Curve-Fitting

The Science and Art of Approximation

...just the color figures for those who got the B&W printed book...

by D. James Benton

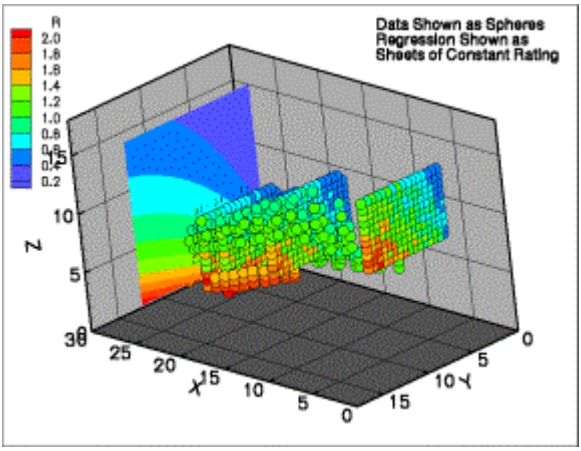
Foreword

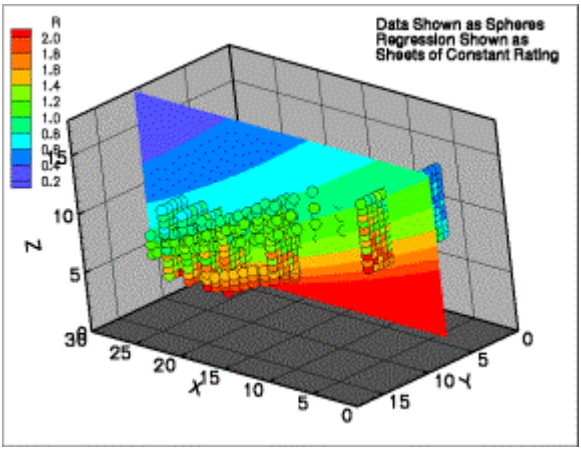
This is a how-to guide on the approximation of data. While this task is often viewed as trivial–simply open Excel® and select Add Trendline–developing an accurate and robust approximation can be quite complex, especially when the shape of the data doesn't correspond to one of the functions built into Excel®. Perhaps the most common example where Excel® is deficient would be data that approaches one or more asymptotic values. The asymptotic values may be constant or infinite. In either case, none of the functions built into Excel® exhibit this behavior.

Efficient and accurate approximation of multi-variable data can be quite challenging. While Excel® can handle such data with the LINEST() function, this capability is limited. Knowing what to feed into the LINEST() function requires an understanding of the principles presented in this book. Many examples are provided and Excel® is used wherever possible to illustrate them.

All of the examples contained in this book,
(as well as a lot of free programs) are available at…

http://www.dudleybenton.altervista.org/software/index.html


R
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
Data Shown as Spheres
Regression Shown as
Sheets of Constant Rating
Z
10
5
0
25
20
15
10
5
0
X
0
5
10
15
Y


R
2.0
1.8
1.6
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Z
10
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20
15
10
5
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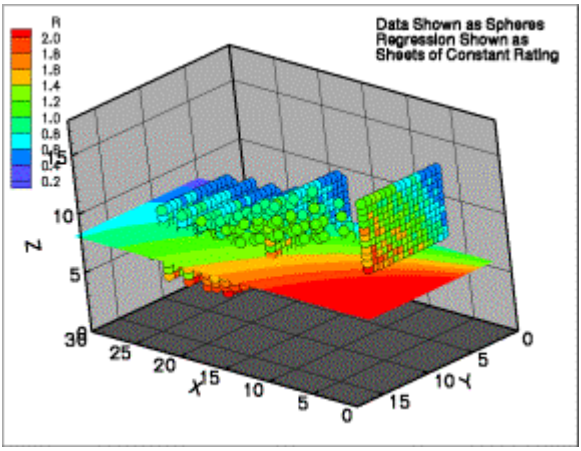
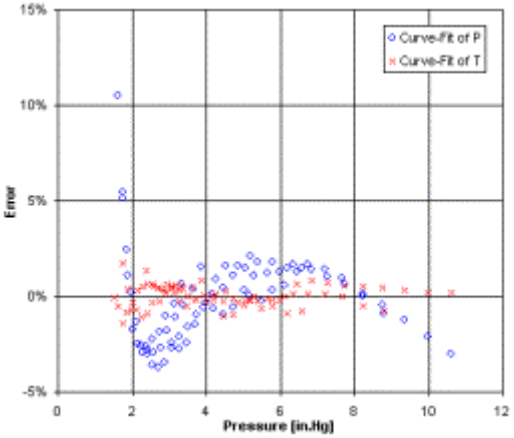
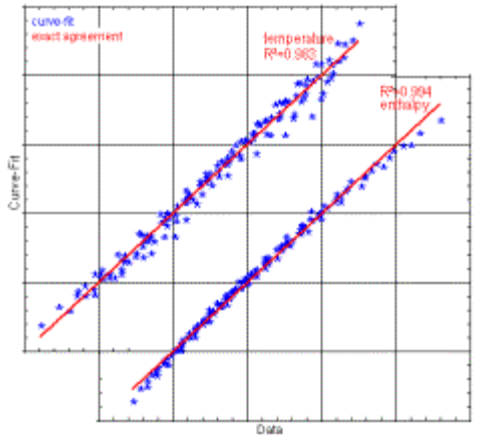
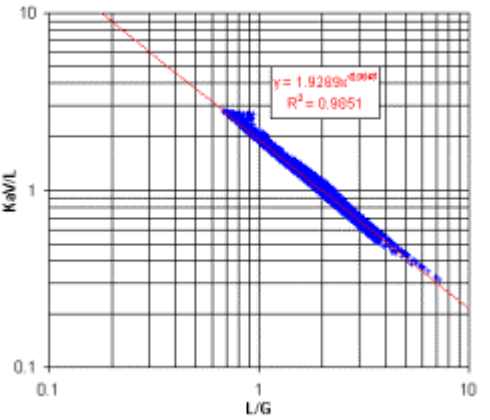
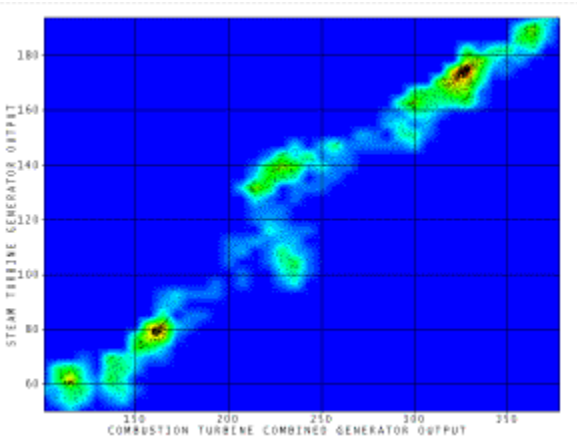
X
0
5
10
15
Y

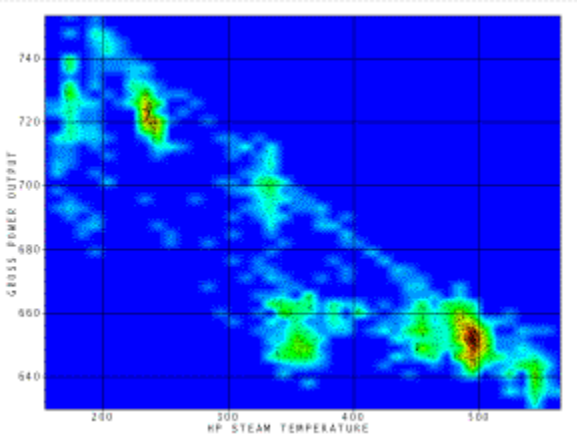

R
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
Data Shown as Spheres
Regression Shown as
Sheets of Constant Rating
Z
10
5
30
25
20
X
15
10
5
0
0
5
10
Y
15

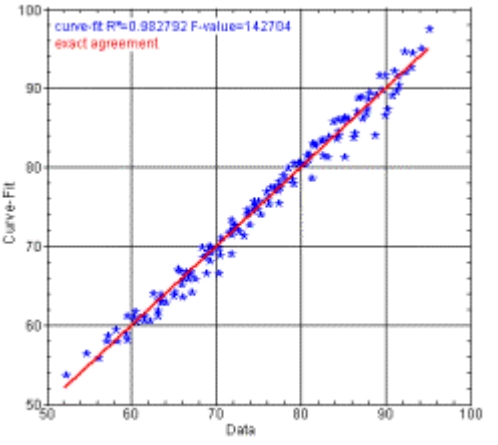

15%
10%
5%
0%
-5%
Error
0
2
4
6
8
10
12
Pressure [in.Hg]
Curve-Fit of P
Curve-Fit of T

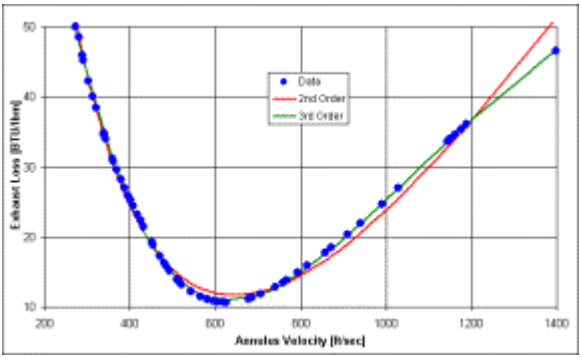

curve-fit
exact agreement
temperature
R²=0.983
R²=0.994
enthalpy
Curve-Fit
Data

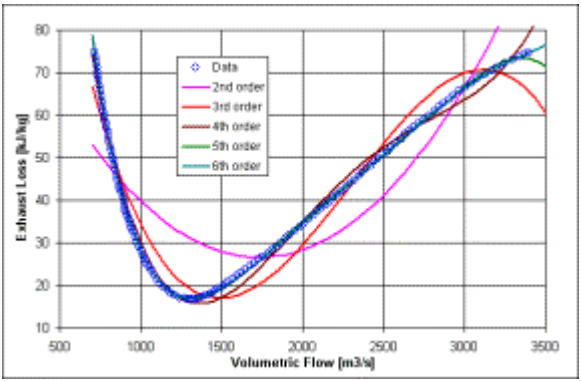

y = 1.9289x-0.9647
R² = 0.9851
KaV/L
L/G
10
1
0.1
0.1
1
10

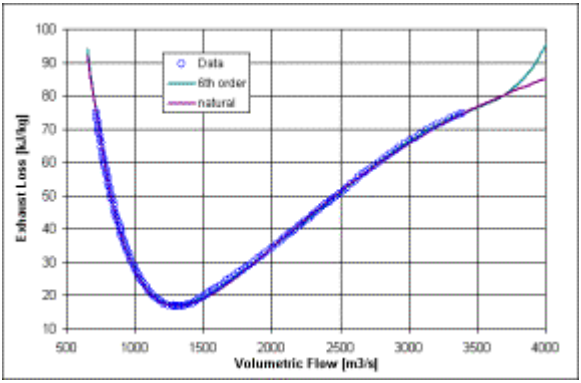

STEAM TURBINE GENERATOR OUTPUT
180
160
140
120
100
80
60
150
200
250
300
350
COMBUSTION TURBINE COMBINED GENERATOR OUTPUT

