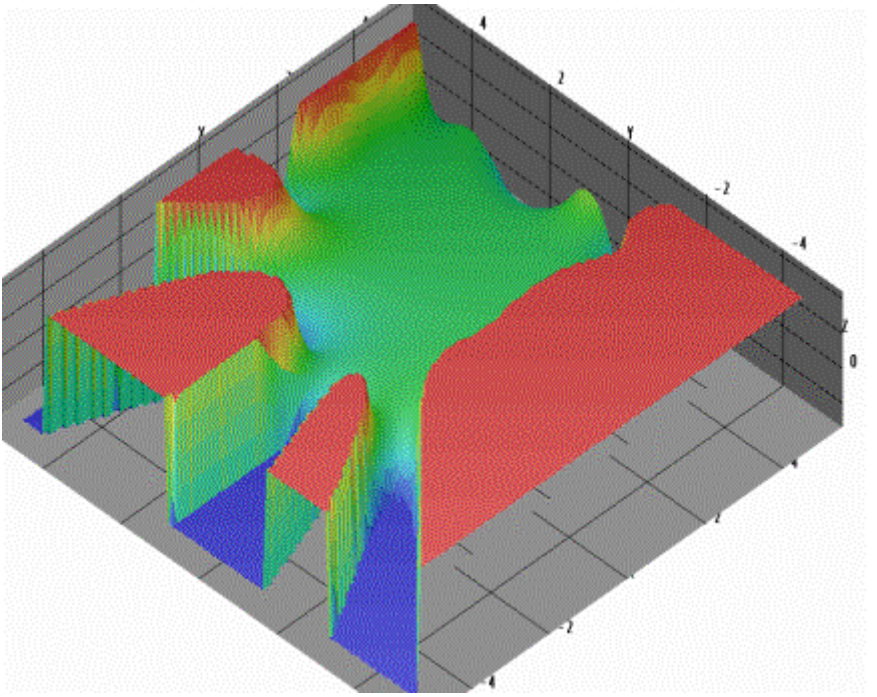

1.0
0.5
0.0
-0.5
-1.0
erf(x)
-3
-2
-1
0
1
2
3
x

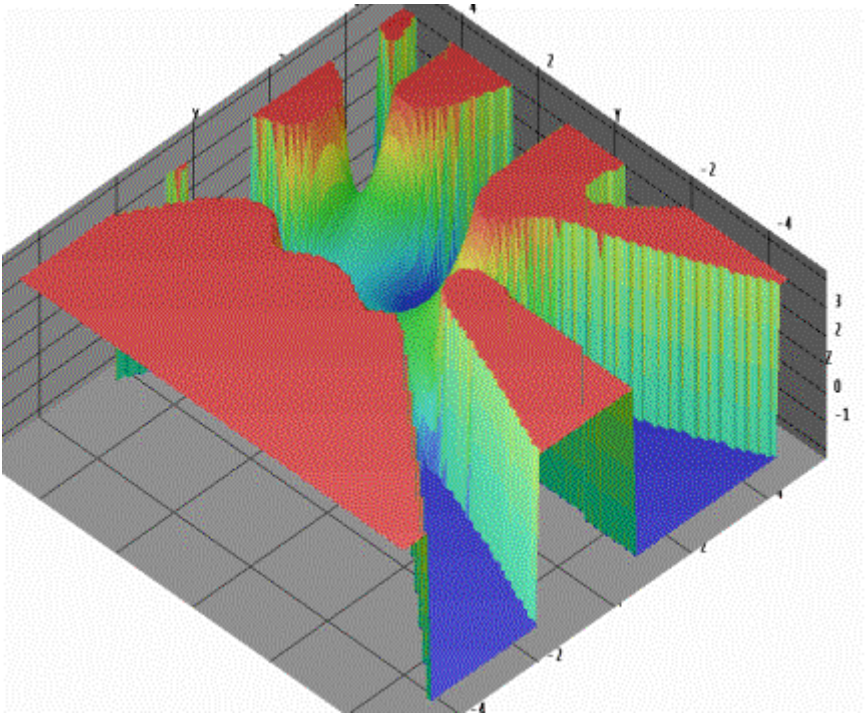

1500
1000
500
0
-500
-1000
-1500
-3
-2
-1
0
1
2
3
x
erf(z).re
y=-3
y=-2.4
y=-1.8
y=-1.2
y=-0.6
y=0
y=0.6
y=1.2
y=1.8
y=2.4
y=3

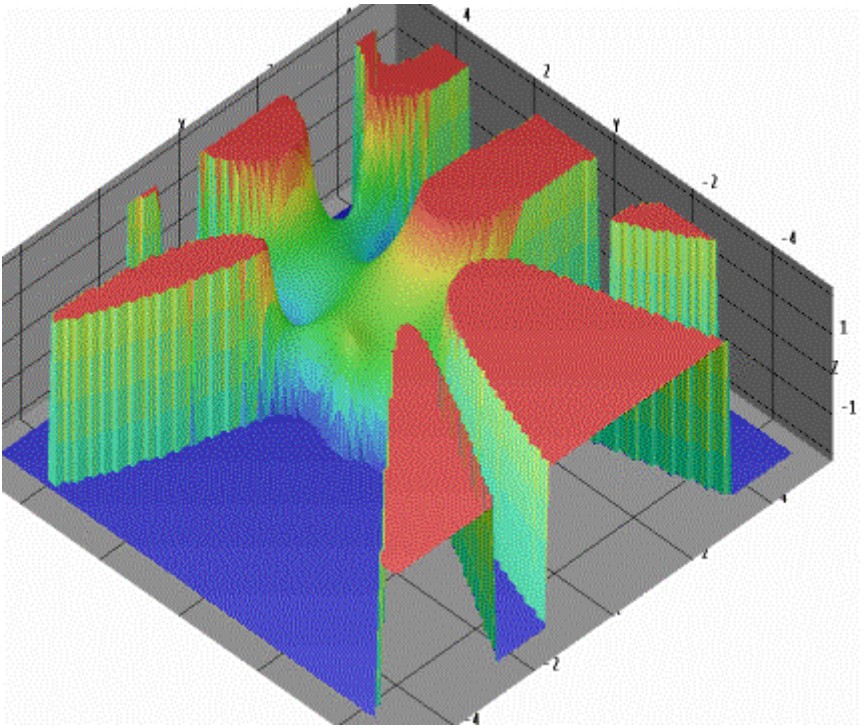

1500
1000
500
0
-500
-1000
-1500
-3
-2
-1
0
1
2
3
x
erf(z).im
y=-3
y=-2.4
y=-1.8
y=-1.2
y=-0.6
y=0
y=0.6
y=1.2
y=1.8
y=2.4
y=3

