Numerical Methods for Solving

Nonlinear Equations

...just the color figures for those who got the B&W printed book...

D. James Benton

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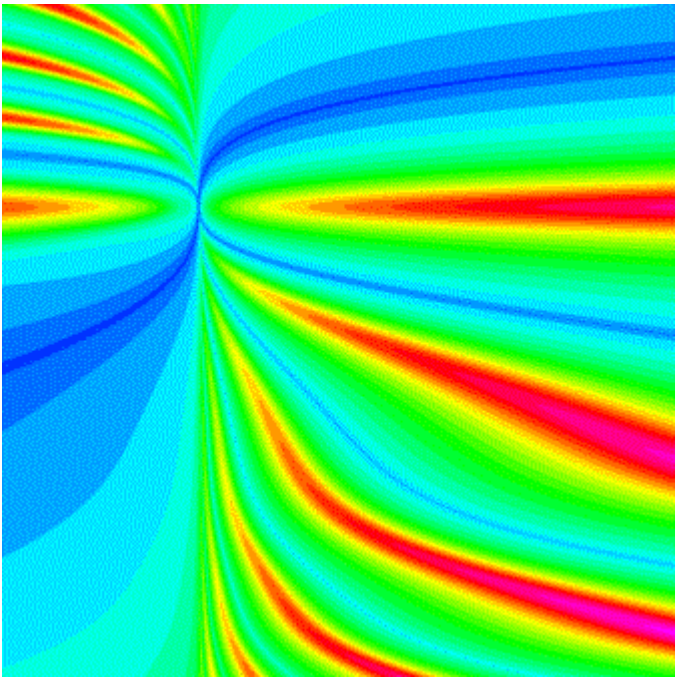
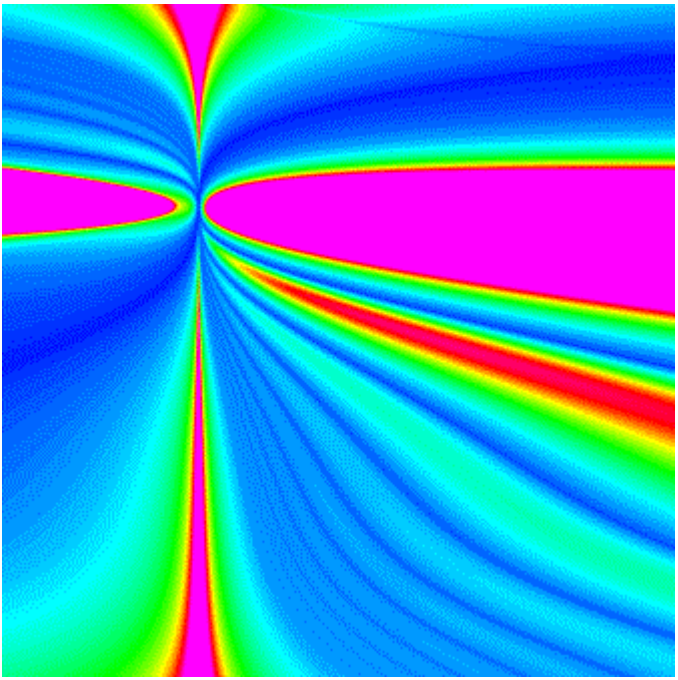
Foreword

Nonlinear equations are found throughout science and engineering across a wide variety of disciplines. These are a significant part of applied mathematics and much effort has been devoted to their study and solution. We will consider single and multi-variable problems. Various theories will be presented, but always with a focus on what works best—that is, the most practical approaches. In the literature of applied mathematics, these are known as *robust* algorithms.