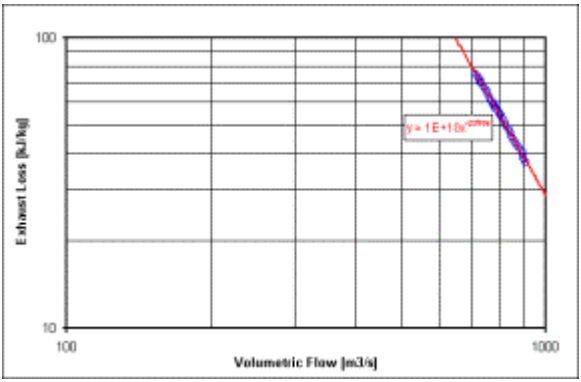

GROSS POWER OUTPUT
740
720
700
680
660
640
200
300
400
500
HP STEAM TEMPERATURE

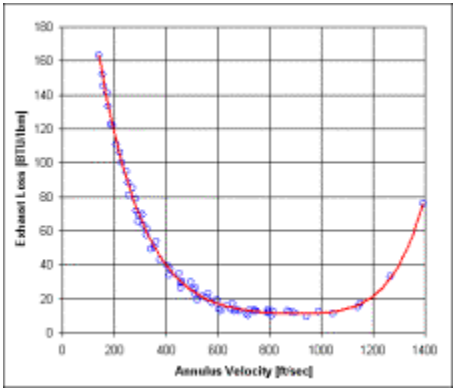

curve-fit R²=0.982792 F-value=14.2704
exact agreement
Curve-Fit
100
90
80
70
60
50
50
60
70
80
90
100
Data

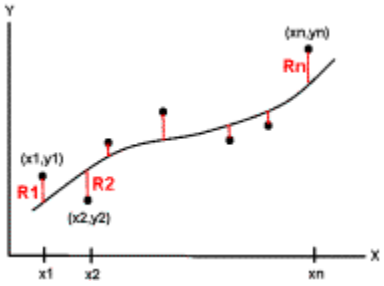

Data
2nd Order
3rd Order
Exhaust Loss [BTU/lbm]
Annulus Velocity [ft/sec]
50
40
30
20
10
200
400
600
800
1000
1200
1400


Data
2nd order
3rd order
4th order
5th order
6th order
Exhaust Loss [kJ/kg]
Volumetric Flow [m3/s]
80
70
60
50
40
30
20
10
500
1000
1500
2000
2500
3000
3500

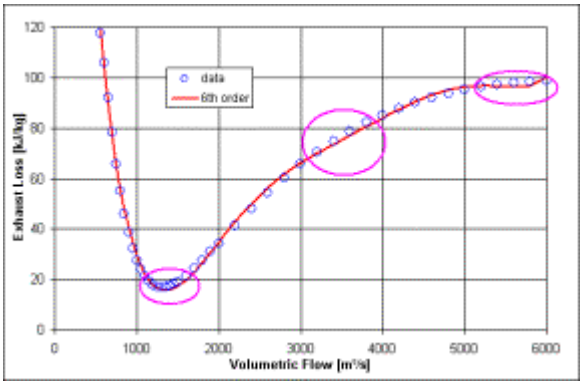

Data
6th order
natural
Exhaust Loss [kJ/kg]
Volumetric Flow [m3/s]
100
90
80
70
60
50
40
30
20
10
500
1000
1500
2000
2500
3000
3500
4000


100
10
100
1000
Exhaust Loss [kJ/kg]
Volumetric Flow [m3/s]

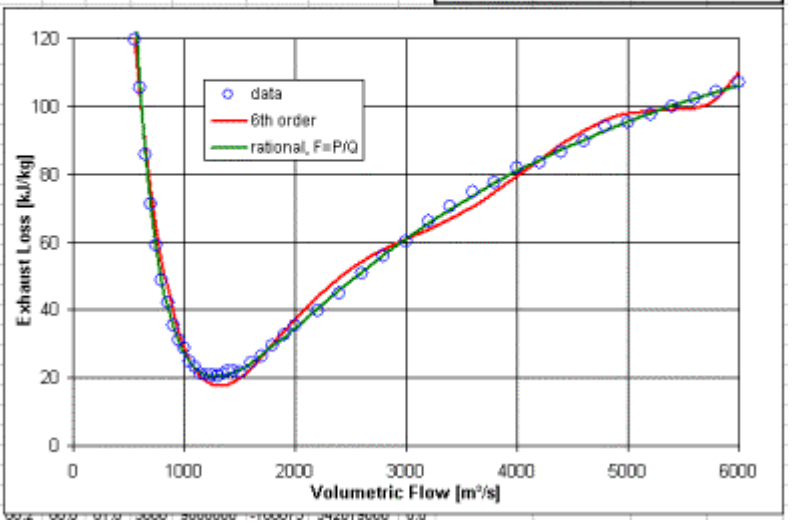

180
160
140
120
100
80
60
40
20
0
0
200
400
600
800
1000
1200
1400
Exhaust Loss [BTU/lbm]
Annulus Velocity [ft/sec]

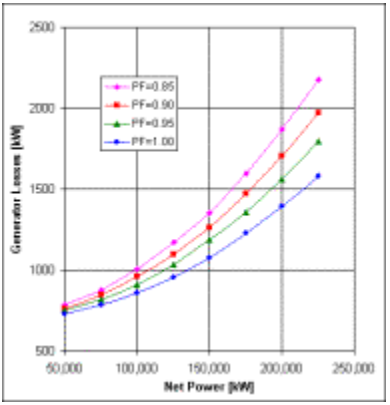

Y
(xn,yn)
Rn
(x1,y1)
R1
R2
(x2,y2)
X
x1
x2
xn

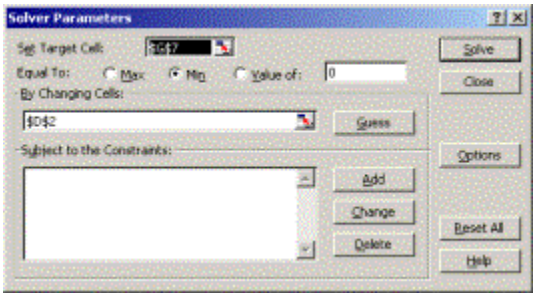

order 0
order 1
order 2
order 3
order 4
order 5


data
6th order
Exhaust Loss [kJ/kg]
Volumetric Flow [m³/s]

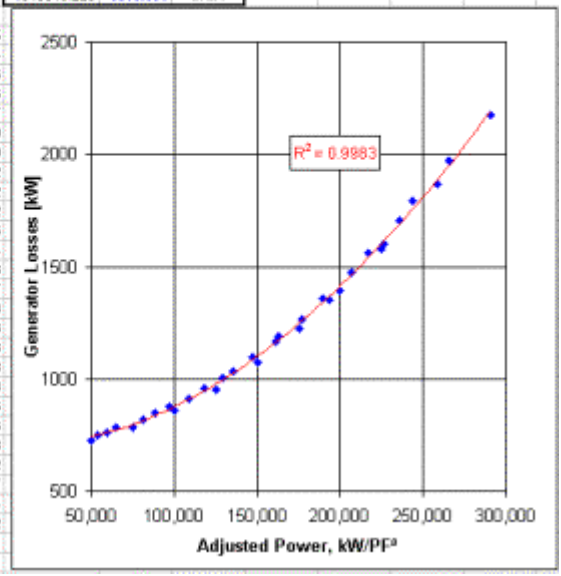
	A	B	C	D	E	F	G	H	I	J	K	L	M
1										LINEST			
2	m^3/s	data	6th ord	F=P/Q	X	X^2	-XY	X^2Y	R	832.2033491	178.7427342	-389064.7	2.6E+08
3	550	119.7	119.9	131.9	550	302500	-65835	36209250	12.2	76.33670916	1.579095267	4027.88	4944363
4	600	105.6	103.1	106.0	600	360000	-63373	38024045	0.4	**0.999930116**	9464981.844	#N/A	#N/A
5	650	85.8	88.2	86.0	650	422500	-55799	36269513	0.1	**195547.8436**	41	#N/A	#N/A
6	700	71.2	75.3	70.3	700	490000	-49808	34865783	-0.8	5.2555E+19	3.67302E+15	#N/A	#N/A
7	750												
8	800												
9	850												
10	900												
11	950												
12	1000												
13	1050												
14	1100												
15	1150												
16	1200												
17	1250												
18	1300												
19	1350												
20	1400												
21	1450												
22	1500												
23	1600												
24	1700												
25	1800												
26	1900												
27	2000												
28	2200												
29	2400												
30	2600												
31	2800												
32	3000												