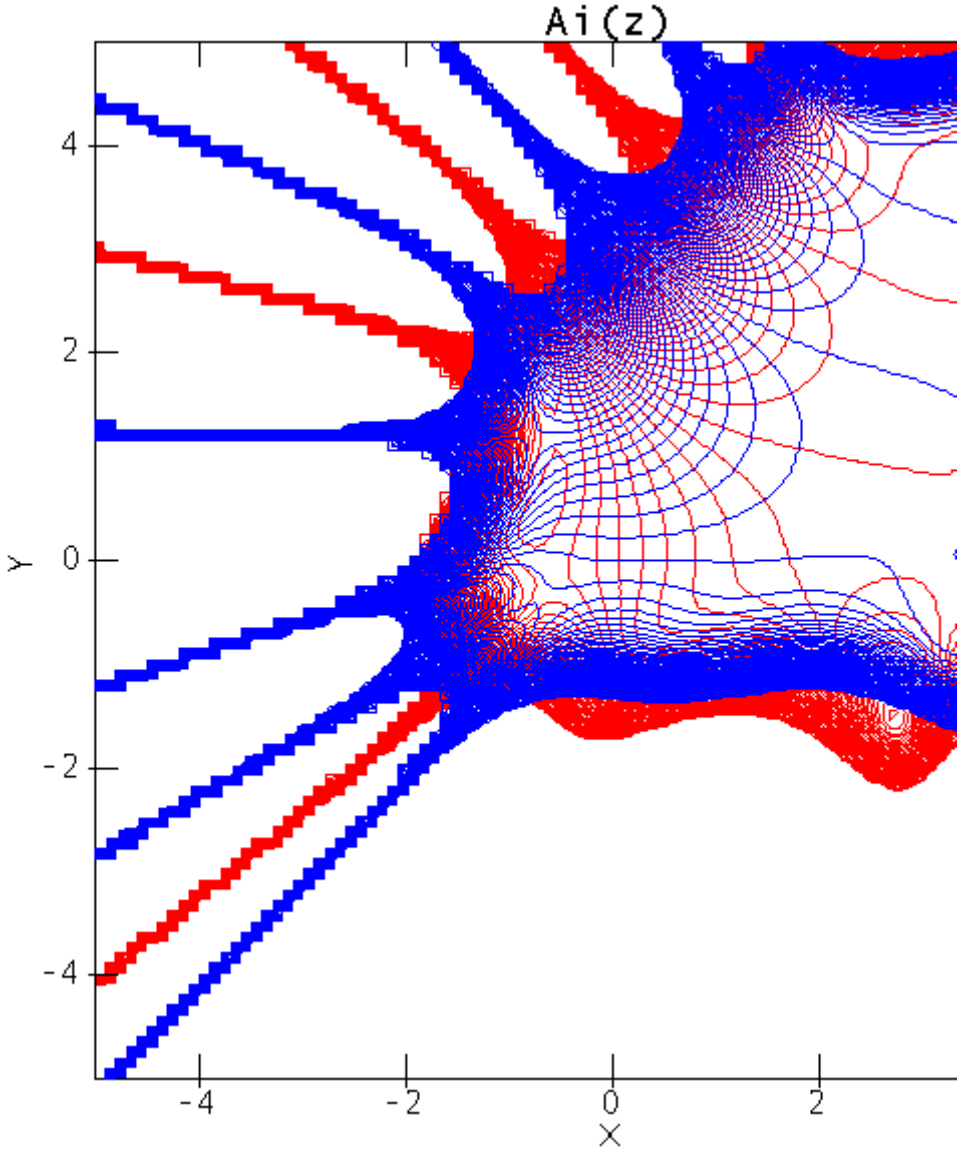

w=erf(z)
3
2
1
0
-1
-2
-3
Y
-3
-2
-1
0
1
2
X

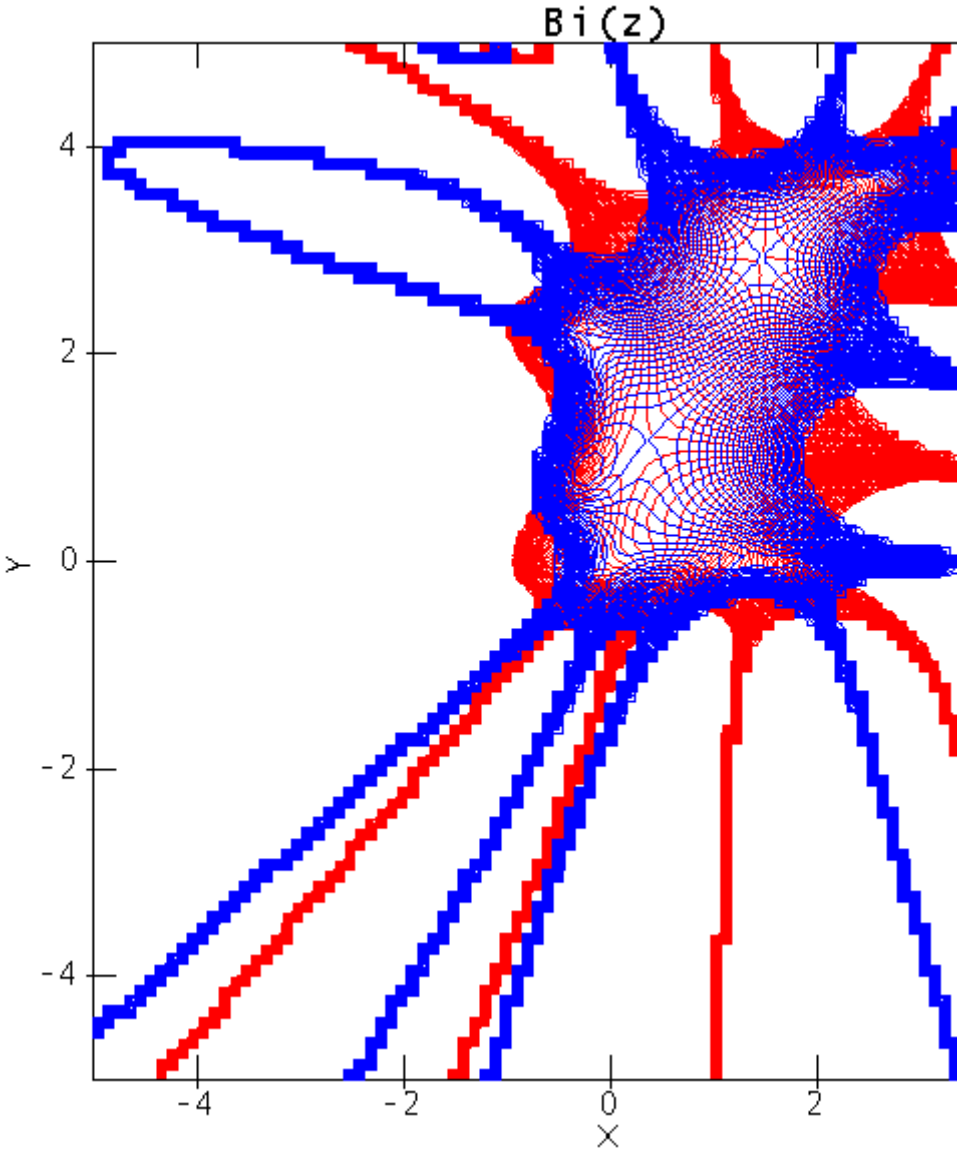

4
2
y
-2
-4
z
0
-2
-4

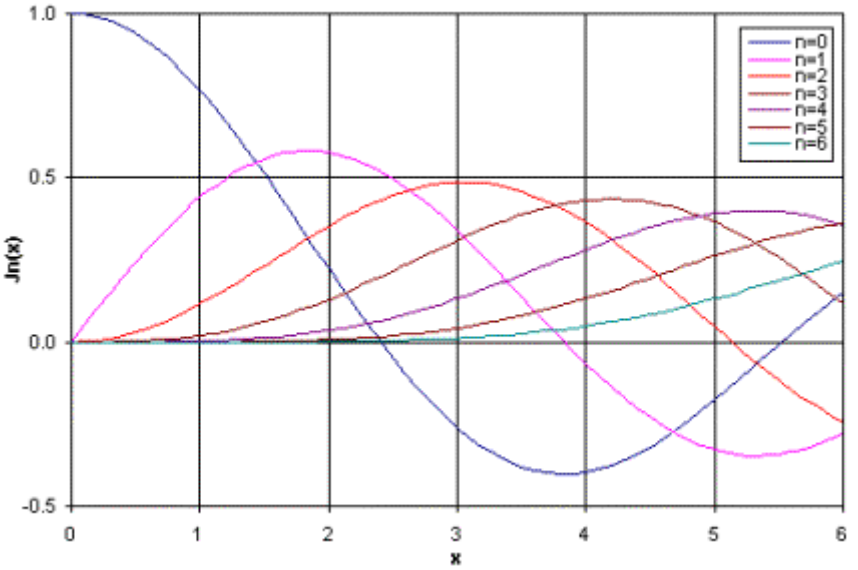

4
2
Y
-2
-4
0
Y
-2
-4

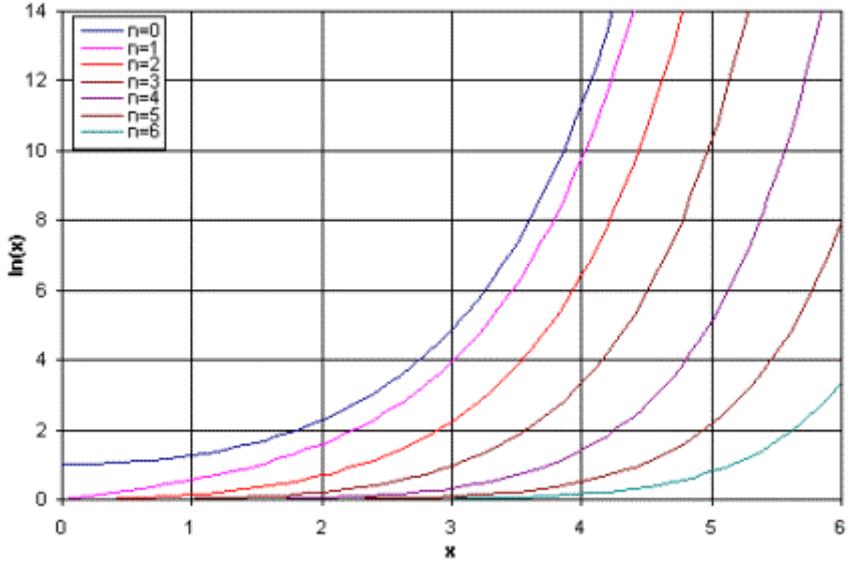

