Numerical Methods for Solving

Differential Equations

...just the color figures for those who got the B&W printed book...

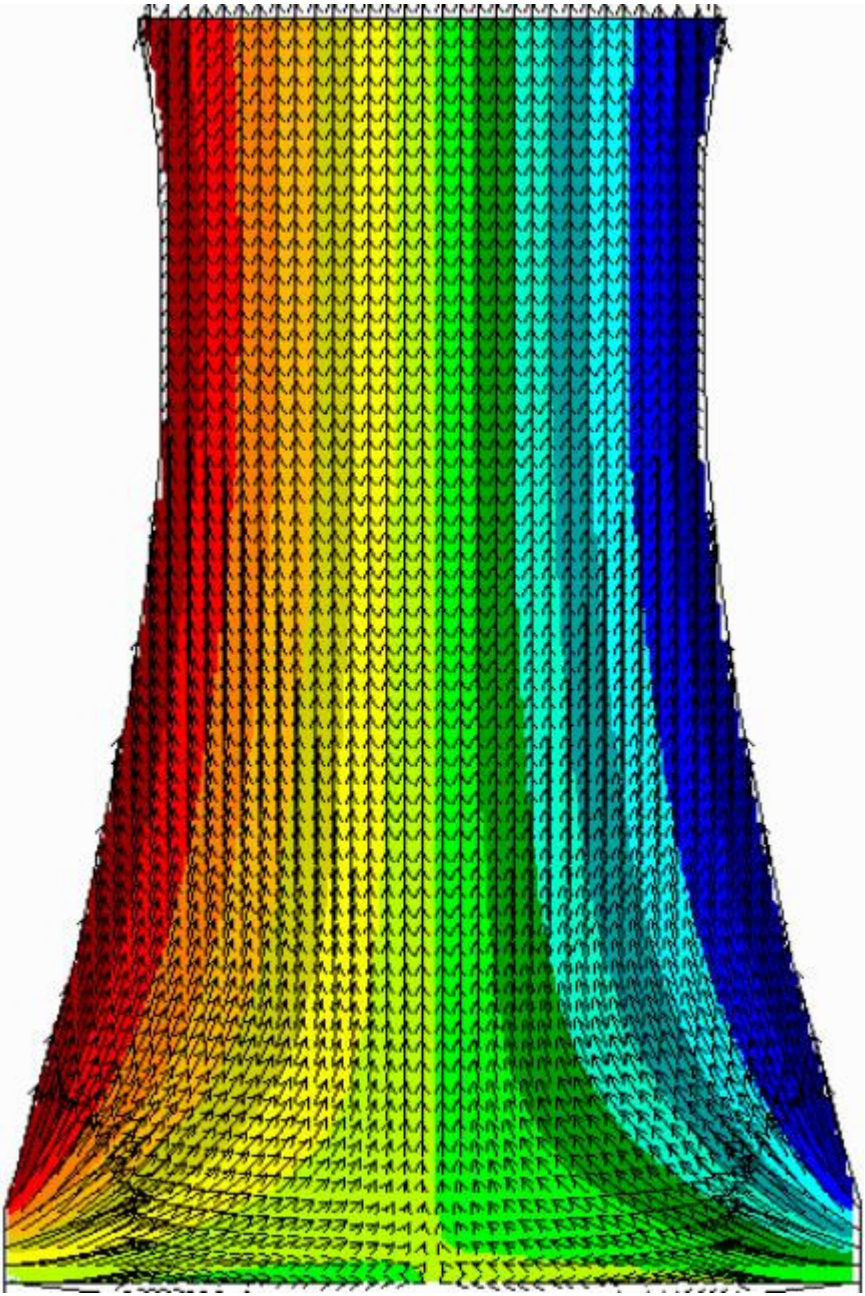
D. James Benton

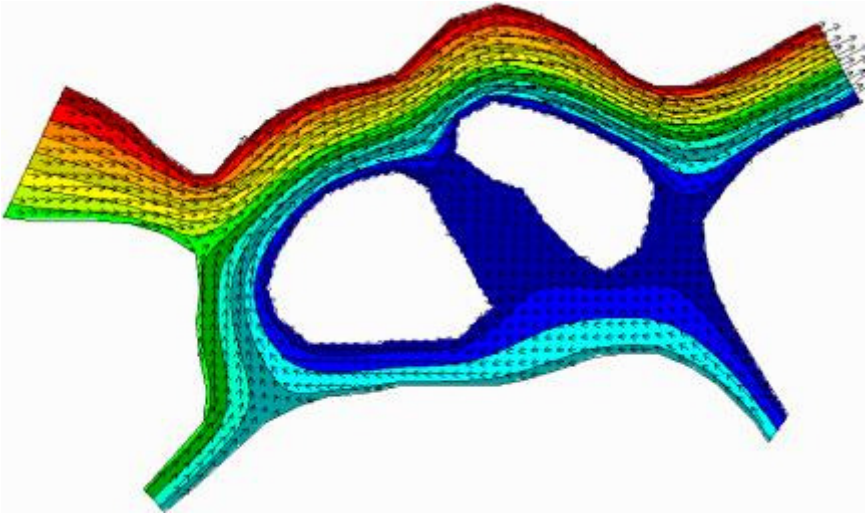
Foreword

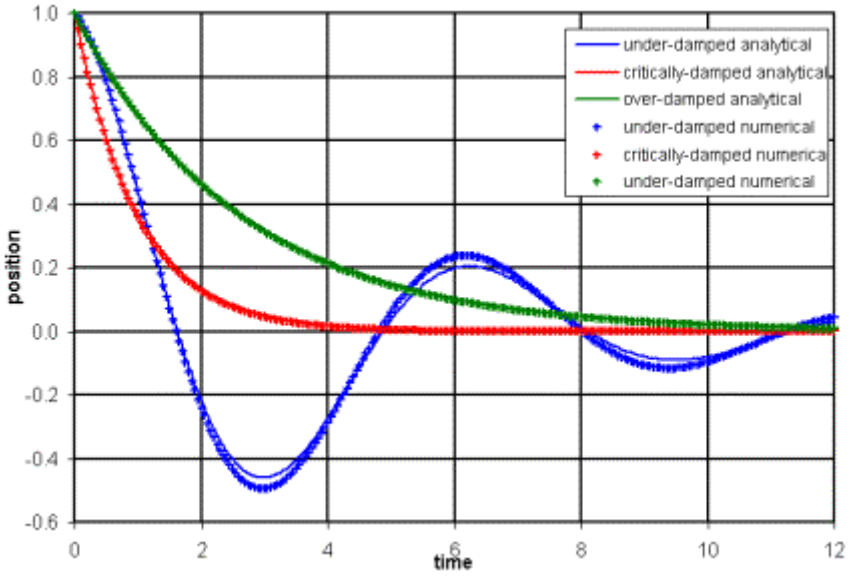
If you've struggled to understand the finite element method, then you must read this book. I don't cover a lot of theory in this text. It's mostly a compilation of examples. The one theoretical aspect of numerical methods for solving differential equations that I will present is the finite element method. This powerful technique is more often than not buried under a mountain of esoteric details that make it inaccessible to most students of applied mathematics. I aim to reveal, not obfuscate. I trust you will find this book helpful in that respect.