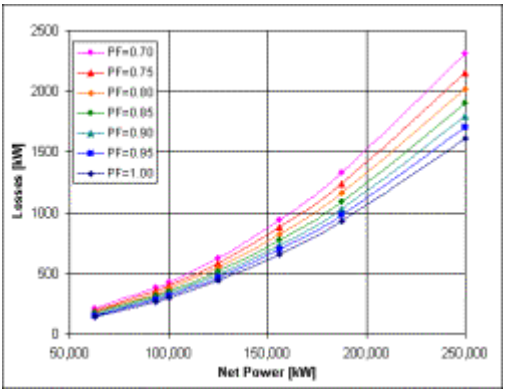




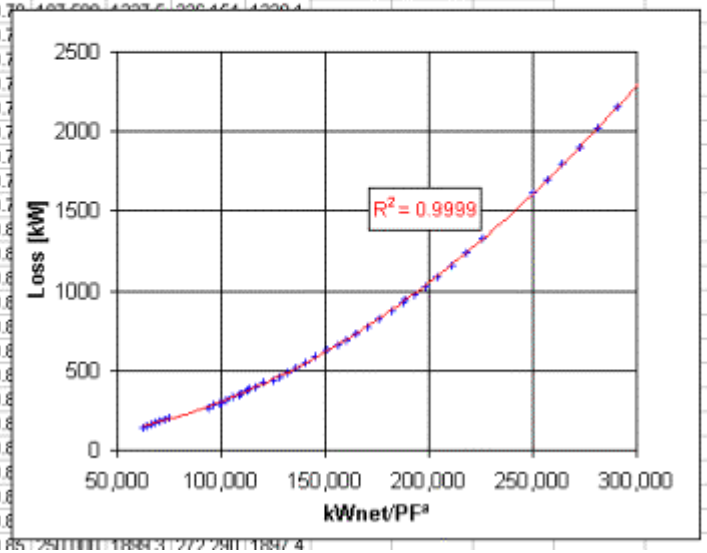


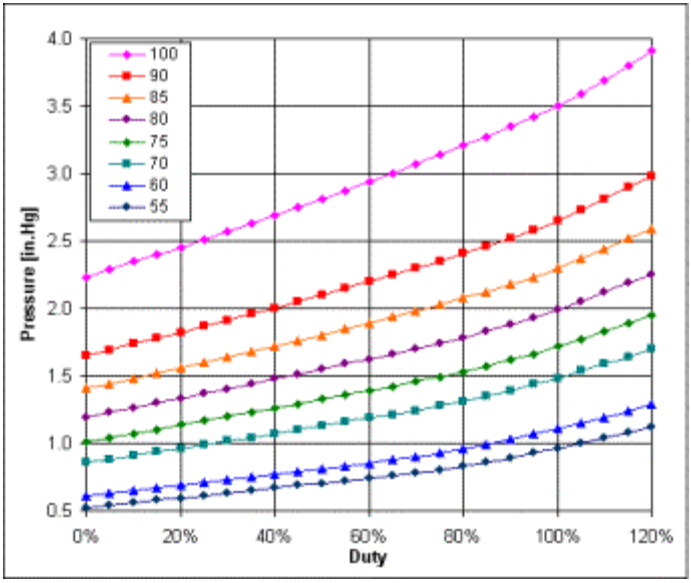
	A	B	C	D	E	F	G	H	I	J	K	L
1	**Typical Generator Losses**											
2	PF	net	loss	1.5753	a		LINEST					
3	0.85	50,000	782	64589	769	-3.0787E-15	1.87E-08	692.0632				
4	0.85	75,000	877	96883	865	2.98888E-15	8.4E-10	7.938175				
5	0.85	100,000	1004	129178	997	**0.998259024**	17.23672	#N/A				
6	0.85	125,000	1168	161472	1166	**8314.160416**	29	#N/A				
7	0.85	150,000	1352	193767	1371	4940349.229	**8616.031**	#N/A				
8	0.85	175,000	1599	226061	1611							
9	0.85	200,000	1871	258356	1885							
10	0.85	225,000	2175	290650	2194							
11	0.90	50,000	763	59027	756							
12	0.90	75,000	846	88541	836							
13	0.90	100,000	959	118055	947							
14	0.90	125,000	1099	147568	1089							
15	0.90	150,000	1263	177082	1260							
16	0.90	175,000	1472	206596	1462							
17	0.90	200,000	1706	236109	1692							
18	0.90	225,000	1972	265623	1952							
19	0.95	50,000	751	54208	746							
20	0.95	75,000	820	81312	814							
21	0.95	100,000	909	108416	908							
22	0.95	125,000	1035	135520	1027							
23	0.95	150,000	1187	162624	1173							
24	0.95	175,000	1358	189728	1343							
25	0.95	200,000	1561	216832	1538							
26	0.95	225,000	1795	243936	1758							
27	1.00	50,000	725	50000	738							
28	1.00	75,000	782	75000	796							
29	1.00	100,000	858	100000	876							
30	1.00	125,000	953	125000	978							
31	1.00	150,000	1073	150000	1102							
32	1.00	175,000	1225	175000	1247							
33	1.00	200,000	1390	200000	1414							
34	1.00	225,000	1580	225000	1602							
35												
36												



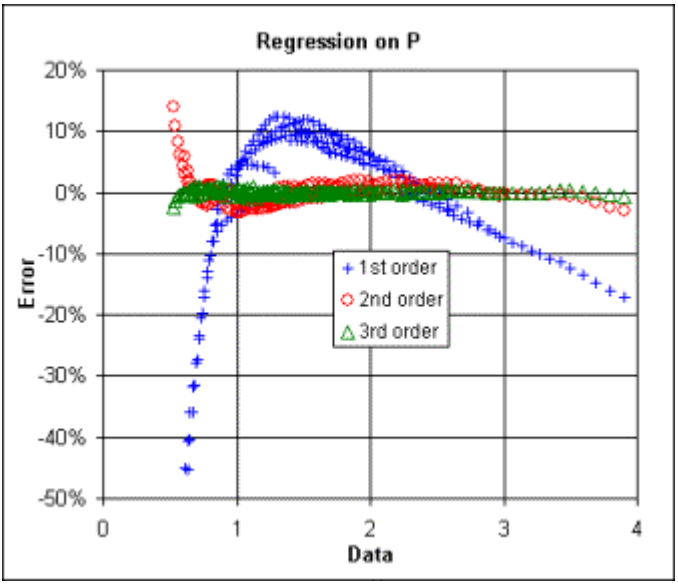


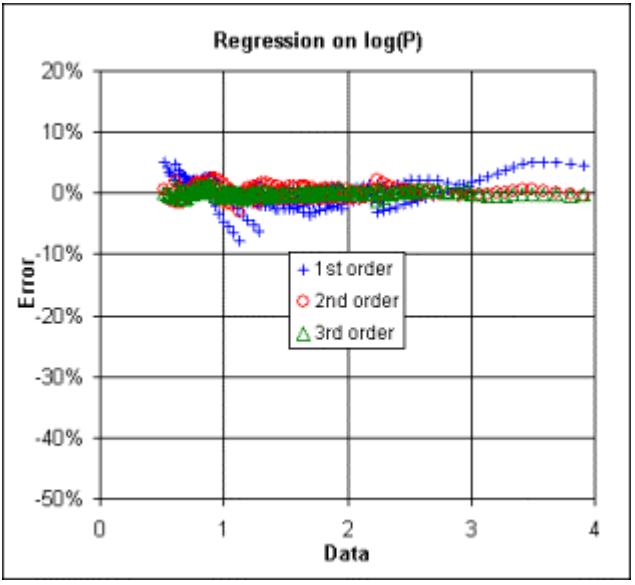
	A	B	C	D	E	F	G	H	I	J
1	**Transformer Losses**									
2	PF	kWhet	Loss	**0.5255**	fit		**LINEST**			
3	0.70	62,500	206.9	75,385	195.8	-1E-15	2.5E-08	53.1174		
4	0.70	93,750	382.2	113,077	373.5	5.8E-16	1.7E-10	1.69472		
5	0.70	100,000	424.8	120,616	417.6	**0.99995**	4.28413	#N/A		
6	0.70	125,000	626.9	150,769	621.8	**443690**	46	#N/A		
7	0.70	156,250	941.6	188,462	940.1	1.6E+07	**844.273**	#N/A		

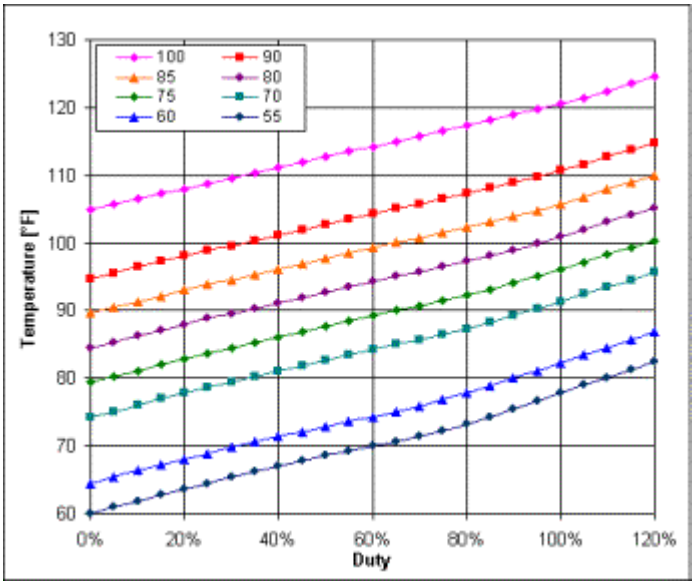


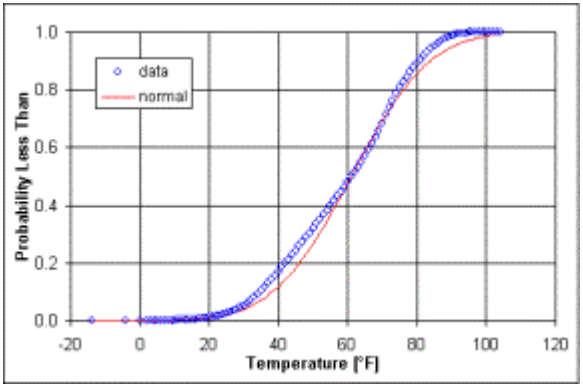


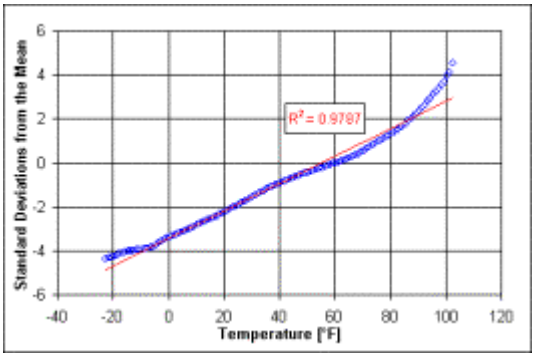
	A	B	C	D	E	AA	AB	AC
1	**Typical Condenser Curves**					1st Order LINEST on P		
2	Tin	Duty	Pres.	1st	2nd	0.828249754	0.047653	-2.524188839
3	°F	%	in.Hg	in.Hg	in.Hg	0.033421384	0.000853	0.069616551
4	55	0%	0.52	0.10	0.59	**0.949906641**	0.170416	#N/A
5	55	5%	0.54	0.14	0.60	**1867.828522**	197	#N/A
6	55	10%	0.56	0.18	0.60	108.4898765	**5.721217**	#N/A
7	55	15%	0.58	0.22	0.61			
8	55	20%	0.59	0.26	0.62			2nd Order LINES
9	55	25%	0.61	0.30	0.63	0.285452599	0.018821	0.000673961
10	55	30%	0.63	0.35	0.64	0.014042601	0.00032	8.22771E-06
11	55	35%	0.65	0.39	0.65	**0.999098606**	0.023036	#N/A
12	55	40%	0.67	0.43	0.67	**43005.64709**	194	#N/A
13	55	45%	0.69	0.47	0.68	114.1081446	**0.102949**	#N/A
14	55	50%	0.70	0.51	0.70			
15	55	55%	0.72	0.55	0.72			
16	55	60%	0.74	0.59	0.74	0.32365156	0.002651	0.000223898
17	55	65%	0.76	0.64	0.76	0.010414942	0.000232	5.3251E-06
18	55	70%	0.78	0.68	0.79	**0.999951926**	0.005376	#N/A
19	55	75%	0.80	0.72	0.81	**439119.0478**	190	#N/A
20	55	80%	0.83	0.76	0.84	114.2056032	**0.005491**	#N/A
21	55	85%	0.86	0.80	0.86			