4
2
y
-2
-4
3
2
z
0
-1
-2
-4

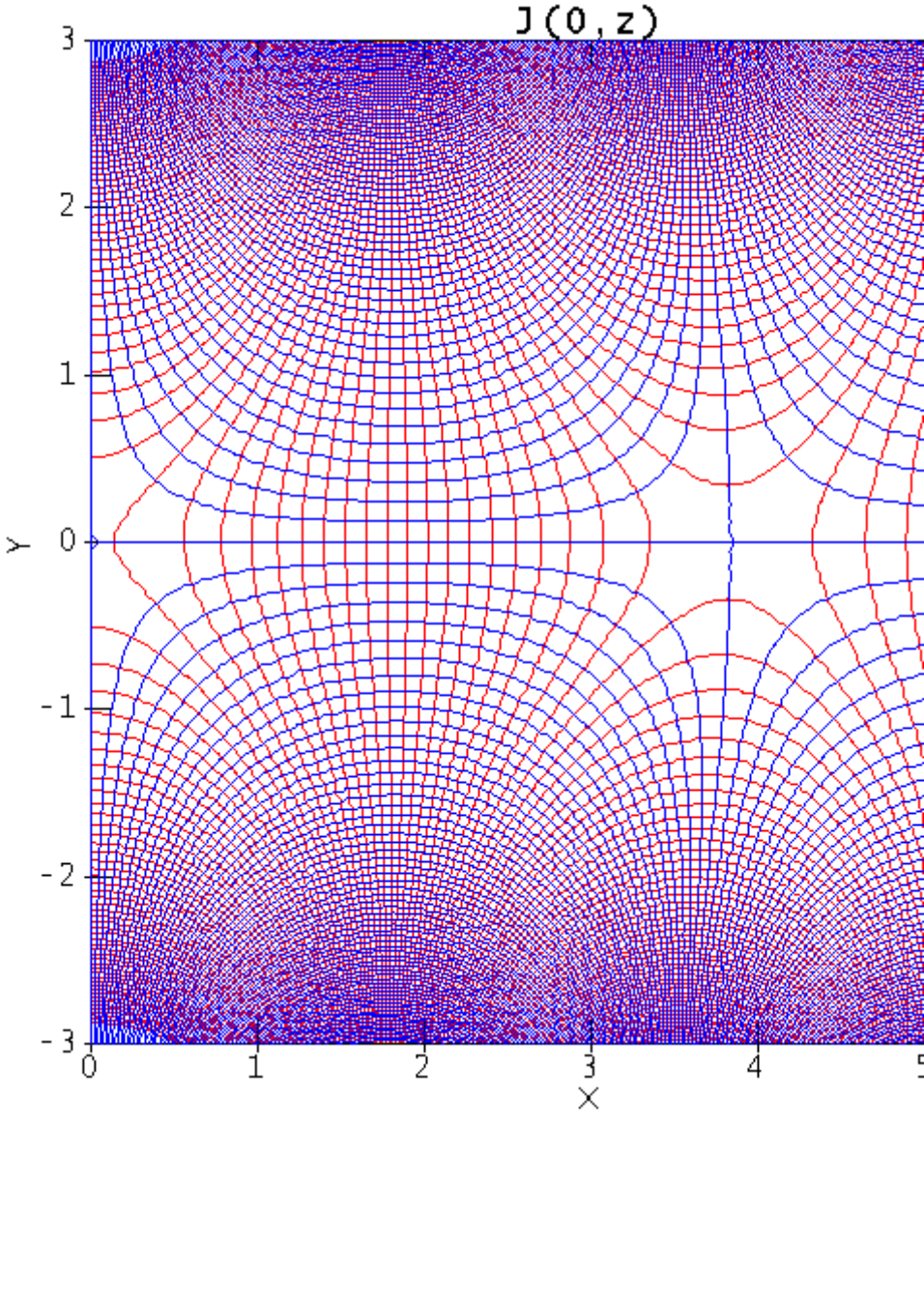

4
2
y
y
-2
-4
1
z
-1
-2
-4

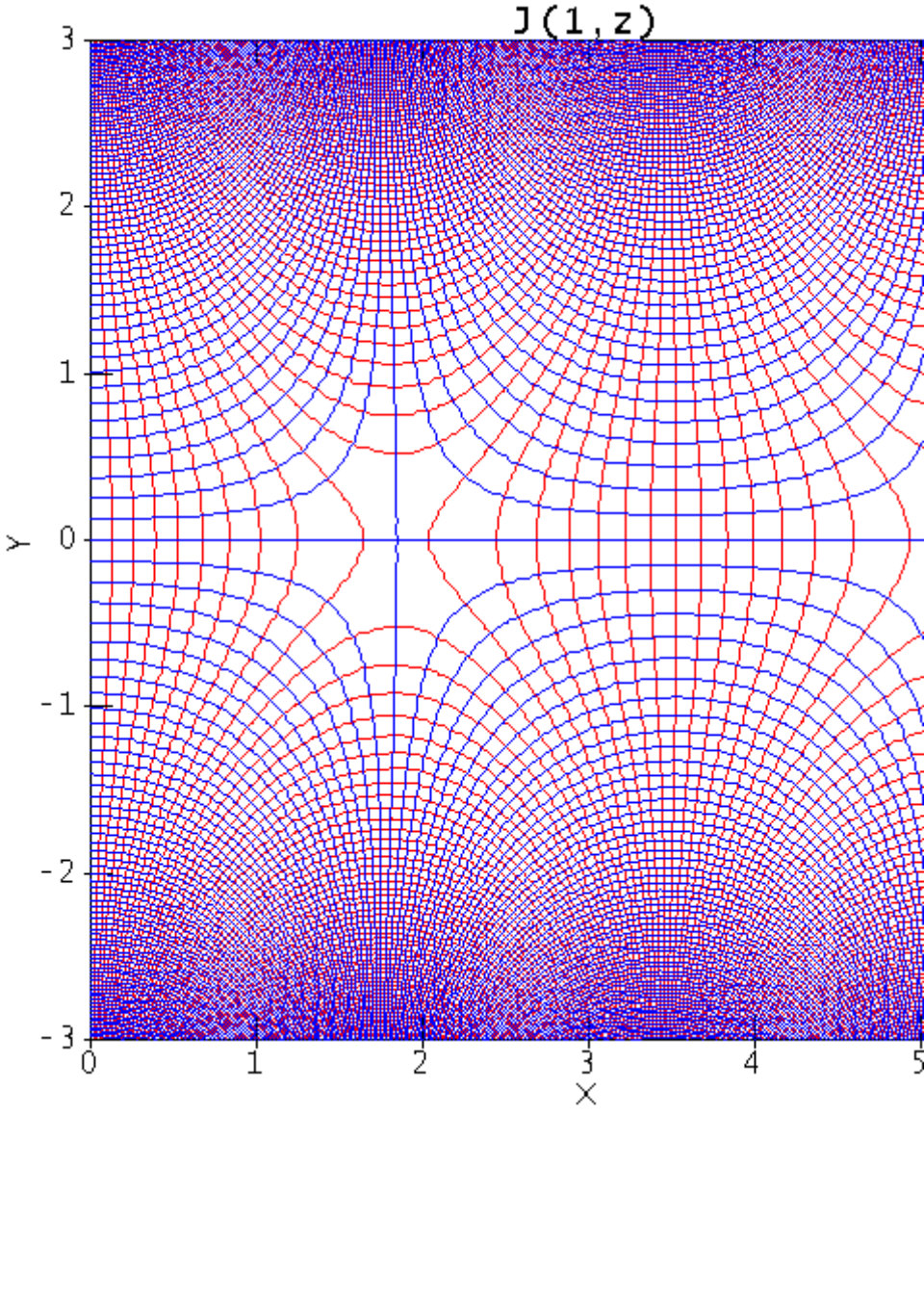

Ai(z)
4
2
0
-2
-4
Y
-4
-2
0
2
X

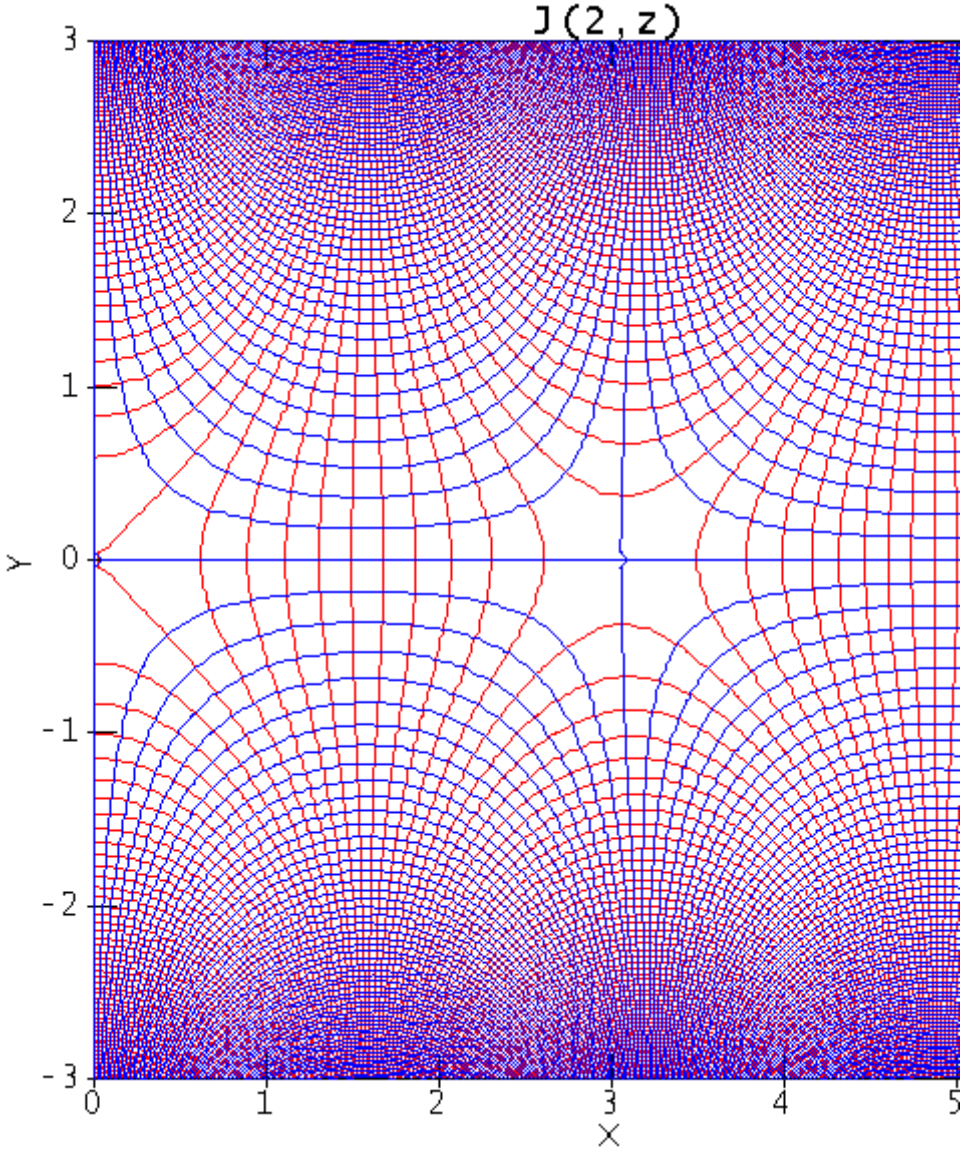

Bi(z)
4
2
0
-2
-4
Y
-4