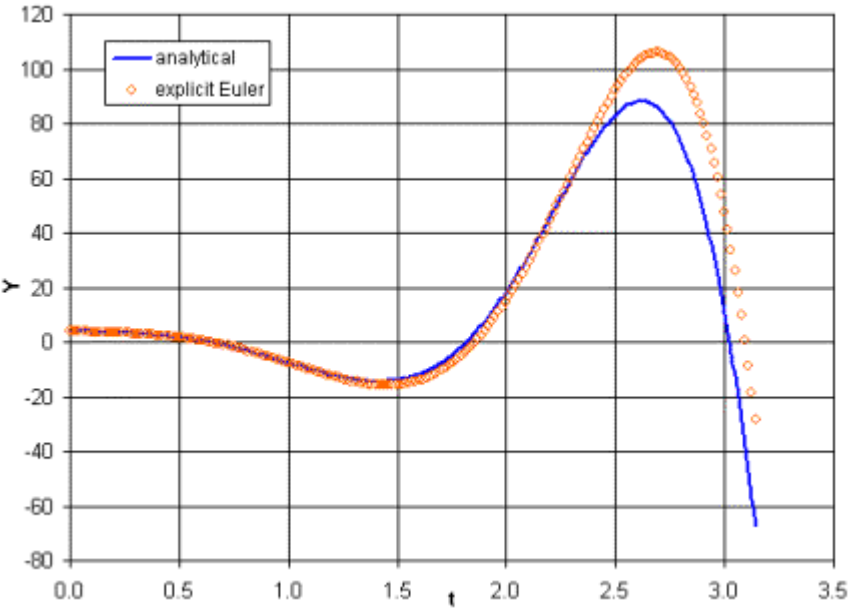
Understanding applied mathematics results in a richer appreciation of the world and how it works. Those who see only disconnected objects and forces miss the elegance with which such things can be described in the versatile language we call calculus. Differential equations describe the way objects and forces interact. While there are many analytical techniques for solving such problems, this book deals with numerical methods. You must understand some of the former in order to appreciate the latter. Rather than covering the minutia of every obscure method, this book will focus on what works consistently and efficiently. This is a compilation of examples, not a textbook on theory. I trust you will find it interesting and useful.

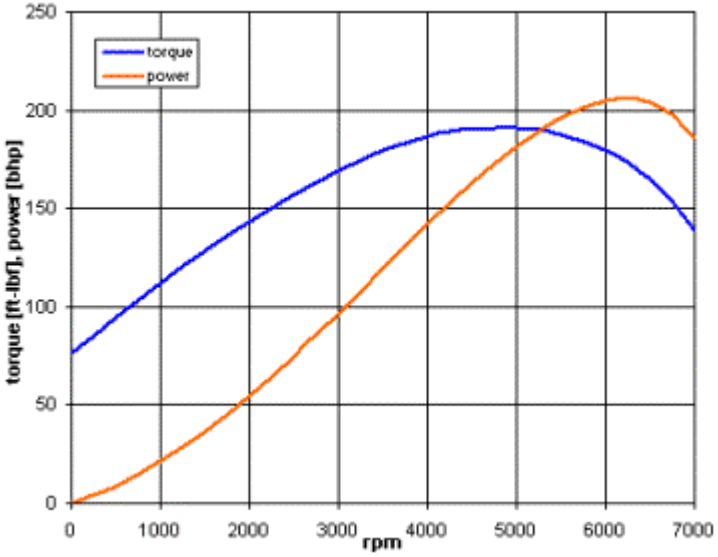
All of the examples contained in this book,
(as well as a lot of free programs) are available at…
http://www.dudleybenton.altervista.org/software/index.html

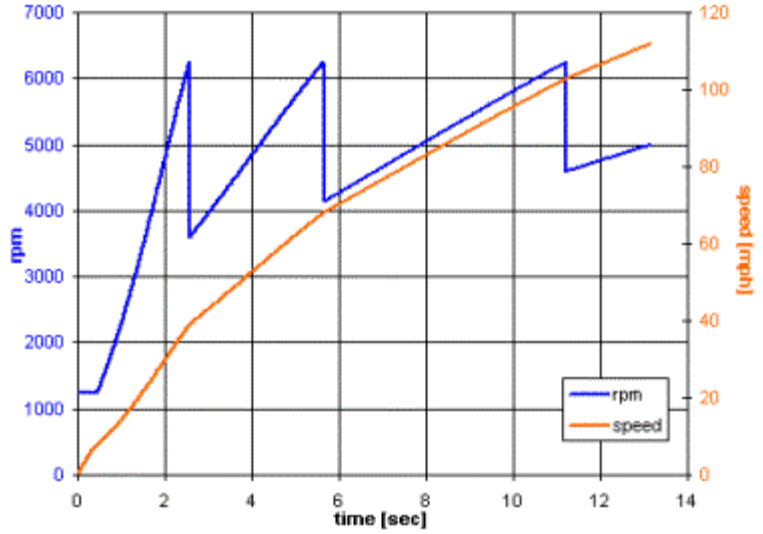





under-damped analytical
critically-damped analytical
over-damped analytical
under-damped numerical
critically-damped numerical
under-damped numerical
position
time
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
0
2
4
6
8
10
12


analytical
explicit Euler
Y
t
120
100
80
60
40
20
0
-20
-40
-60
-80
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5


torque
power
torque [ft-lbf], power [bhp]
rpm
0
50
100
150
200
250
0
1000
2000
3000
4000
5000
6000
7000