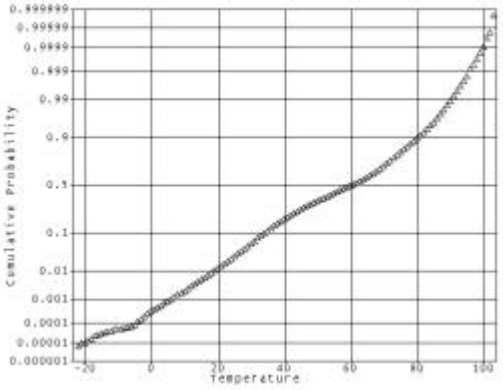

Regression on P
20%
10%
0%
-10%
-20%
-30%
-40%
-50%
Error
0
1
2
3
4
Data
1st order
2nd order
3rd order


Regression on log(P)
20%
10%
0%
-10%
-20%
-30%
-40%
-50%
Error
0
1
2
3
4
Data
1st order
2nd order
3rd order

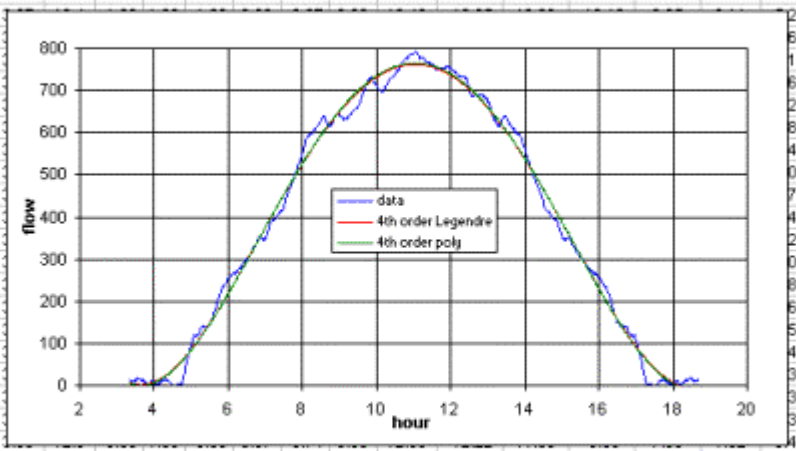

100
90
85
80
75
70
60
55
130
120
110
100
90
80
70
60
Temperature [°F]
0%
20%
40%
60%
80%
100%
120%
Duty


data
normal
1.0
0.8
0.6
0.4
0.2
0.0
Probability Less Than
-20
0
20
40
60
80
100
120
Temperature [°F]

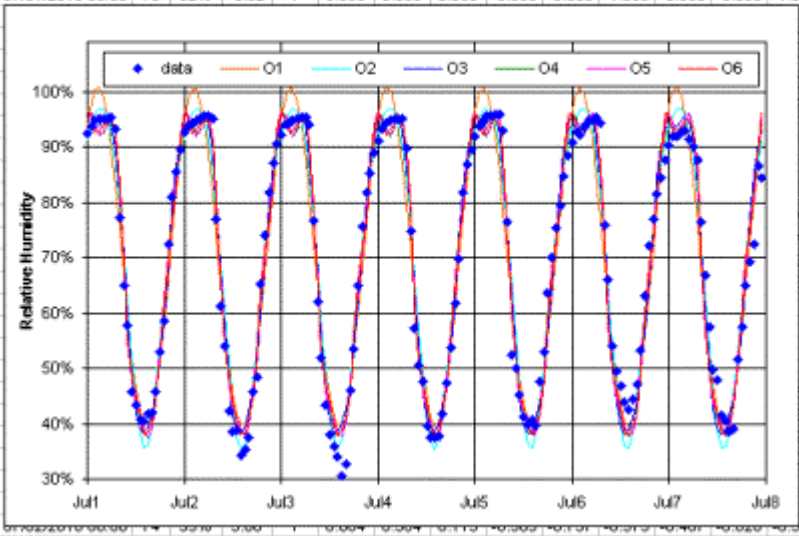

Standard Deviations from the Mean
6
4
2
0
-2
-4
-6
-40
-20
0
20
40
60
80
100
120
Temperature [°F]
R² = 0.9797


Cumulative Probability
0.999999
0.99999
0.9999
0.999
0.99
0.9
0.5
0.1
0.01
0.001
0.0001
0.00001
0.000001
-20
0
20
40
60
80
100
Temperature

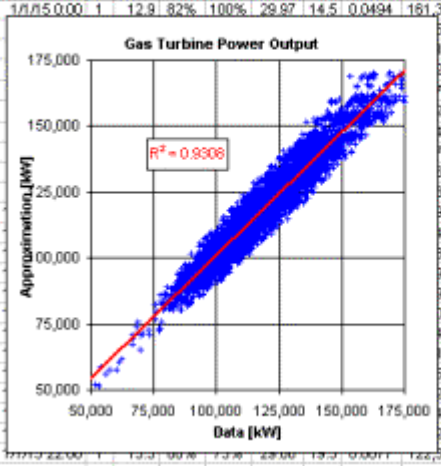
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	data			Legendre Polynomials, Le(Xi)					Yi*Le(Xi)					approximations	
2	time	flow	x	0	1	2	3	4	0	1	2	3	4	4Leg	4Poly
3	3.33	13.3	-1.00	1.00	-1.00	1.00	-1.00	1.00	13.30	-13.30	13.30	-13.30	13.30	7.28	6.44
4	3.35	12.8	-1.00	1.00	-1.00	0.99	-0.99	0.98	12.80	-12.77	12.72	-12.63	12.52	6.67	5.81
924	18.68	12.8	1.00	1.00	1.00	0.99	0.99	0.98	12.80	12.77	12.72	12.63	12.52	-1.65	-2.49
925	18.70	13.3	1.00	1.00	1.00	1.00	1.00	1.00	13.30	13.30	13.30	13.30	13.30	-1.19	-2.02
926				923	308	185	133	103	4E+05	579.44	-1E+05	-811.4	20915		
927				sumsq(Le(Xi))					sum(Yi*Le(Xi))						
928									391.12	1.88	-590.25	-6.11	202.17		
929									coefficients=sum(Yi*Le(Xi))/sumsq(Le(Xi))						



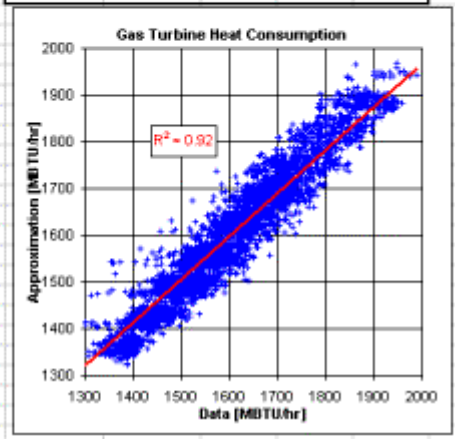
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	date:time	T	RH	x	cos0	cos1	cos2	cos3	cos4	cos5	cos6	sin1	sin2	sin3
2	07/01/2016 00:00	75	92%	-0.52	1	0.866	0.500	0.000	-0.500	-0.866	-1.000	-0.500	-0.866	-1.000
27	07/02/2016 01:00	73	94%	6.06	1	0.975	0.903	0.785	0.629	0.442	0.233	-0.221	-0.431	-0.619

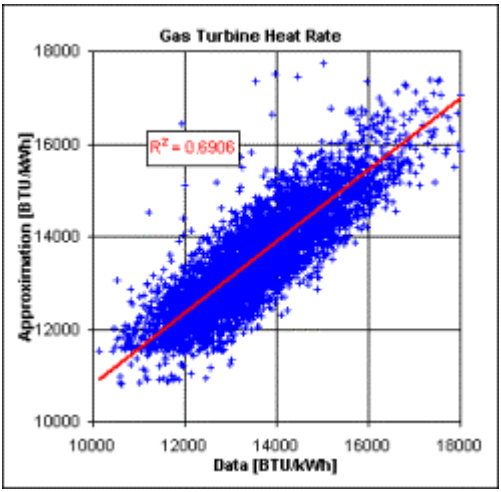


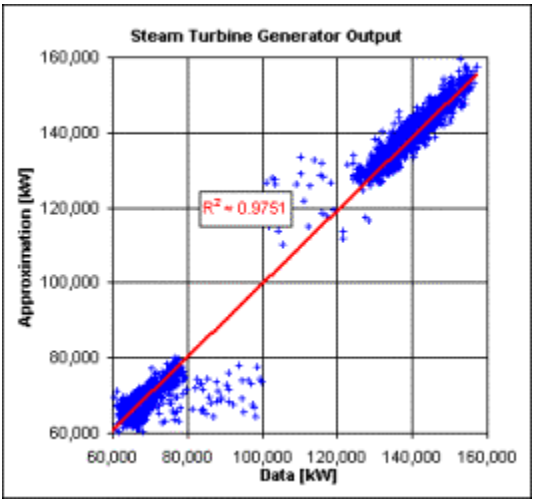
	A	B	C	D	E	F	G	H	I	J	K	L
1	date:time	GT	Tdb	RH	load	baro	CIT	humidity	power	GT1	LHV	approx
2		#	°F	%	%	in.Hg/A	°F	lb/lb	kW	lb/hr	BTU/lb	kW
3	1/1/15 0:00	1	12.9	82%	100%	29.97	14.5	0.0494	161,335	73,660	25,830	169,092
4									574	74,106	25,629	168,816
5									192	72,976	25,627	168,995
6									225	73,767	25,690	169,650
7									788	74,038	25,593	169,823
8									885	68,217	25,559	146,933
9									601	73,000	25,584	176,916
10									705	74,424	25,707	168,709
11									645	74,471	25,714	168,086
12									895	72,806	25,708	160,098
13									979	69,009	25,646	151,946
14									476	66,759	25,628	143,817
15									838	65,590	25,577	143,501
16									975	64,242	25,564	143,131
17									949	64,990	25,707	142,913
18									026	65,267	25,607	142,737
19									481	65,686	25,658	142,735
20									108	72,503	25,671	165,533
21									559	74,499	25,751	165,767
22									222	68,847	25,788	143,388
23									679	60,983	25,795	126,503
24									238	61,340	25,643	129,072
25									363	61,808	25,599	129,533
26	1/1/15 23:00	1	15.4	85%	75%	29.79	19.0	0.0653	125,789	61,792	25,526	129,628