-2
0
2
X

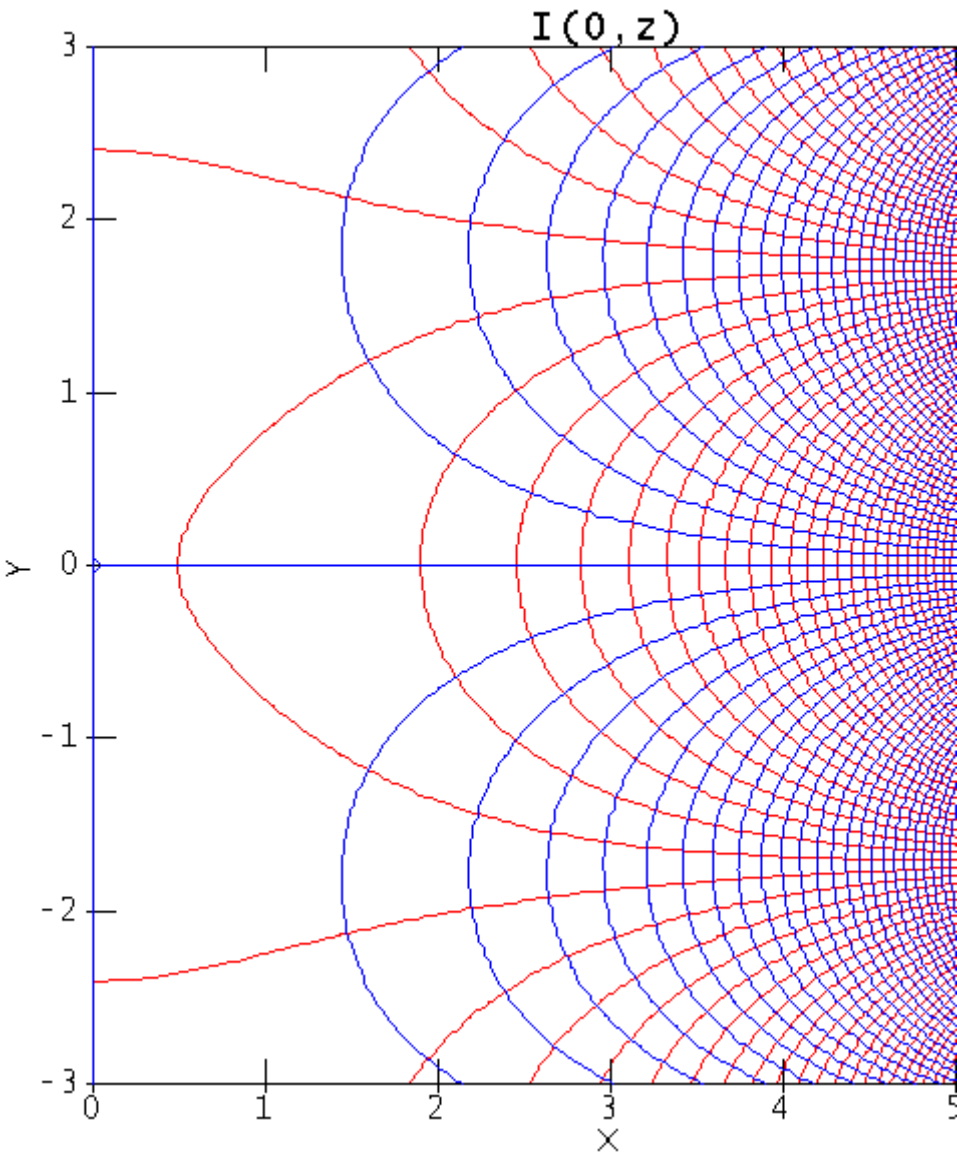

1.0
0.5
0.0
-0.5
Jn(x)
0
1
2
3
4
5
6
x
n=0
n=1
n=2
n=3
n=4
n=5
n=6

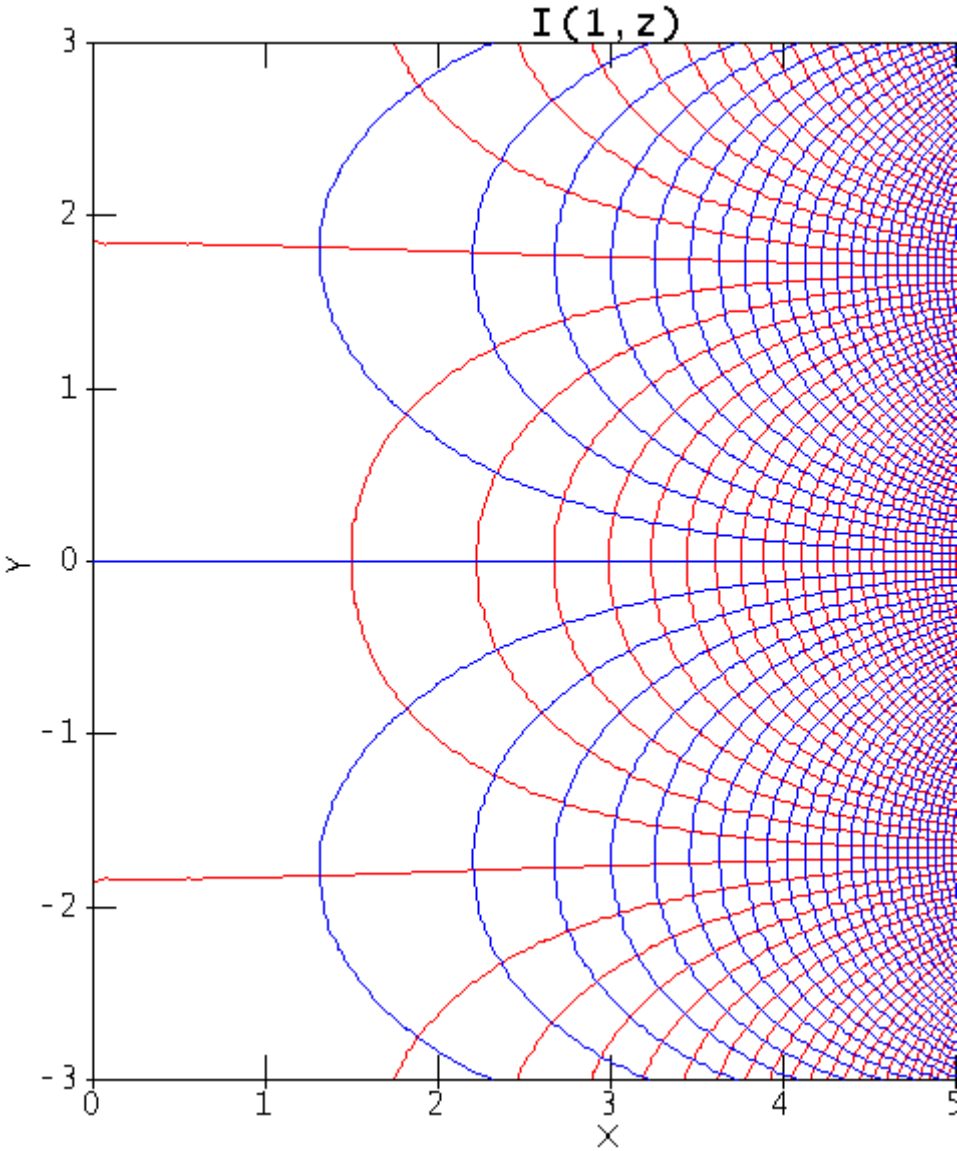

14
12
10
8
6
4
2
0
In(x)
0
1
2
3
4
5
6
x
n=0
n=1
n=2
n=3
n=4
n=5
n=6