rpm
speed [mph]
time [sec]
rpm
speed
0
1000
2000
3000
4000
5000
6000
7000
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20
40
60
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100
120
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4
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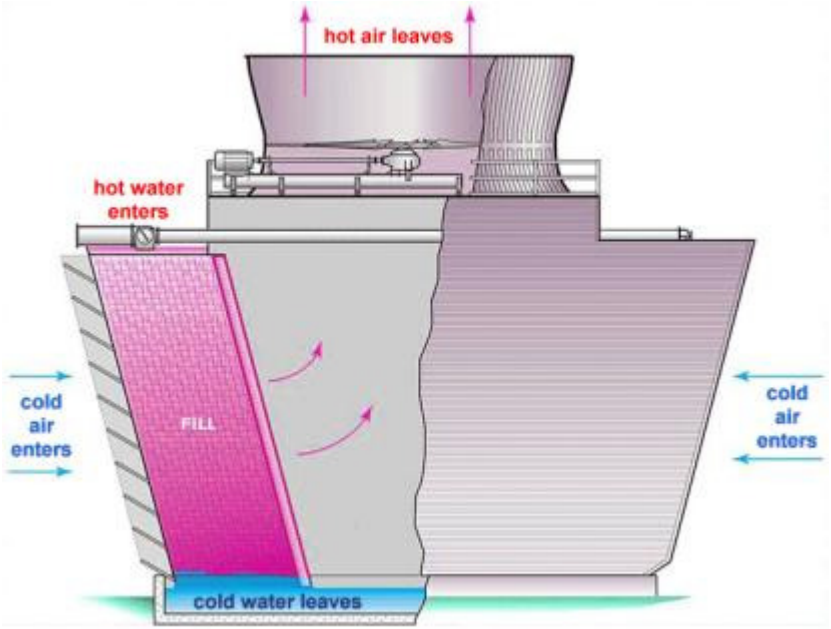
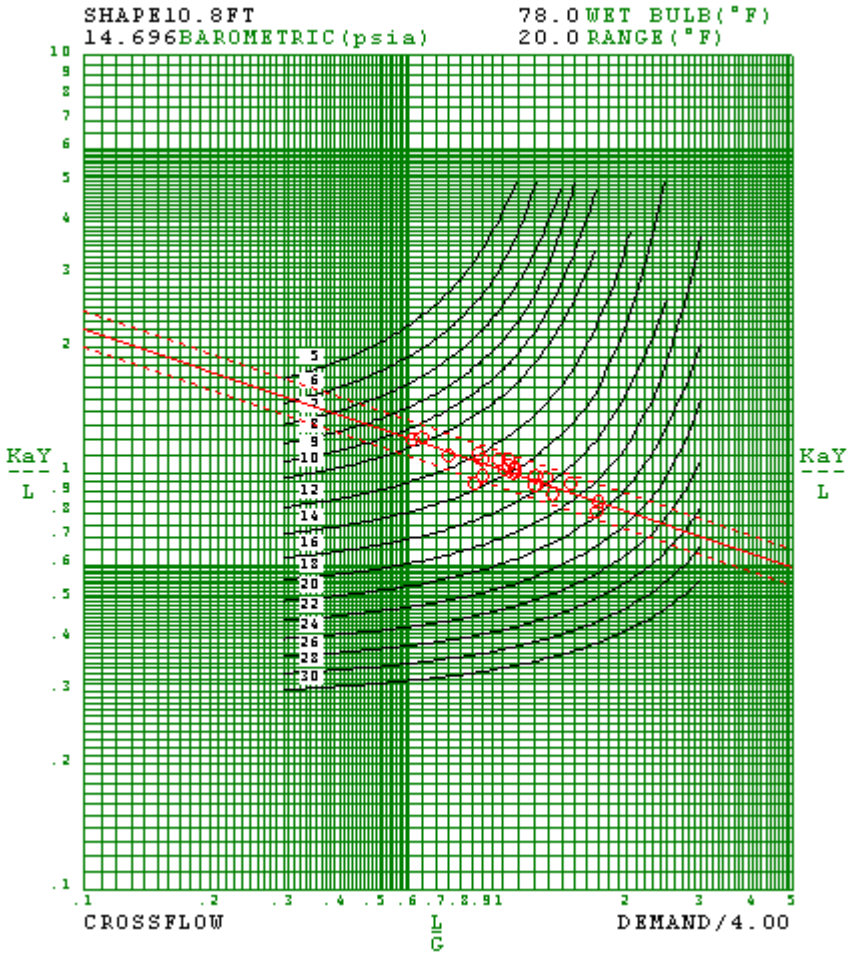
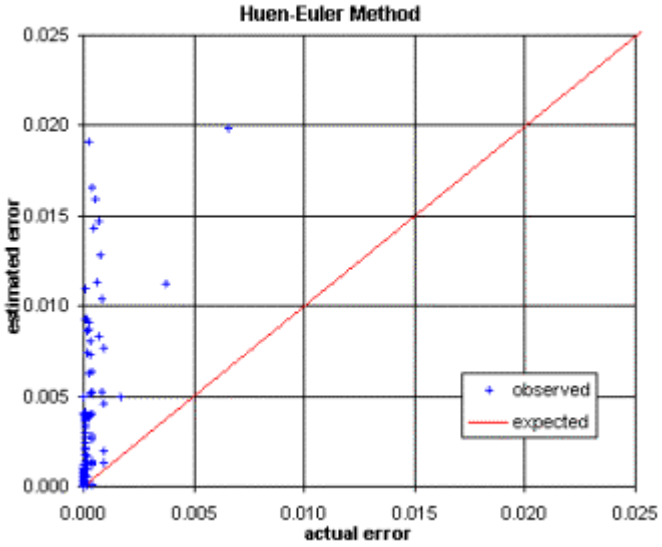
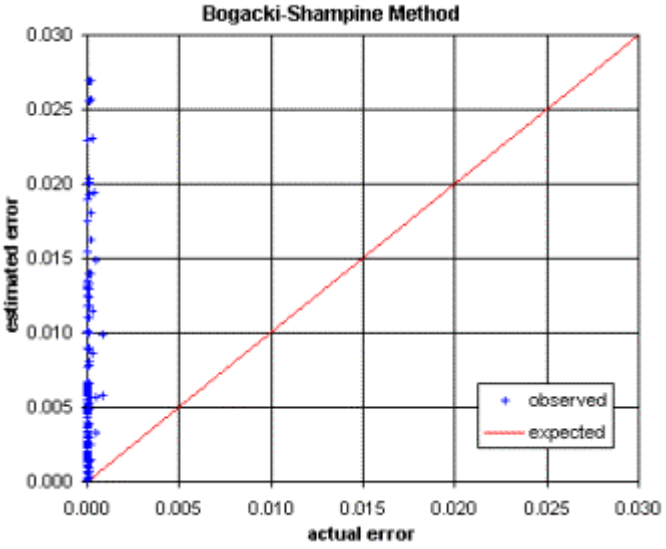
10
12
14