All of the examples contained in this book,
(as well as a lot of free programs) are available at…
http://www.dudleybenton.altervista.org/software/index.html





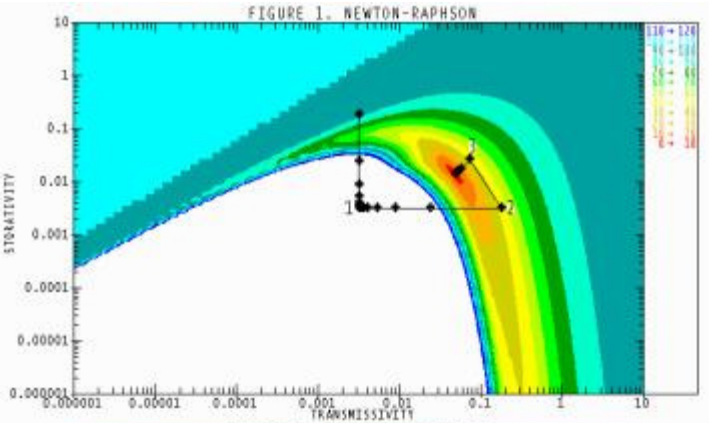


FIGURE 1. NEWTON-RAPHSON
STORATIVITY
TRANSMISSIVITY
1
2

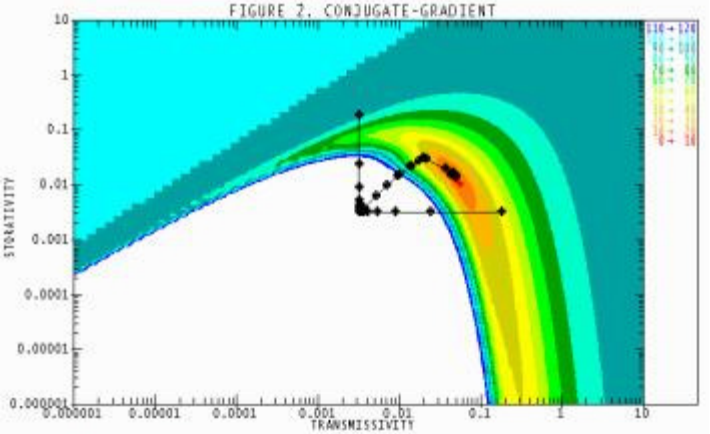


FIGURE 2. CONJUGATE-GRADIENT
STORATIVITY
TRANSMISSIVITY

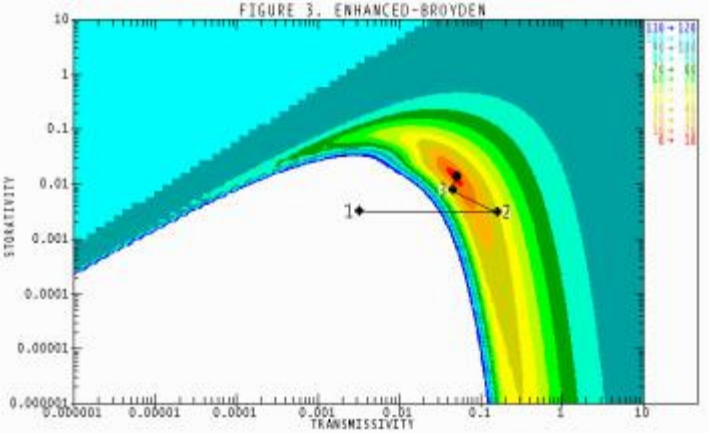
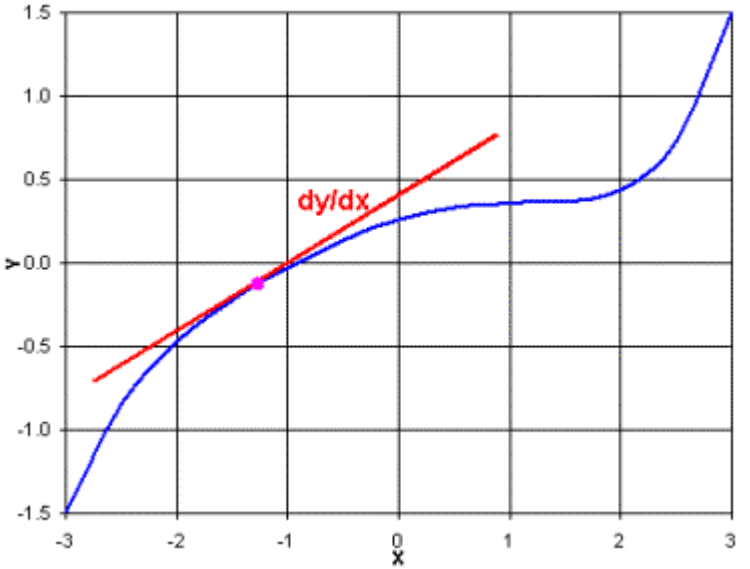