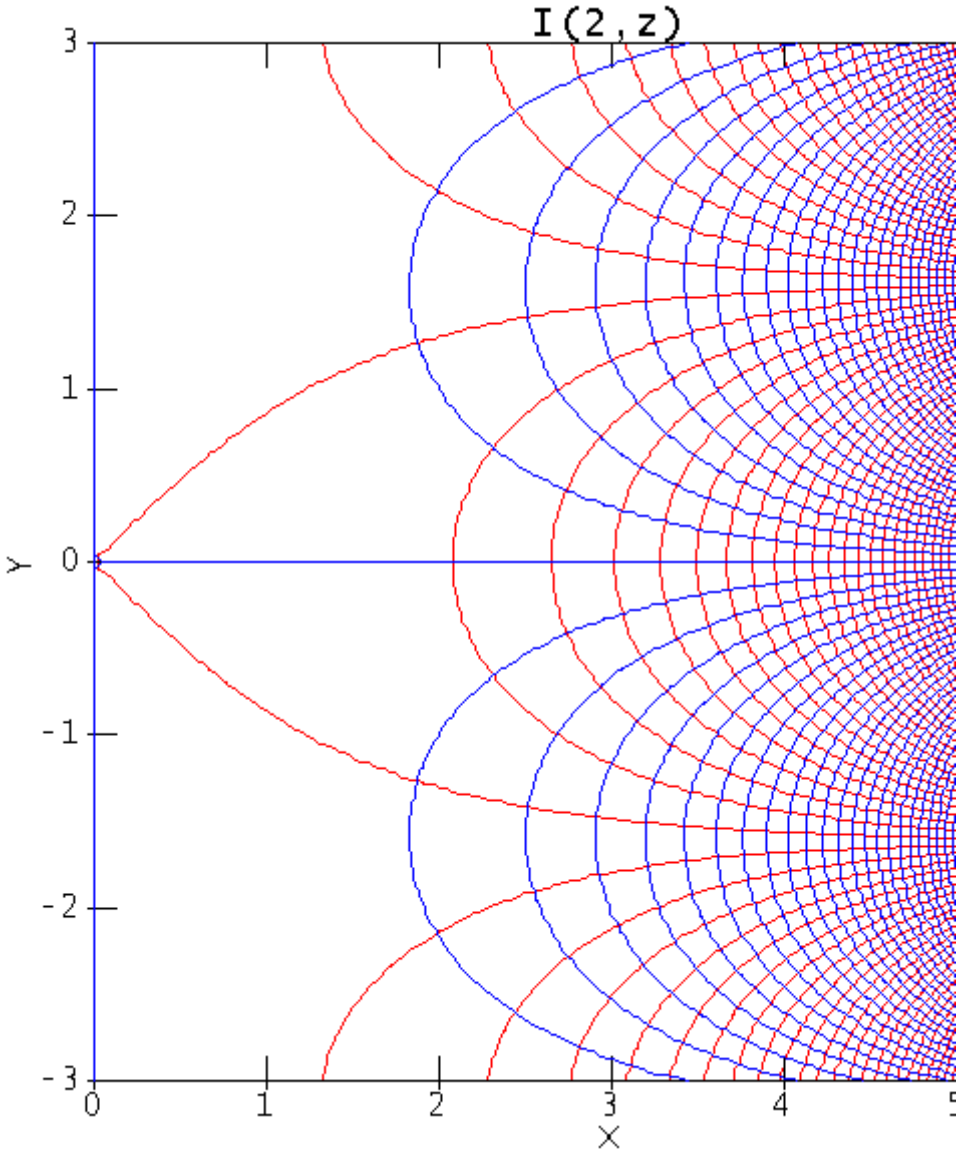

J(0,z)
3
2
1
0
-1
-2
-3
Y
0
1
2
3
4
X

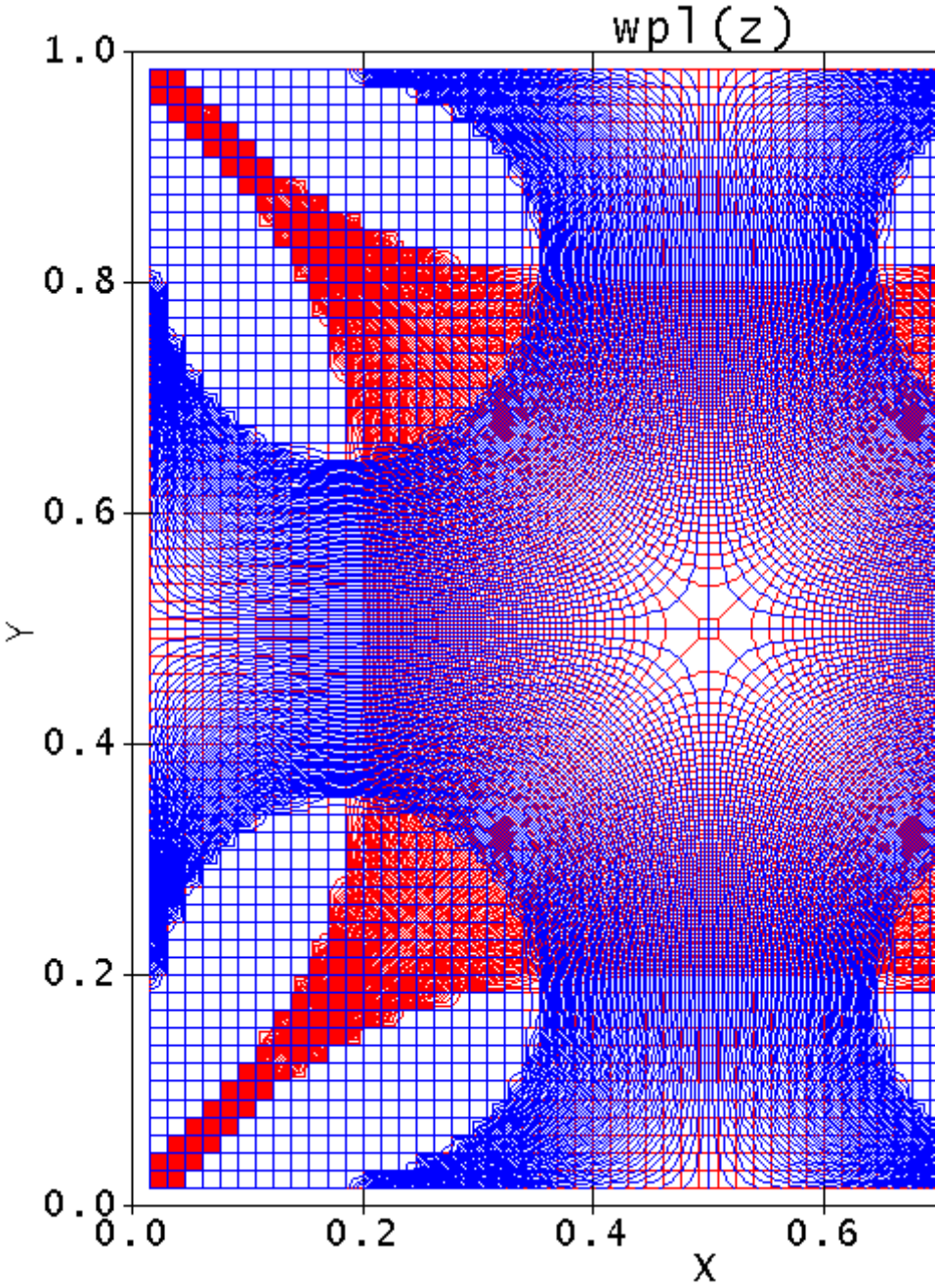

J(1,z)
3
2
1
0
-1
-2
-3
Y
0
1
2
3
4
5
X

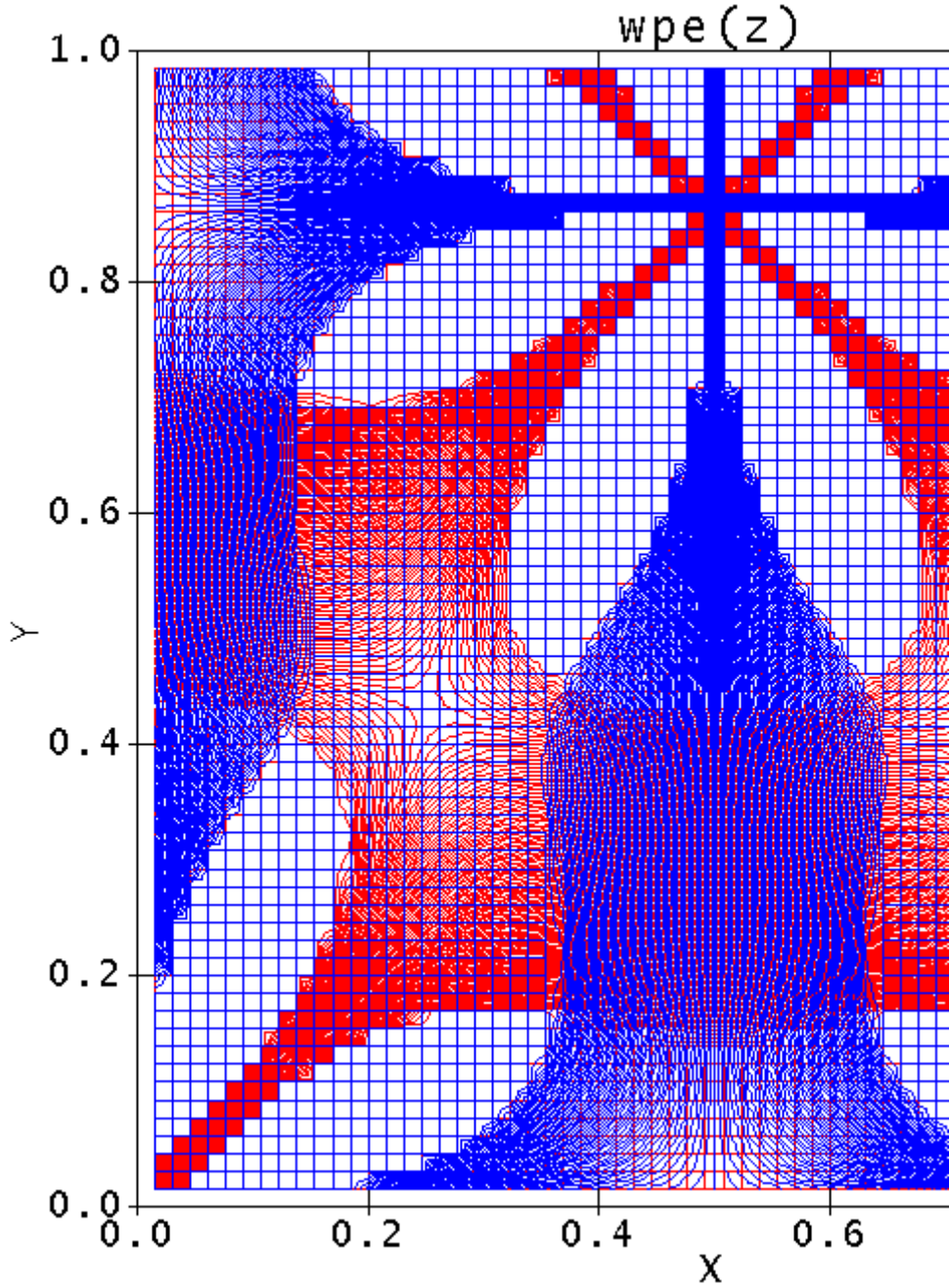

J(2,z)
3
2
1
0
-1
-2
-3
Y
0
1
2
3
4
5
X