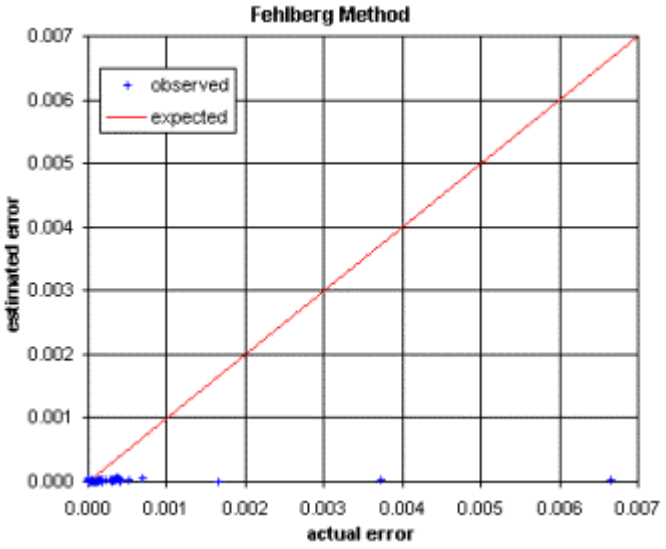


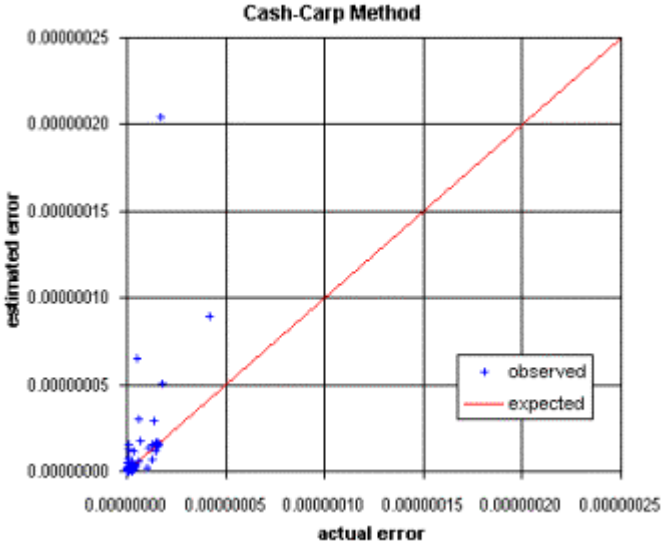
						Tw				
Ta						148	148	148	148	148
78	102	116	124	128	132	115	115	124	130	131
78	92	99	108	115	118	97	104	108	113	121
78	86	92	98	103	108	88	96	99	104	110
78	82	88	92	96	101	84	89	93	98	102
78	80	84	87	91	94	81	85	89	92	97
						11 approach				89
						Hw				
Ha						260.1	260.1	260.1	260.1	260.1
41.6	74.7	107.4	131.5	149.2	165.8	104.7	105.8	132.7	157.3	162.5
41.6	59.3	70.8	86.9	103.7	114.1	66.8	79.0	87.5	100.9	122.9
41.6	50.5	58.7	68.0	77.6	88.2	53.6	64.4	69.2	79.0	93.4
41.6	46.0	52.8	58.6	64.5	73.4	48.0	54.5	59.9	68.0	74.5
41.6	44.0	48.0	51.8	57.5	62.2	45.2	49.3	54.6	59.1	66.2

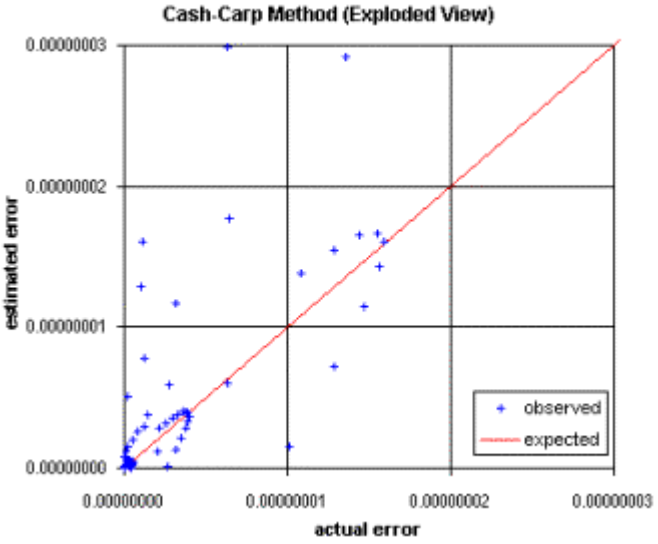

SHAPE10.8FT
14.696BAROMETRIC(psia)
78.0WET BULB(°F)
20.0RANGE(°F)
KaY / L
KaY / L
CROSSFLOW
L / G
DEMAND/4.00


Huen-Euler Method
0.025
0.020
0.015
0.010
0.005
0.000
estimated error
0.000
0.005
0.010
0.015
0.020
0.025
actual error
observed
expected

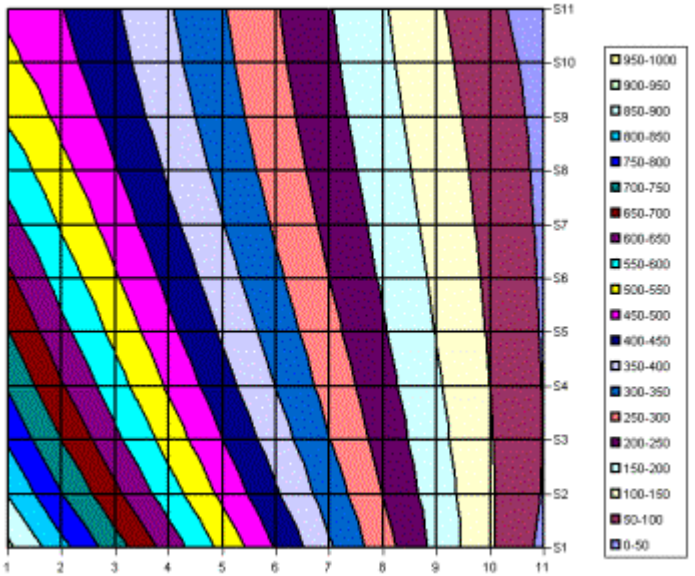

Bogacki-Shampine Method
0.030
0.025
0.020
0.015
0.010
0.005
0.000
estimated error
0.000
0.005
0.010
0.015
0.020
0.025
0.030
actual error
observed
expected