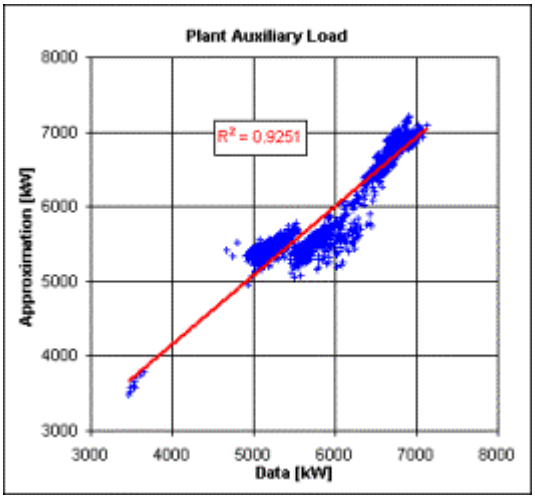


M	N	O	P	Q	R	S
heat	approx	LINEST(Power Output)				
MBTU/hr	MBTU/hr	758.2145352	-358.386771	1534.219	150262.2	-22030.57
1902.6	1851.9	311.9544883	6.362818035	458.5893	711.3123	13771.8
1899.3	1852.2	0.930832432	5215.646461	#N/A	#N/A	#N/A
1870.2	1852.0	16835.51132	5004	#N/A	#N/A	#N/A
1895.0	1849.8	1.8319E+12	1.36124E+11	#N/A	#N/A	#N/A
1894.8	1849.0	826.7688191	-367.184341	1021.697	150258.9	-6434.051
1743.5	1649.2	LINEST(Heat Input)				
1867.6	1917.0	-12.8047653	1.167420869	14.52678	1337.817	62.40589
1913.2	1852.3	2.66470271	0.054350936	3.917251	6.076001	117.6382
1915.0	1854.4	0.919976756	44.55184259	#N/A	#N/A	#N/A
1871.7	1789.2	14381.95787	5004	#N/A	#N/A	#N/A
1769.8	1724.4	114185075.7	9932272.858	#N/A	#N/A	#N/A
1710.9	1658.9					
1677.6	1659.5					
1642.3	1660.4					
1670.7	1660.8					
1671.3	1661.1					
1685.4	1661.1					
1861.2	1860.8					
1918.4	1859.8					
1775.4	1658.6					
1573.1	1524.1					
1573.0	1522.1					
1582.2	1520.4					
1577.3	1519.9					
1586.3	1519.2					
1573.8	1585.0					
1576.2	1584.7					
1570.6	1584.4					
1747.1	1783.5					
1541.2	1585.3					
1670.7	1653.8					
1708.2	1722.1					
1723.9	1723.2					

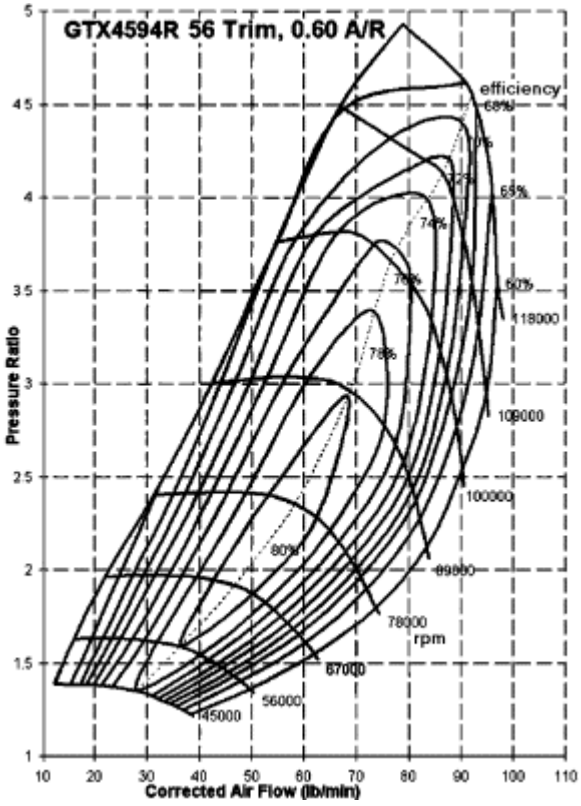


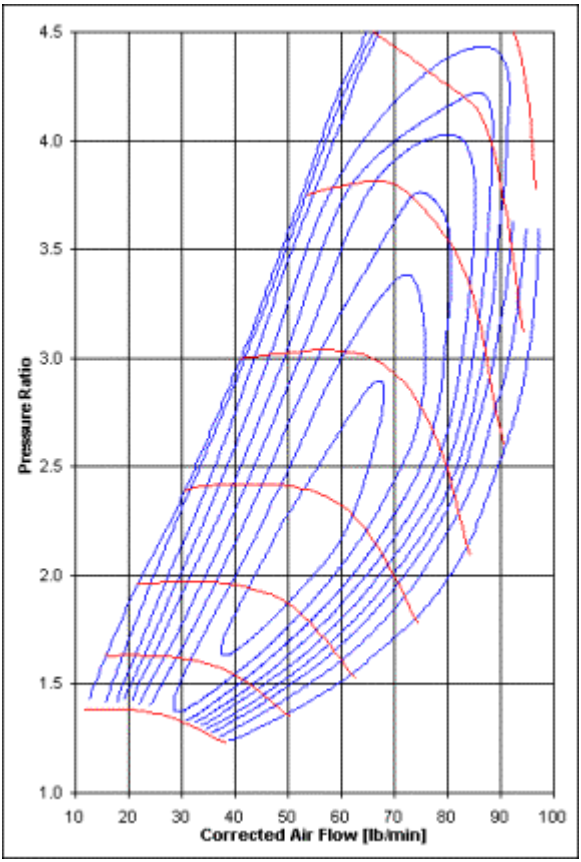

Gas Turbine Heat Rate
Approximation [BTU/kWh]
Data [BTU/kWh]
R² = 0.6906
18000
16000
14000
12000
10000
10000
12000
14000
16000
18000


Steam Turbine Generator Output
Approximation [kW]
Data [kW]
R² = 0.9751
160,000
140,000
120,000
100,000
80,000
60,000
60,000
80,000
100,000
120,000
140,000
160,000

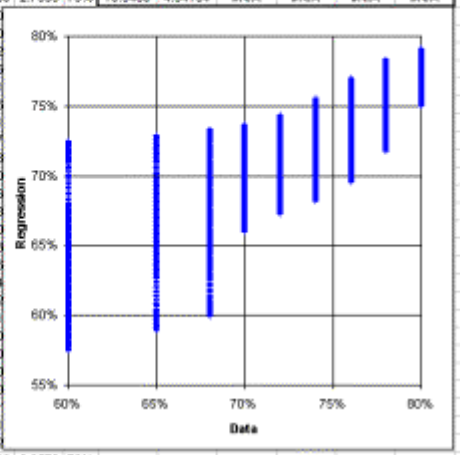


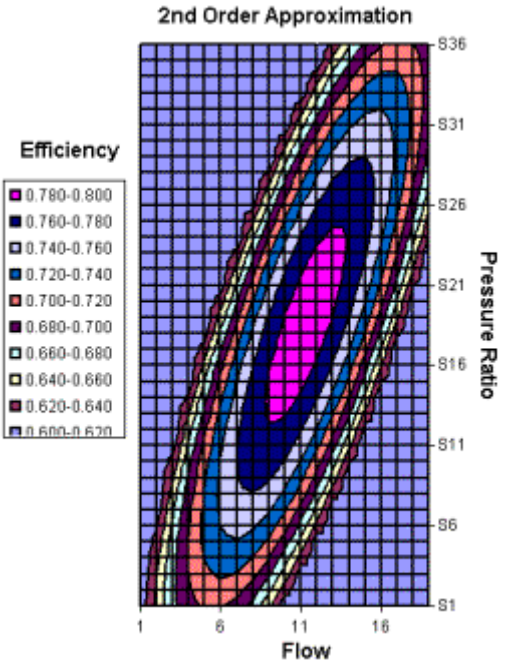
Chapter 11. Contours

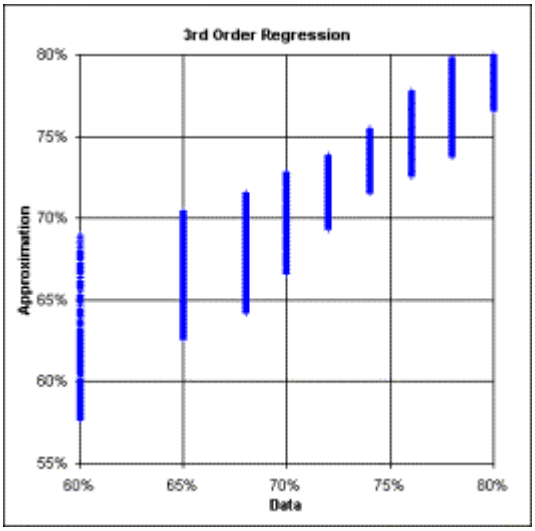

GTX4594R 56 Trim, 0.60 A/R
efficiency
68%
74%
78%
80%
118000
109000
100000
89000
78000
rpm
67000
56000
45000
Pressure Ratio
Corrected Air Flow (lb/min)
1
1.5
2
2.5
3
3.5
4
4.5
5
10
20
30
40
50
60
70
80
90
100
110