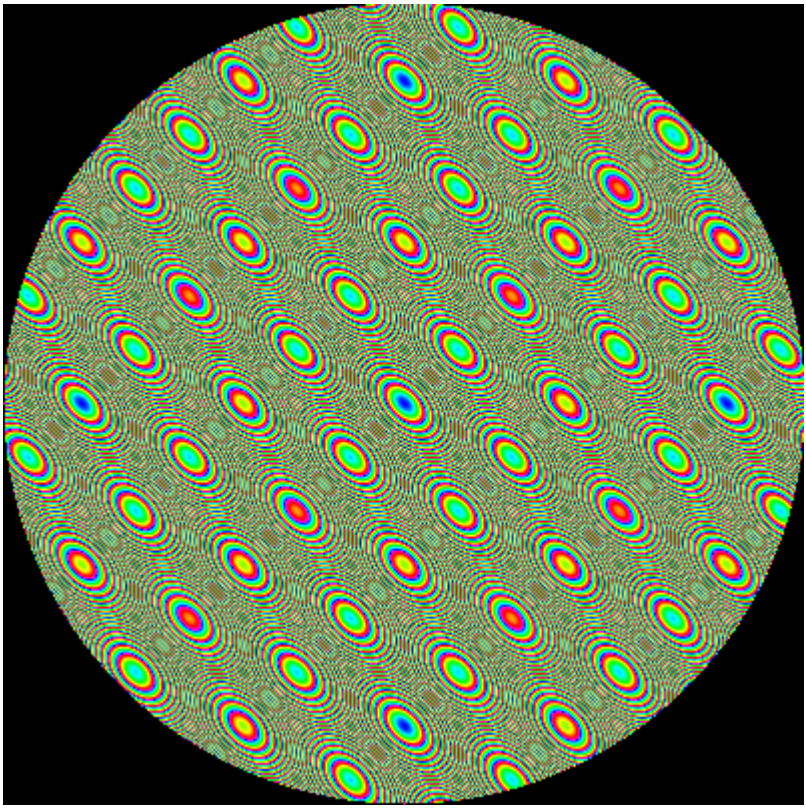

I(0,z)
3
2
1
0
-1
-2
-3
Y
0
1
2
3
4
5
X


I(1,z)
3
2
1
0
-1
-2
-3
Y
0
1
2
3
4
5
X

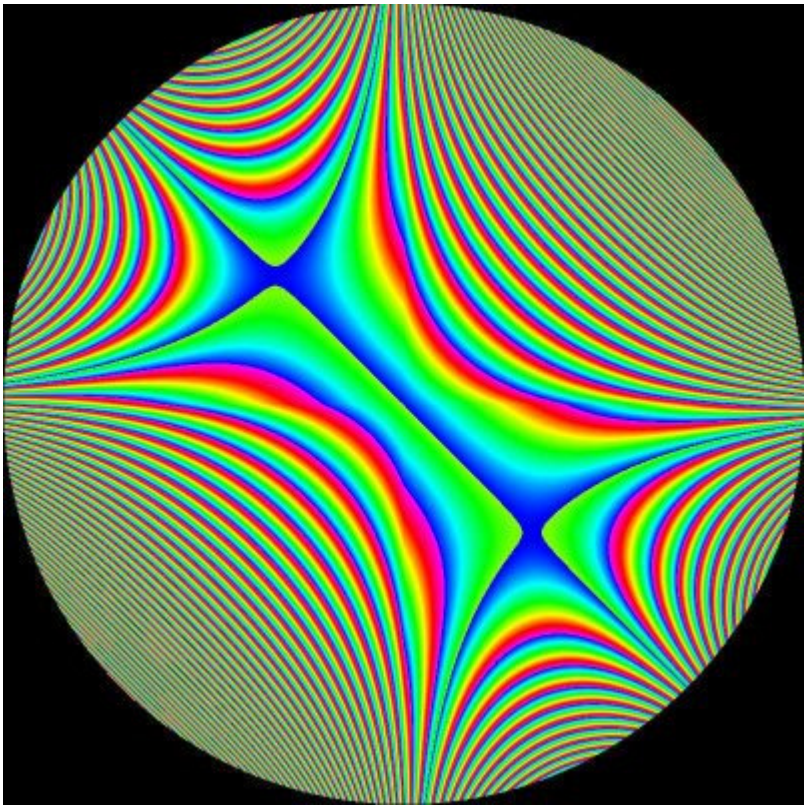

I(2,z)
3
2
1
0
-1
-2
-3
Y
0
1
2
3
4
X


wpl(z)
1.0
0.8
0.6
0.4
0.2
0.0
Y
0.0
0.2
0.4
0.6
X