Fehlberg Method
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
estimated error
observed
expected
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
actual error


Cash-Carp Method
0.00000025
0.00000020
0.00000015
0.00000010
0.00000005
0.00000000
estimated error
observed
expected
0.00000000
0.00000005
0.00000010
0.00000015
0.00000020
0.00000025
actual error


Cash-Carp Method (Exploded View)
0.00000003
0.00000002
0.00000001
0.00000000
estimated error
0.00000000
0.00000001
0.00000002
0.00000003
actual error
observed
expected

	A	B	C	D	E	F	G	H	I	J	K	L	M
1						Temperatures							
2		1000	900	800	700	600	500	400	300	200	100	0	
3	1000	911	816	722	629	537	446	357	270	185	105	37	0
4	950	847	751	661	573	489	406	325	248	174	106	47	0
5	900	794	699	612	529	450	374	301	231	165	103	49	0
6	850	746	654	571	493	419	348	281	217	156	99	48	0
7	800	701	614	536	462	393	327	264	204	148	95	46	0
8	750	659	578	505	436	371	309	250	193	140	90	44	0
9	700	618	545	477	413	352	293	237	184	133	86	42	0
10	650	579	514	453	393	335	280	226	175	127	82	39	0
11	600	543	487	432	376	322	269	217	167	120	77	37	0
12	550	511	466	415	363	311	260	209	160	114	71	32	0
13	500	490	452	405	355	304	253	203	154	107	62	23	0
14		500	450	400	350	300	250	200	150	100	50	0	
15						Thermal Conductivities							
16		0.50	0.54	0.57	0.62	0.66	0.71	0.76	0.81	0.87	0.93	1.00	
17	0.50	0.53	0.57	0.61	0.65	0.69	0.73	0.78	0.83	0.88	0.93	0.98	1.00
18	0.52	0.56	0.59	0.63	0.67	0.71	0.75	0.80	0.84	0.89	0.93	0.97	1.00
19	0.54	0.58	0.62	0.65	0.69	0.73	0.77	0.81	0.85	0.89	0.93	0.97	1.00
20	0.56	0.60	0.64	0.67	0.71	0.75	0.79	0.82	0.86	0.90	0.93	0.97	1.00
21	0.57	0.62	0.65	0.69	0.73	0.76	0.80	0.83	0.87	0.90	0.94	0.97	1.00
22	0.59	0.63	0.67	0.70	0.74	0.77	0.81	0.84	0.87	0.91	0.94	0.97	1.00
23	0.62	0.65	0.69	0.72	0.75	0.78	0.82	0.85	0.88	0.91	0.94	0.97	1.00
24	0.64	0.67	0.70	0.73	0.76	0.79	0.82	0.86	0.89	0.92	0.95	0.97	1.00
25	0.66	0.69	0.71	0.74	0.77	0.80	0.83	0.86	0.89	0.92	0.95	0.98	1.00
26	0.68	0.70	0.72	0.75	0.78	0.81	0.84	0.87	0.90	0.92	0.95	0.98	1.00
27	0.71	0.71	0.73	0.76	0.78	0.81	0.84	0.87	0.90	0.93	0.96	0.98	1.00
28		0.71	0.73	0.76	0.79	0.81	0.84	0.87	0.90	0.93	0.97	1.00	


S11
S10
S9
S8
S7
S6
S5
S4
S3
S2
S1
1
2
3
4
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6
7
8
9
10
11
950-1000
900-950
850-900
800-850
750-800
700-750
650-700
600-650
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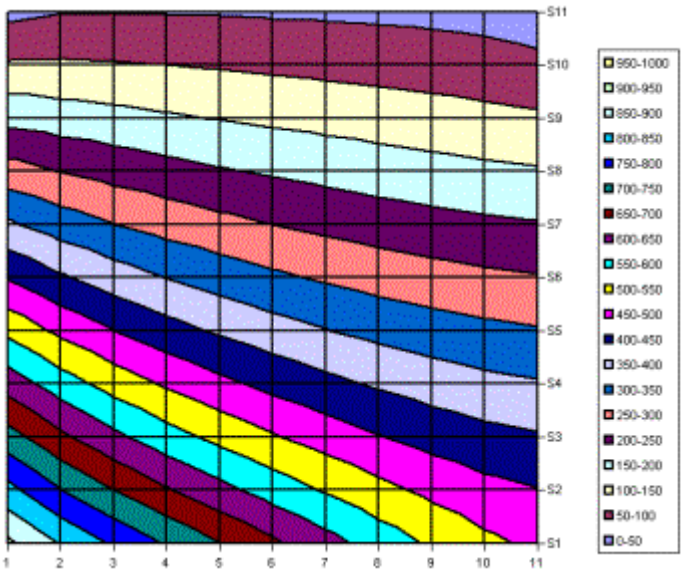
250-300
200-250
150-200
100-150
50-100
0-50