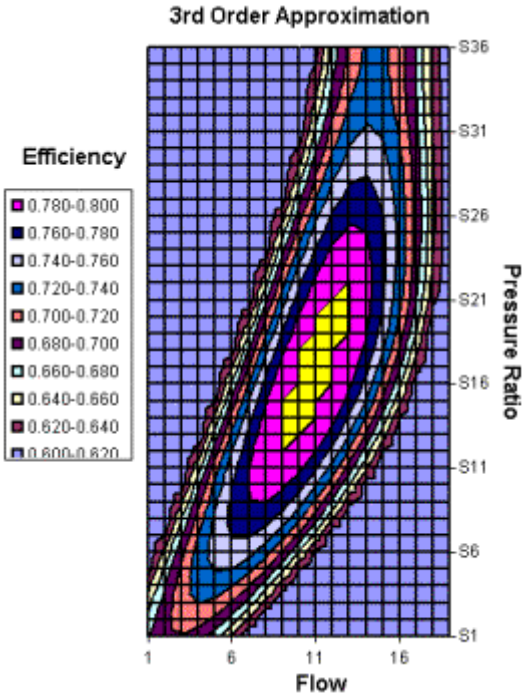

4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
Pressure Ratio
10
20
30
40
50
60
70
80
90
100
Corrected Air Flow [lb/min]

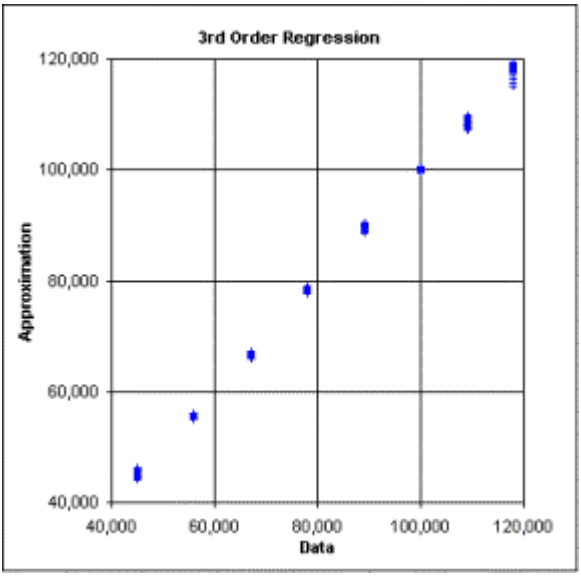
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	data			A matrix						2nd Order LINEST					
2	flow	pr	eff	1	p	ff	fp	pp	ff	-0.0817	0.00844	-0.0002	0.05446	0.00616	0.52662
3	37.5	1.63	80%	37.49	1.6335	1405.5	61.24	2.6684	75%	0.00086	6E-05	1.8E-06	0.00314	0.0001	0.00468
4	37.5	1.65	80%	37.49	1.6462	1405.5	61.714	2.7098	75%	0.71602	0.02821	#N/A	#N/A	#N/A	#N/A
5	37.5	1.66	80%	37.503	1.6588	1406.4	62.209	2.7516	75%	2750.85	5455	#N/A	#N/A	#N/A	#N/A
6	37.5	1.67	80%	37.536	1.6713	1409.1	62.736	2.7933	75%	10.9468	4.34154	#N/A	#N/A	#N/A	#N/A
7	37.6	1.68	80%	37.602	1.6835	1413.9	63.30								
8	37.7	1.70	80%	37.695	1.6953	1420.9	63.90								
9	37.8	1.71	80%	37.814	1.7065	1429.9	64.52								
10	38.0	1.72	80%	37.952	1.717	1440.4	65.16								
11	38.1	1.73	80%	38.106	1.7271	1452.1	65.81								
12	38.3	1.74	80%	38.27	1.7367	1464.6	66.46								
13	38.4	1.75	80%	38.441	1.746	1477.7	67.11								
14	38.6	1.76	80%	38.618	1.7551	1491.3	67.77								
15	38.8	1.76	80%	38.797	1.764	1505.2	68.43								
16	39.0	1.77	80%	38.978	1.7728	1519.3	69.10								
17	39.2	1.78	80%	39.162	1.7815	1533.6	69.76								
18	39.3	1.79	80%	39.345	1.7902	1548.1	70.43								
19	39.5	1.80	80%	39.53	1.7988	1562.6	71.10								
20	39.7	1.81	80%	39.715	1.8075	1577.3	71.76								
21	39.9	1.82	80%	39.899	1.8161	1592	72.46								
22	40.1	1.82	80%	40.084	1.8248	1606.7	73.14								
23	40.3	1.83	80%	40.267	1.8335	1621.5	73.82								
24	40.5	1.84	80%	40.45	1.8422	1636.2	74.51								
25	40.6	1.85	80%	40.633	1.8509	1651	75.20								
26	40.8	1.86	80%	40.815	1.8597	1665.9	75.90								
27	41.0	1.87	80%	40.997	1.8685	1680.7	76.60								
28	41.2	1.88	80%	41.178	1.8773	1695.6	77.30								
29	41.4	1.89	80%	41.36	1.8861	1710.6	78.01								
30	41.5	1.89	80%	41.542	1.8949	1725.7	78.71								
31	41.7	1.90	80%	41.724	1.9037	1740.9	79.42								

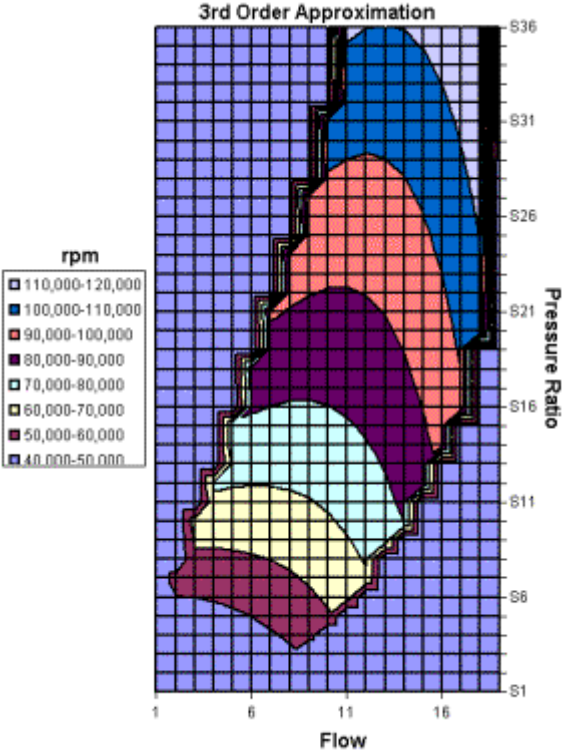


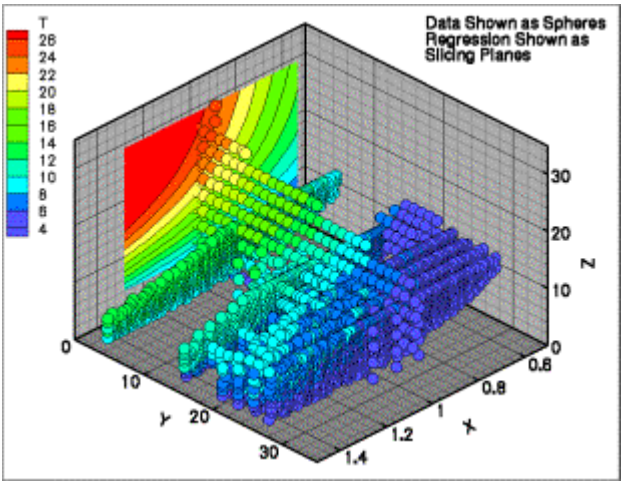



3rd Order Regression
80%
75%
70%
65%
60%
55%
Approximation
60%
65%
70%
75%
80%
Data

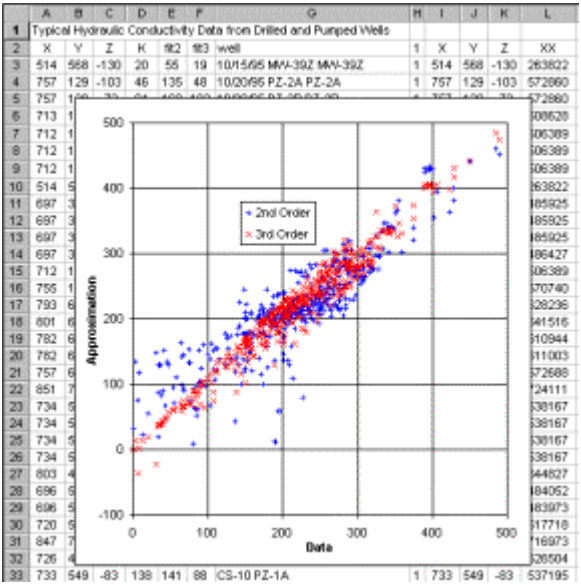

3rd Order Approximation
Efficiency
0.780-0.800
0.760-0.780
0.740-0.760
0.720-0.740
0.700-0.720
0.680-0.700
0.660-0.680
0.640-0.660
0.620-0.640
S36
S31
S26
S21
S16
S11
S6
S1
Pressure Ratio
1
6
11
16
Flow

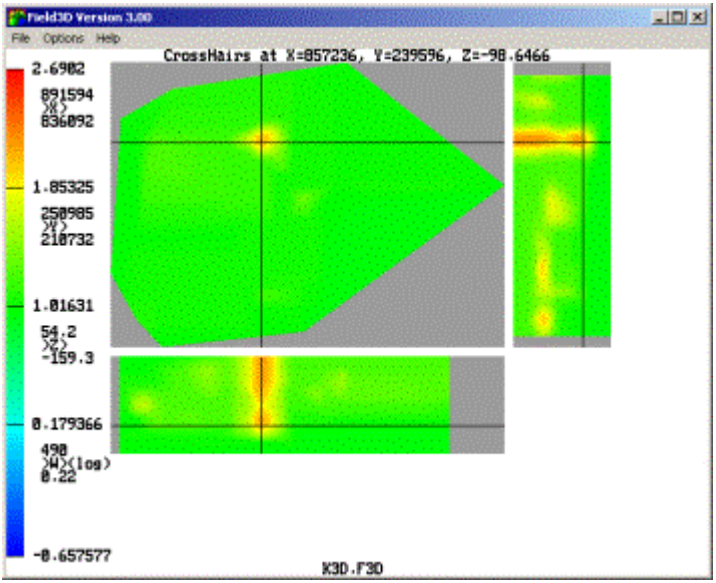

3rd Order Regression
120,000
100,000
80,000
60,000
40,000
Approximation
40,000
60,000
80,000
100,000
120,000
Data