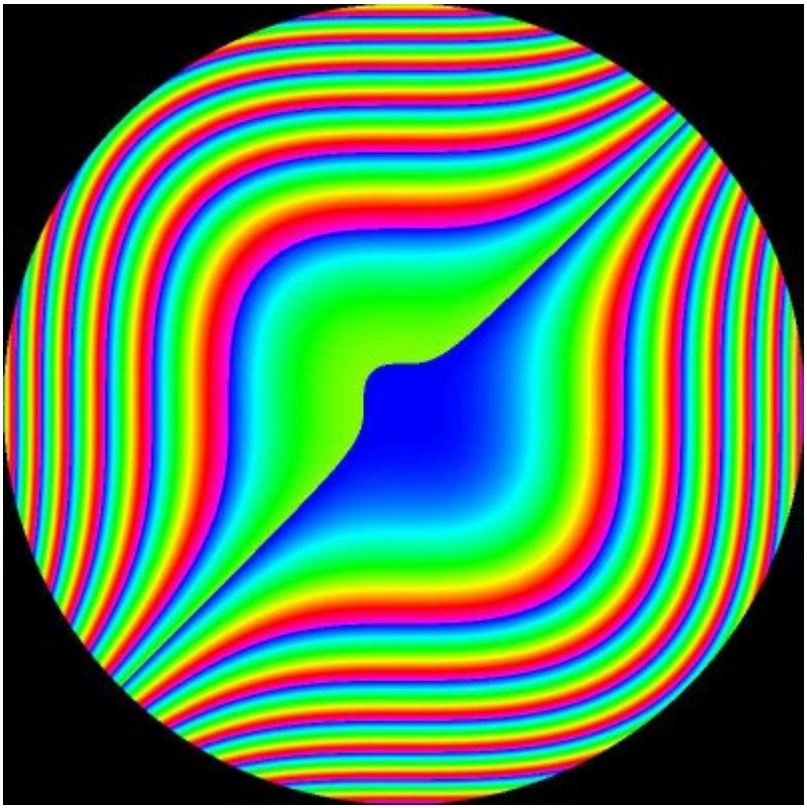

wpe(z)
1.0
0.8
0.6
0.4
0.2
0.0
Y
0.0
0.2
0.4
0.6
X



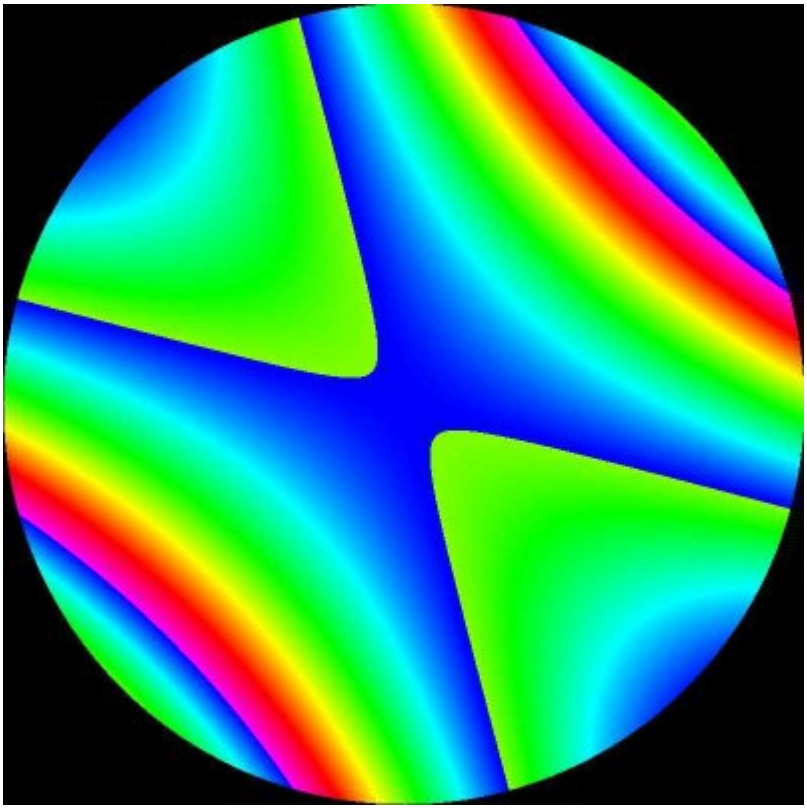
(x*x+x*y+y*y)/2



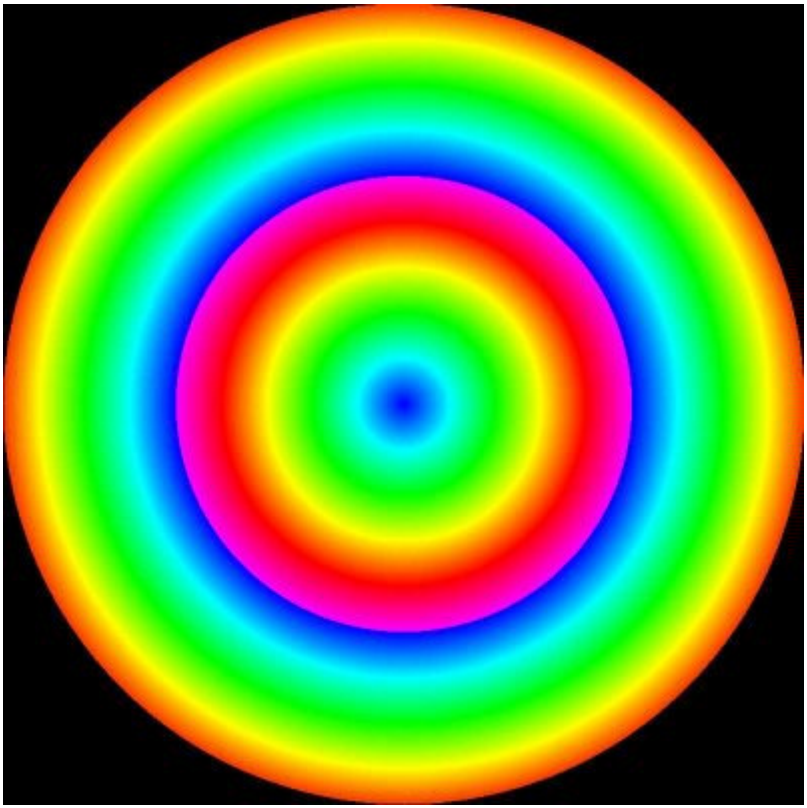
x/exp(-abs(y)/128.)+y/exp(-abs(x)/128.)