S11
S10
S9
S8
S7
S6
S5
S4
S3
S2
S1
1
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4
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6
7
8
9
10
11
950-1000
900-950
850-900
800-850
750-800
700-750
650-700
600-650
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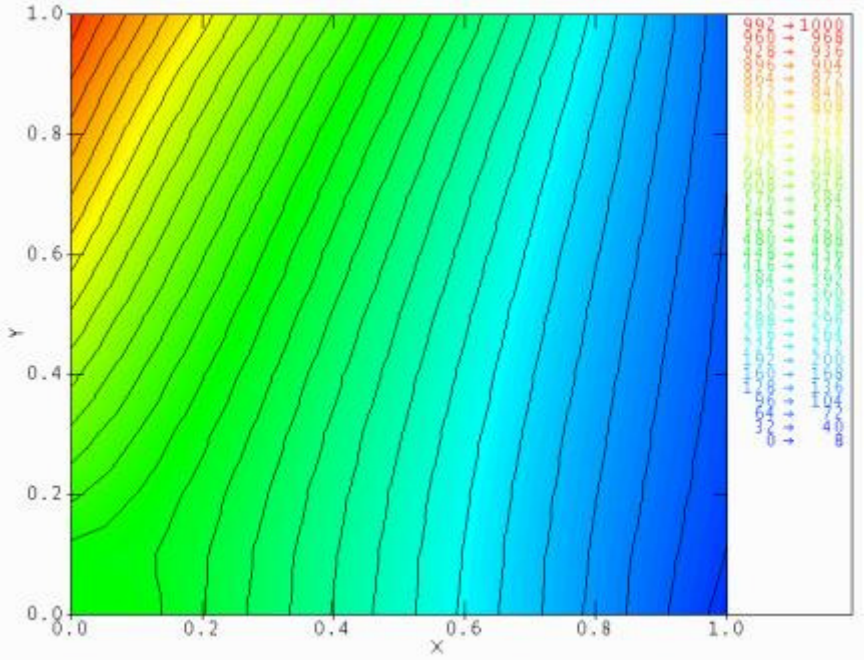
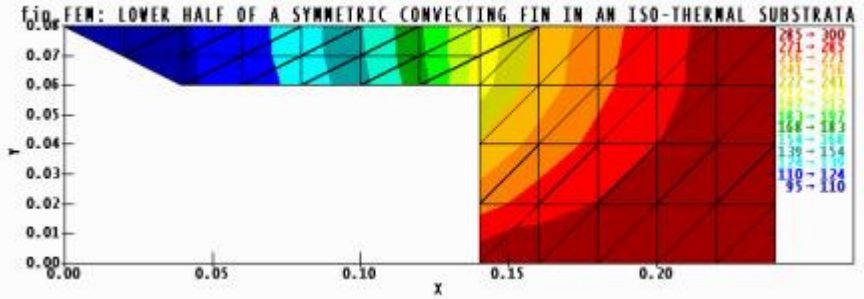
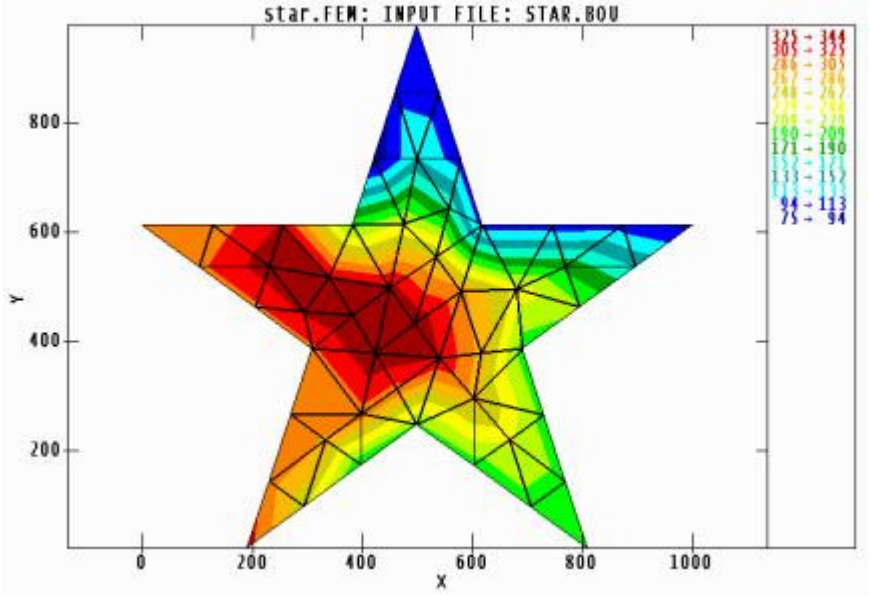
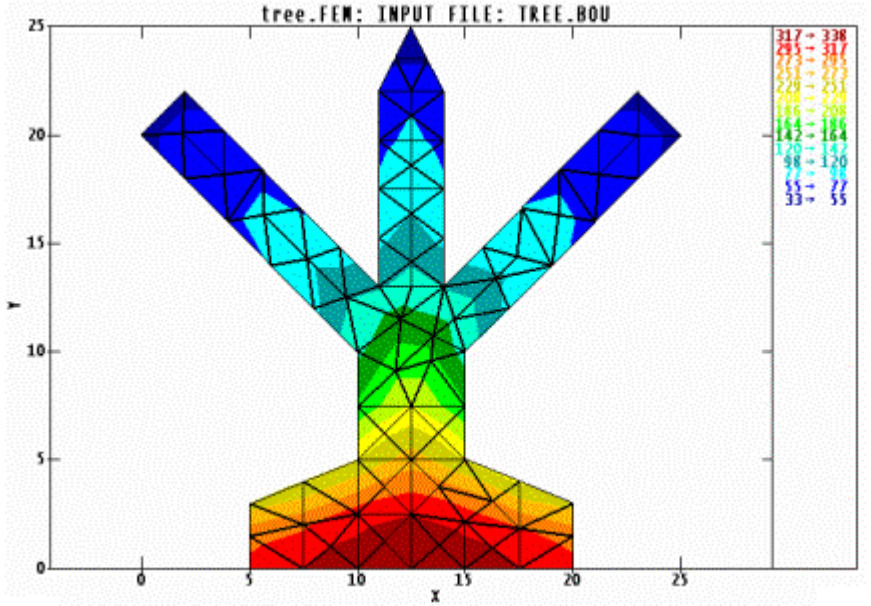
250-300
200-250
150-200
100-150
50-100
0-50

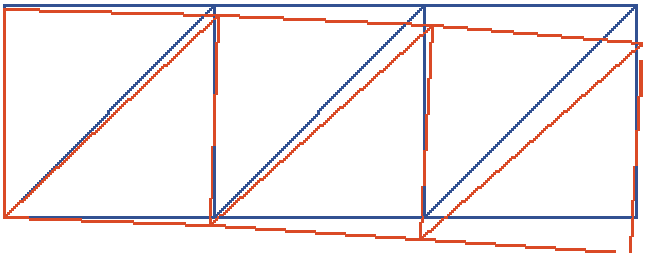


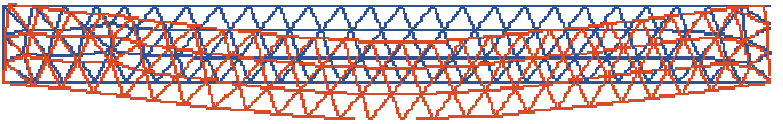
Crank-Nicholson Matrices								properties	
matrix A				matrix B				Δt	1.00
2.0	-0.5	0.0	0.0	0.0	0.5	0.0	0.0	Δx	1.00
-0.5	2.0	-0.5	0.0	0.5	0.0	0.5	0.0	ρ	1.00
0.0	-0.5	2.0	-0.5	0.0	0.5	0.0	0.5	C	1.00
0.0	0.0	-1.0	2.0	0.0	0.0	1.0	0.0	k	1.00
calculated parameter, $\beta = k\Delta t/(\rho C \Delta x^2)$									1.00