3rd Order Approximation
rpm
110,000-120,000
100,000-110,000
90,000-100,000
80,000-90,000
70,000-80,000
60,000-70,000
50,000-60,000
40,000-50,000
S36
S31
S26
S21
S16
S11
S6
S1
Pressure Ratio
1
6
11
16
Flow

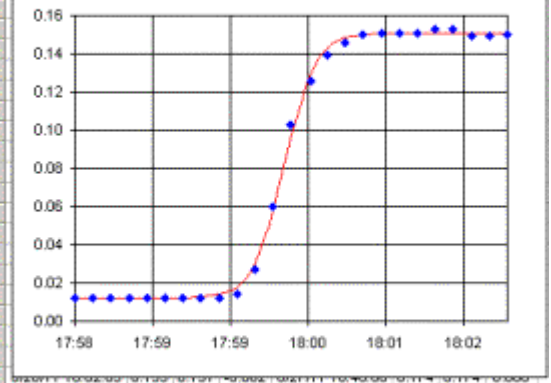


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Typical Cooling Tower Performance Data														
2	range=Thw-Tcw, approach=Tcw-Twb														
3	flow	range	Twb	app.	tf2	err	f	r	T	tf	fr	ft	rr	rt	tt
4	90%	5.0	84.9	3.3	3.4	0.1	0.9	5	84.9	0.81	4.5	76.41	25	424.5	7208
5	90													411	6757
6	90													410	6724
7	90													409.5	6708
8	100													425	7225
9	90													399.5	6384
10	90													394.5	6225
11	100													410	6724
12	90													389.5	6068
13	90													387.5	6006
14	110													425	7225
15	90													374.5	5610
16	90													375.5	5640
17	90													370	5476
18	100													399.5	6384
19	100													394.5	6225
20	110													400	6400
21	100													380	5776
22	110													410	6724
23	110													409.5	6708
24	90													354.5	5027
25	110													390	6084
26	90													364	5300
27	90													863	7448
28	100%	5.0	75.9	5.1	5.3	0.1	1	5	75.9	1	5	75.9	25	379.5	5761


Typical Hydraulic Conductivity Data from Drilled and Pumped Wells
2nd Order
3rd Order
Approximation
Data
500
400
300
200
100
0
-100
0
100
200
300
400
500

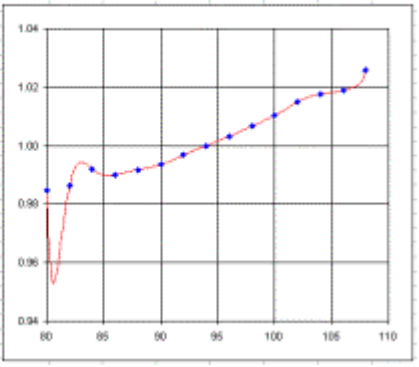

Field3D Version 3.00
File Options Help
CrossHairs at X=857236, Y=239596, Z=-98.6466
2.6902
691594
636092
1.85325
250985
210732
1.01631
54.2
-159.3
0.179366
498
0.22
-0.657577
K3D.F3D

	A	B	C	D	E	F	G	H
1	Typical Plateau Data							
2	date:time	volts	approx	error	date:time	volts	approx	error
3	8/26/11 17:58:33	0.012	0.012	0.000	8/27/11 16:42:30	0.027	0.027	0.000
4	8/26/11 17:58:43	0.012	0.012	0.000	8/27/11 16:42:40	0.027	0.027	0.000
25	8/26/11 18:02:13	0.149	0.151	0.002	8/27/11 16:46:10	0.174	0.174	0.000
26	8/26/11 18:02:23	0.149	0.151	0.002	8/27/11 16:46:20	0.173	0.174	0.001
27	8/26/11 18:02:33	0.150	0.151	0.001	8/27/11 16:46:30	0.175	0.174	-0.001
29	8/26/11 18:00:29	5026	sumsq	1.388	8/27/11 16:44:30	5816	sumsq	0.044

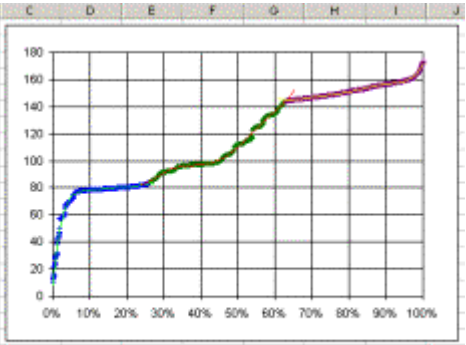


Lagrange Interpolation

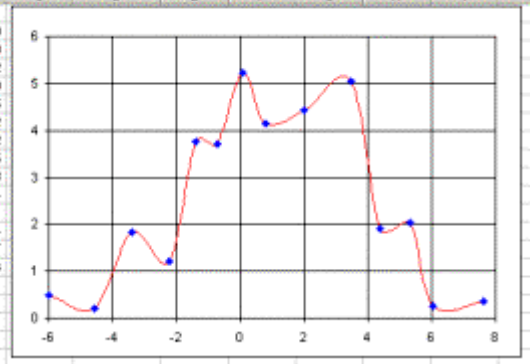
	data		interpolation	
	x	y	x	y
4	80	0.9847	80.0	0.9847
5	82	0.9895	80.1	0.9729
6	84	0.9920	80.2	0.9644
7	86	0.9900	80.3	0.9588
8	88	0.9918	80.4	0.9550
9	90	0.9936	80.5	0.9532
10	92	0.9967	80.6	0.9528
11	94	1.0000	80.7	0.9535
12	96	1.0033	80.8	0.9551
13	98	1.0057	80.9	0.9572
14	100	1.0104	81.0	0.9598
15	102	1.0149	81.1	0.9627
16	104	1.0177	81.2	0.9657
17	106	1.0190	81.3	0.9688
18	108	1.0257	81.4	0.9718
19			81.5	0.9747
20			81.6	0.9775
21			81.7	0.9801
22			81.8	0.9824
23			81.9	0.9845
24			82.0	0.9865
25			82.1	0.9881
26			82.2	0.9896
27			82.3	0.9908
28			82.4	0.9918



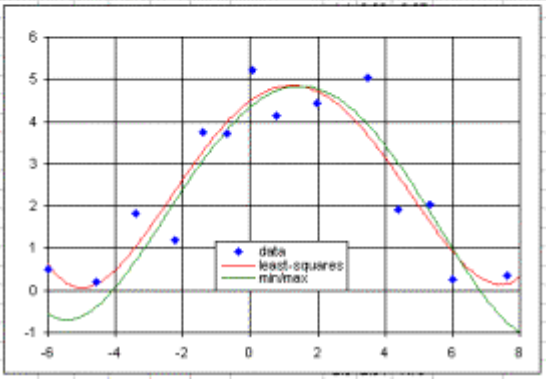
	A	B
1	x	y
2	0.0003	9.88
3	0.0006	12.21
4	0.0009	12.30
5	0.0012	12.93
6	0.0016	13.47
7	0.0019	13.70
8	0.0022	14.23
9	0.0025	15.16
10	0.0028	15.25
11	0.0031	16.33
12	0.0034	17.08
13	0.0037	21.94
14	0.0041	21.95
15	0.0044	22.46
16	0.0047	22.92
17	0.0050	23.79
18	0.0053	24.79
19	0.0056	27.01
20	0.0059	27.03
21	0.0062	27.56
22	0.0065	28.21
23	0.0069	28.53



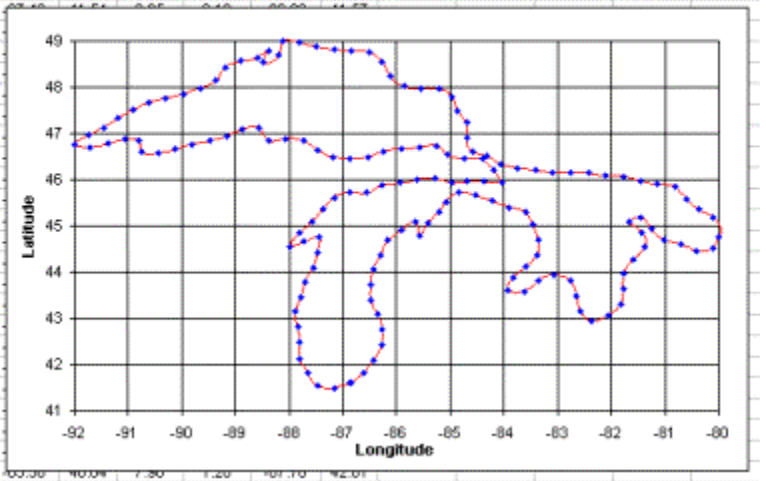
	A	B
1	x	y
2	-5.99	0.49
3	-4.56	0.19
4	-3.40	1.82
5	-2.24	1.20
6	-1.41	3.75
7	-0.70	3.72
8	0.07	5.22
9	0.79	4.15
10	1.99	4.43
11	3.49	5.04
12	4.39	1.91
13	5.32	2.04
14	6.02	0.27
15	7.61	0.36
16		
17		
18		
19		
20		