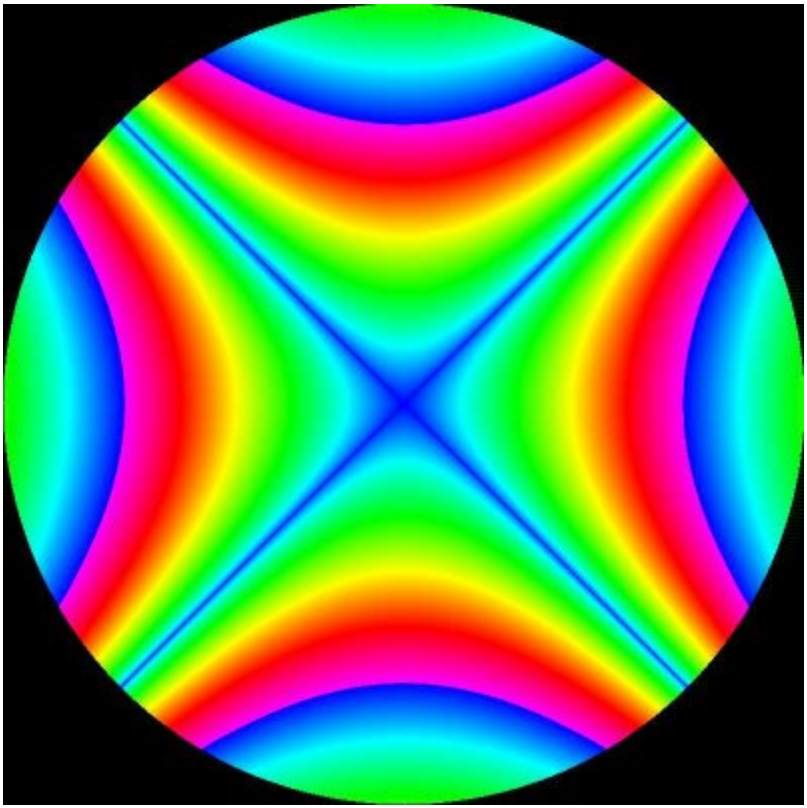
(x*x*x-y*y*y)/65536



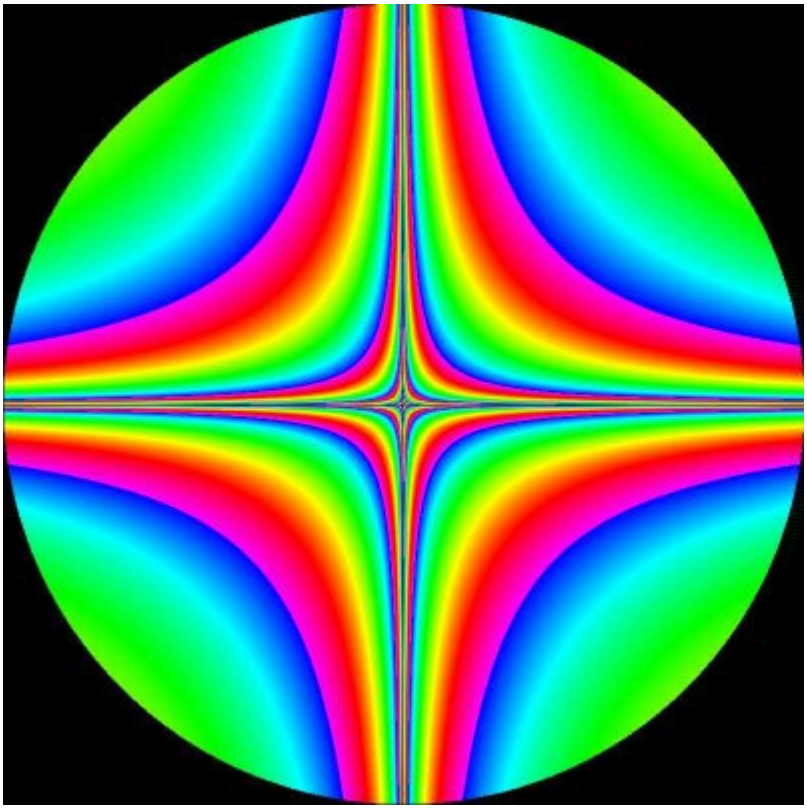
(x*x+4*x*y+y*y)/2048



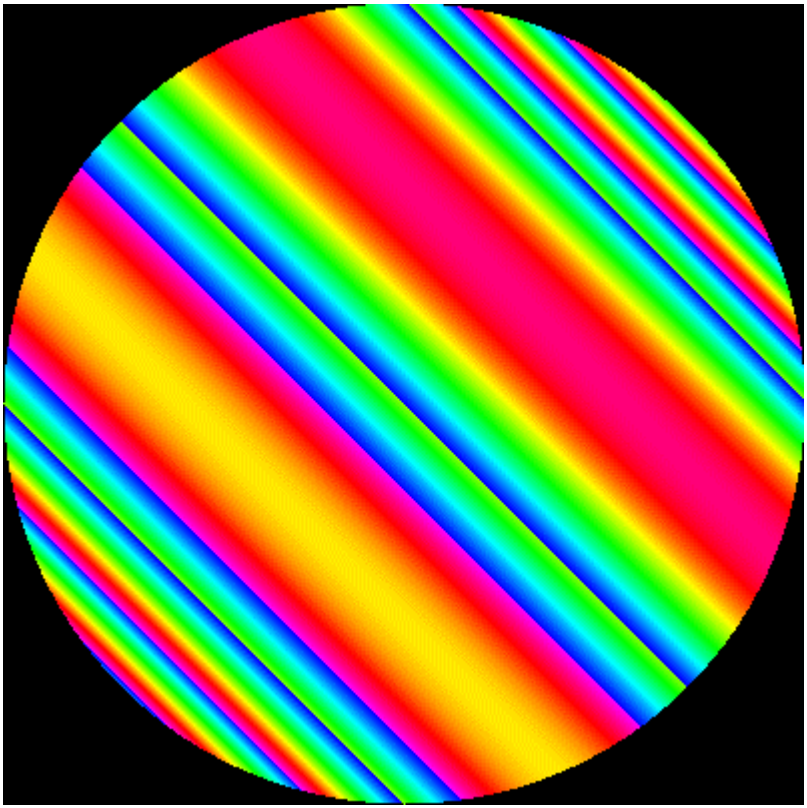
sqrt(fabs(x*x+y*y)/2)



sqrt(fabs(x*x-y*y)/3)



64.*log(fabs((x+x*y+y)))



(x+y)*cos((x+y)/256.)