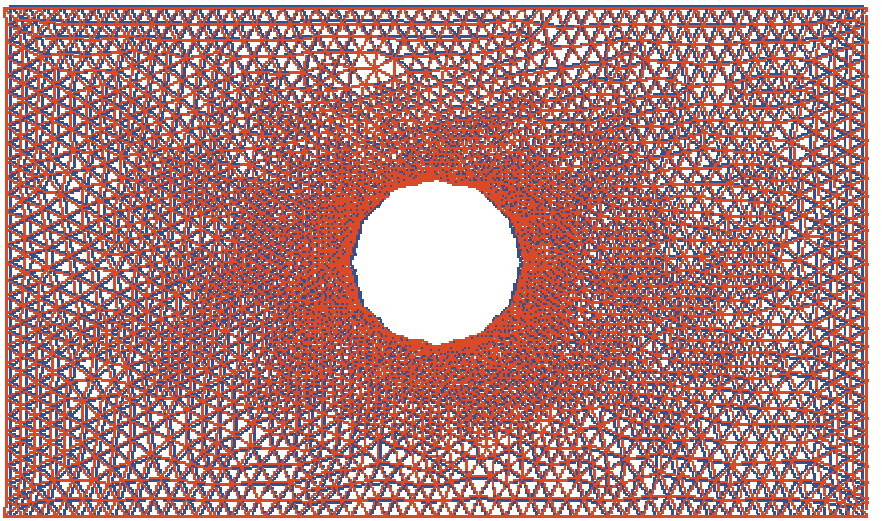


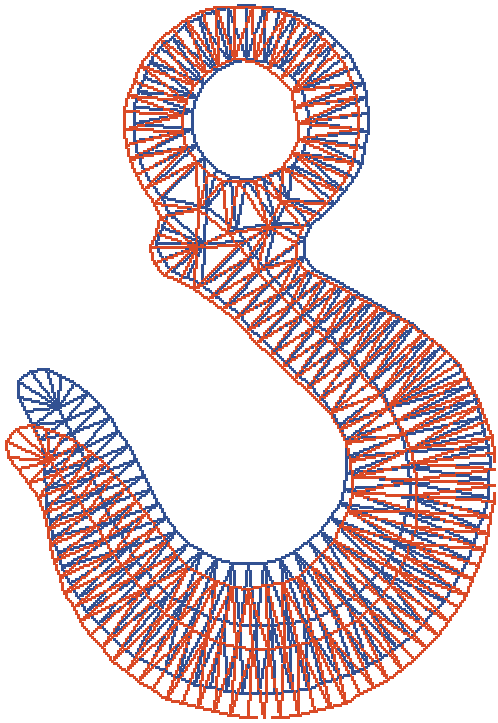

star.FEM: INPUT FILE: STAR.BOU
325 → 344
305 → 325
286 → 305
267 → 286
248 → 267
190 → 209
171 → 190
94 → 113
75 → 94
800
600
400
200
Y
0
200
400
600
800
1000
X

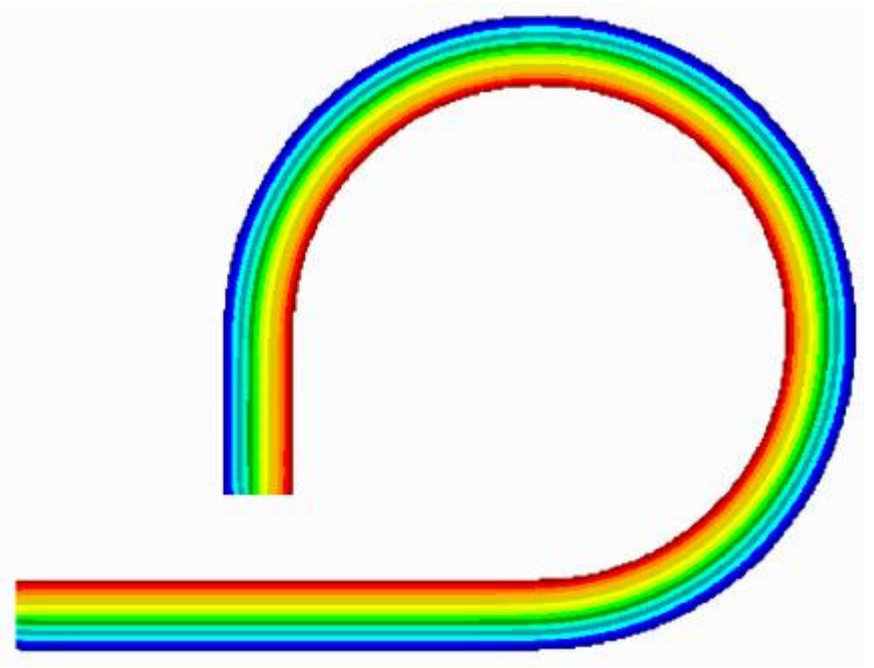

tree.FEM: INPUT FILE: TREE.BOU
317 → 338
295 → 317
273 → 295
251 → 273
229 → 251
207 → 229
185 → 208
164 → 186
142 → 164
120 → 142
98 → 120
77 → 98
55 → 77
33 → 55
25
20
15
10
5
0
Y
0
5
10
15
20
25
X