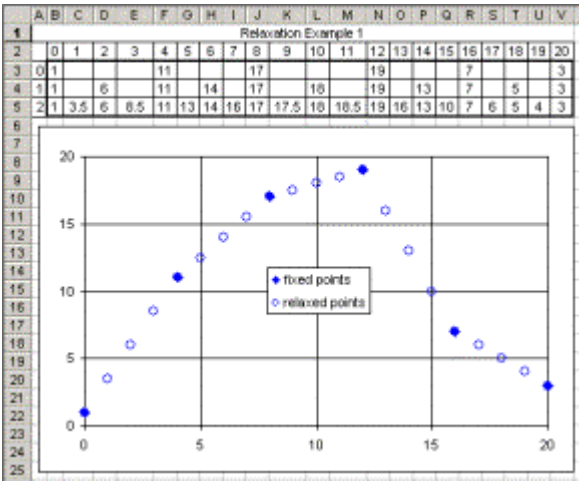


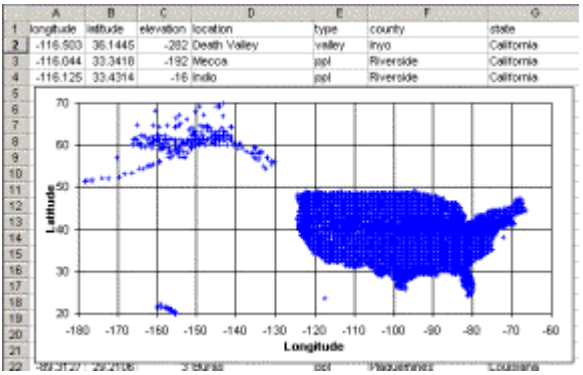
	A	B	C	D	E	F	G	H	I	J	K	L
1	data		least sq.		min/max		4th order	approximations				
2	x	y	fit	err	fit	err	least-squares	x	sq	min		
3	-5.99	0.49	0.61	0.12	-0.56	1.05	4.491900028	-6.0	0.62	-0.55		
4	-4.56	0.19	0.15	0.04	-0.38	0.58	0.580399094	-5.9	0.51	-0.60		
5	-3.40	1.62	1.04	0.76	0.71	1.11	-0.21769863	-5.8	0.41	-0.64		
6	-2.24	1.20	2.36	1.16	2.10	0.90	-0.01543159	-5.7	0.32	-0.67		
7	-1.41	3.75	3.30	0.45	3.06	0.67	0.003181404	-5.6	0.25	-0.69		
8	-0.70	3.72	3.98	0.26	3.79	0.07	4th order	-5.5	0.19	-0.70		
9	0.07	5.22	4.53	0.69	4.39	0.83	min/max	-5.4	0.14	-0.70		
10	0.79	4.15	4.81	0.66	4.73	0.58	4.345230041	-5.3	0.10	-0.69		
11	1.99	4.43	4.71	0.28	4.76	0.35	0.655767257	-5.2	0.08	-0.67		
12	3.49	5.04	3.68	1.36	3.94	1.10	-0.19945184	-5.1	0.06	-0.65		
13	4.39	1.91	2.72	0.81	3.02	1.12	-0.01463946	-5.0	0.06	-0.62		
14	5.32	2.04	1.64	0.40	1.88	0.16	0.002358918	-4.9	0.07	-0.57		
15	6.02	0.27	0.90	0.63	0.97	0.70		-4.8	0.08	-0.53		
16	7.61	0.36	0.16	0.20	-0.76	1.12		-4.7	0.11	-0.47		
17			max=	1.36	>	1.12		-4.6	0.14	-0.41		
18			sumsq=	6.32	<	9.31		-4.5	0.18	-0.34		

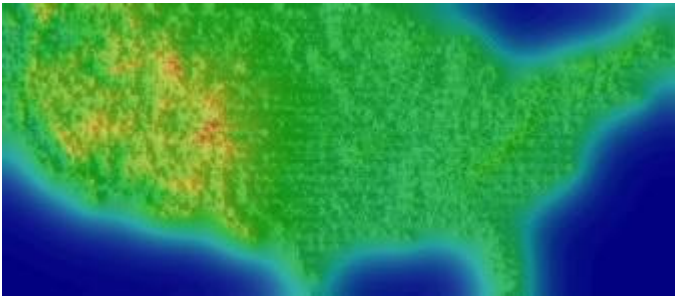


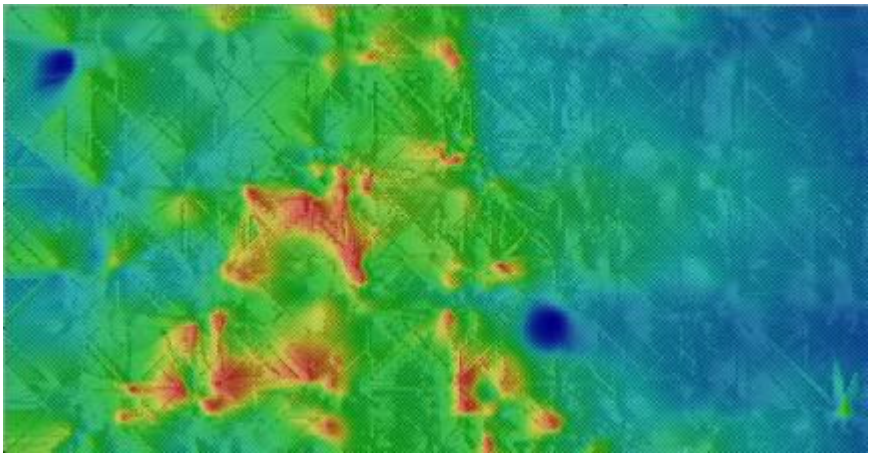
	A	B	C	D	E	F	G	H	I	J	K
1	longitude	latitude	distance	distance	longitude	latitude					
2	-86.85	41.61	0	0.00	-86.85	41.61					
3	-87.16	41.50	0.33	0.05	-86.89	41.59					

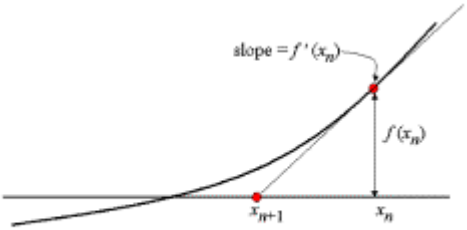



Relaxation Example 1
fixed points
relaxed points


longitude
latitude
elevation
location
type
county
state
-116.503
36.1445
-282
Death Valley
valley
Inyo
California
-116.044
33.3418
-192
Mecca
ppl
Riverside
California
-116.125
33.4314
-16
Indio
ppl
Riverside
California
Latitude
Longitude

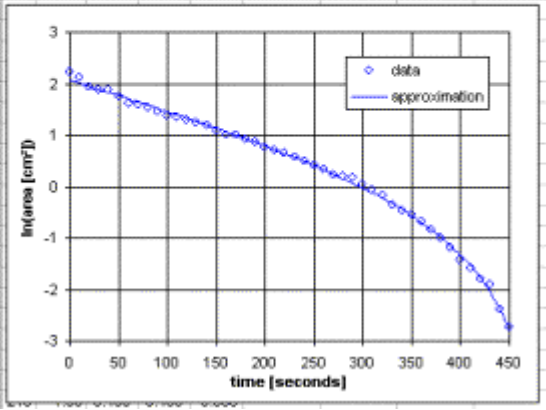


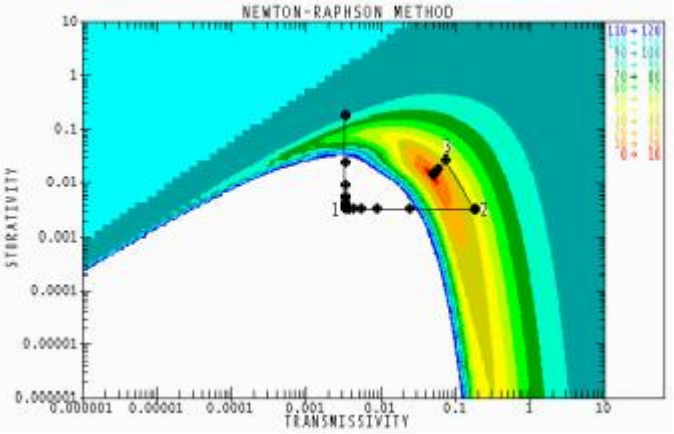


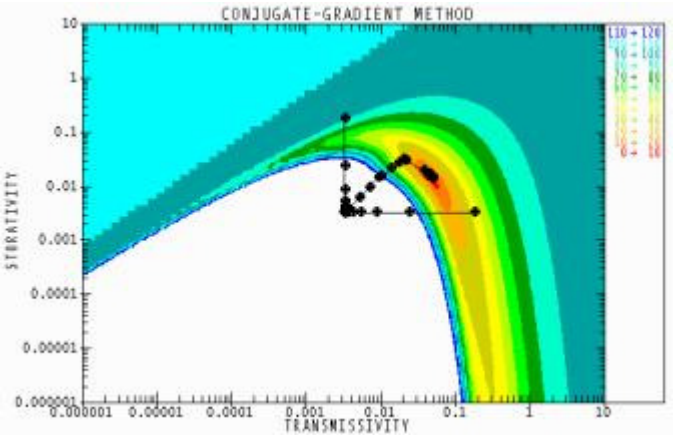


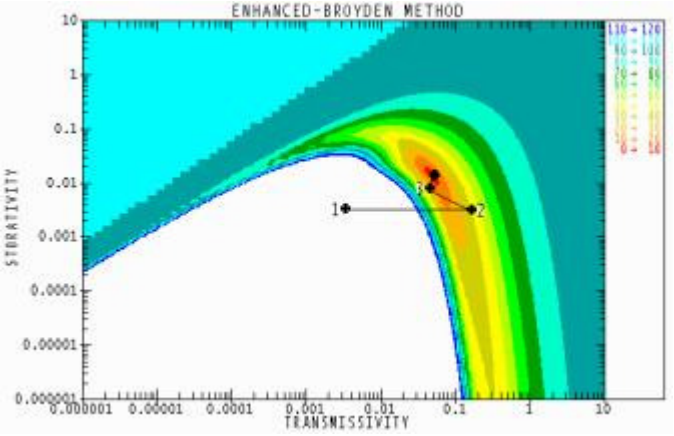
$$x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$$

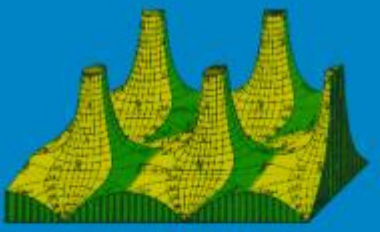
	A	B	C	D	E	F	G	H	I
1	approximating: ln(A)=C0*(C1-t)*(C2-t)/(C3-t)								
2	subject to: C0>0,C1<C3<C2					C0...C3			
3	0	9.24	2.224	2.062	-0.161	0.005068			
4	10	8.43	2.132	2.003	-0.129	296.7227			
5	20	6.98	1.943	1.944	0.001	722.8769			
6	30	6.51	1.873	1.884	0.011	527.0832			
7	40	6.52	1.875	1.824	-0.051				






CONJUGATE-GRADIENT METHOD
STORATIVITY
TRANSMISSIVITY
10
1
0.1
0.01
0.001
0.0001
0.00001
0.000001
0.000001
0.00001
0.0001
0.001
0.01
0.1
1
10


ENHANCED-BROYDEN METHOD
STORATIVITY
TRANSMISSIVITY
1
2
10
1
0.1
0.01
0.001
0.0001
0.00001
0.000001
0.000001
0.00001
0.0001
0.001
0.01
0.1
1
10



HANDBOOK OF MATHEMATICAL FUNCTIONS

with Formulas, Graphs, and Mathematical Tables

Edited by Milton Abramowitz and Irene A. Stegun