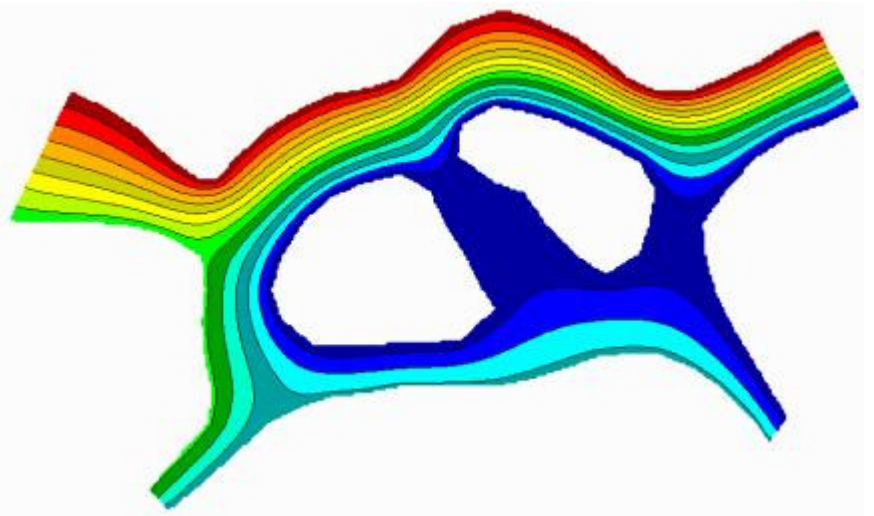


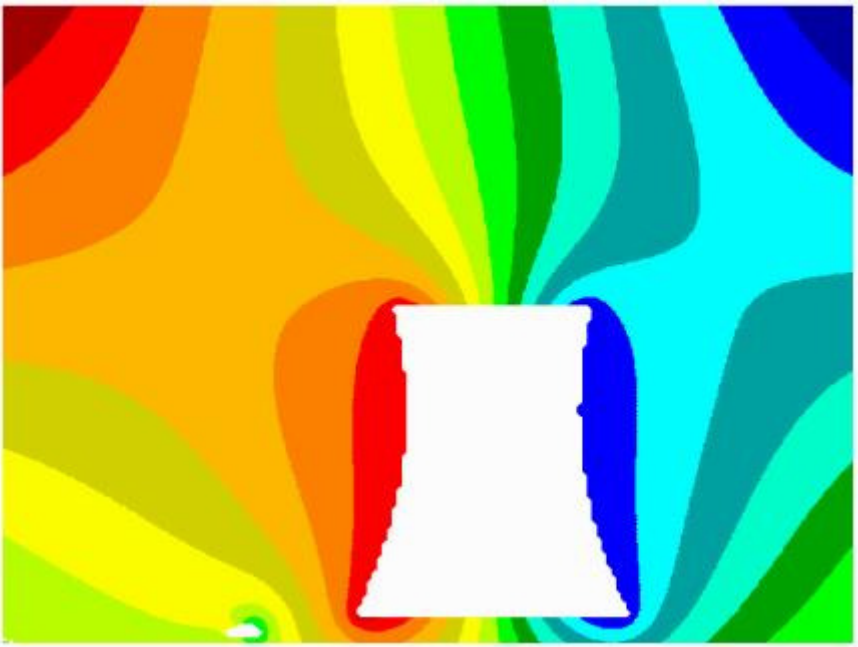


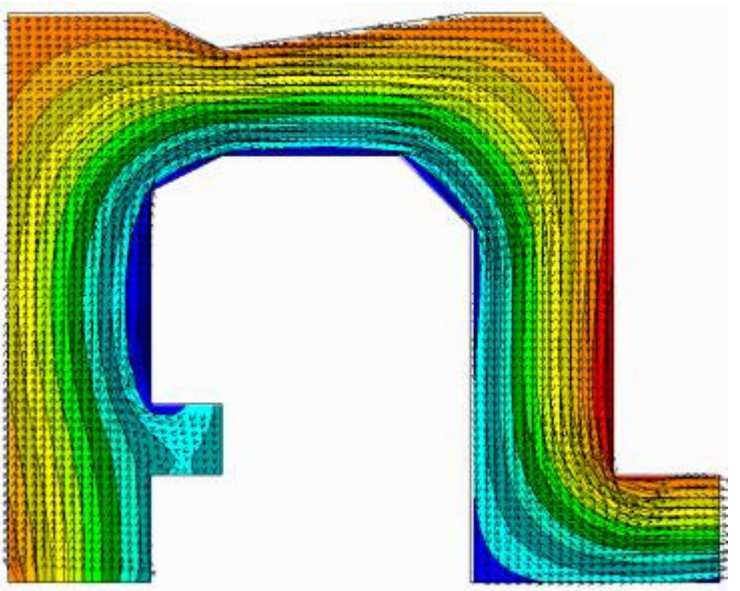


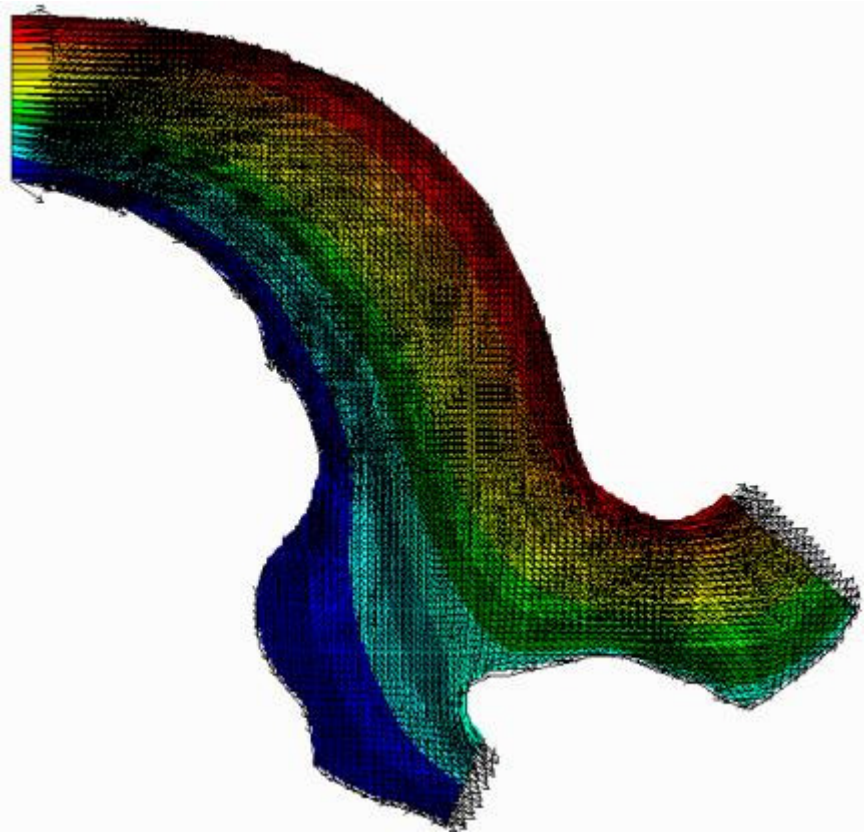












NTRC