FIGURE 3. ENHANCED-BROYDEN
STORATIVITY
TRANSMISSIVITY
1
2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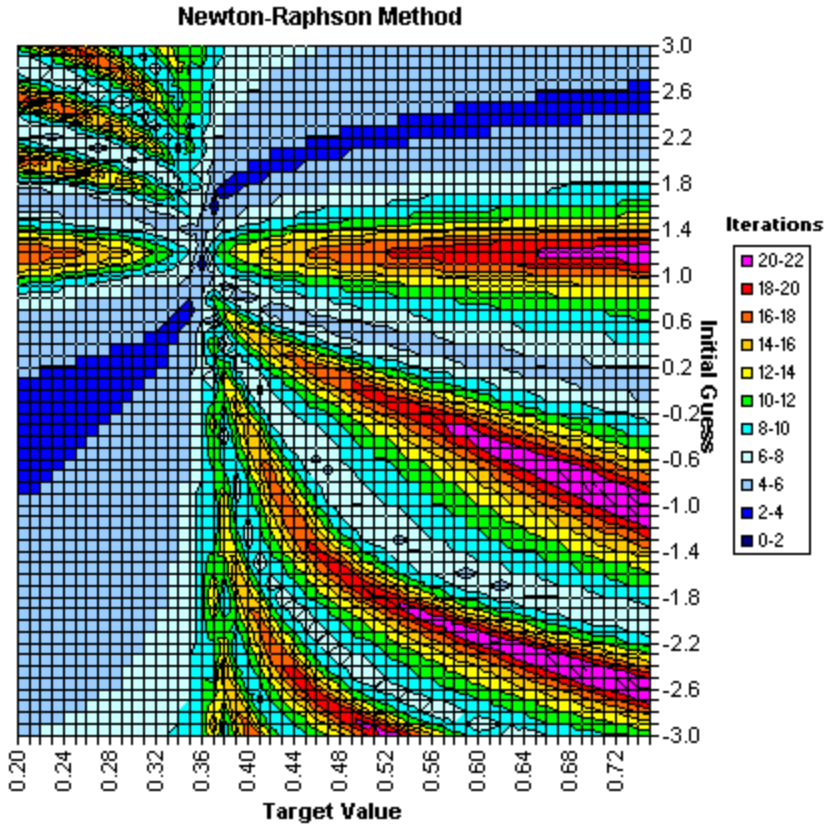
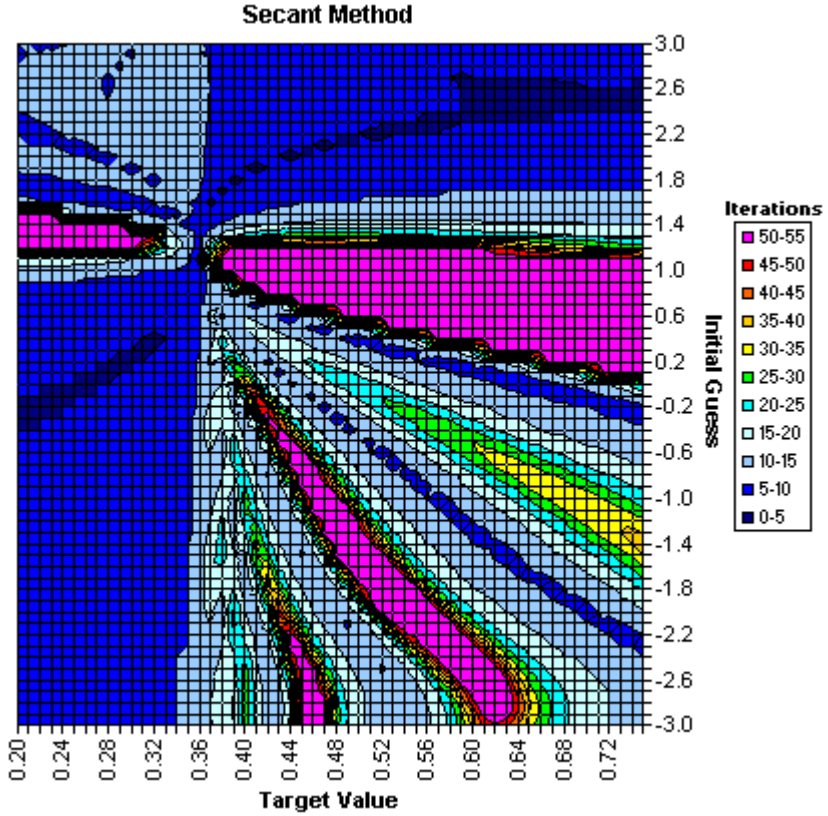
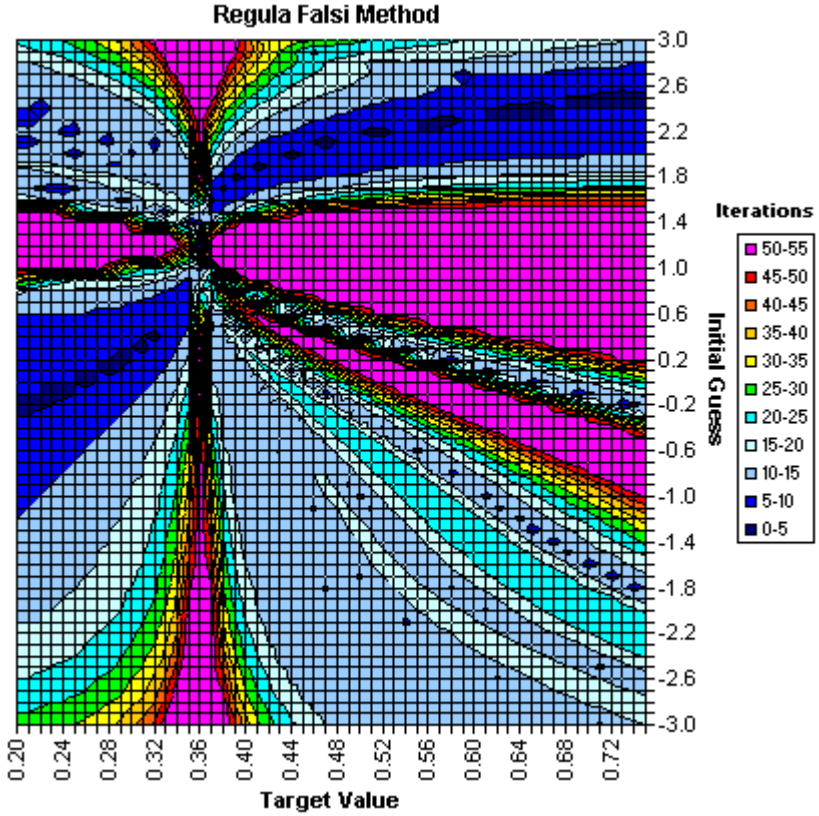
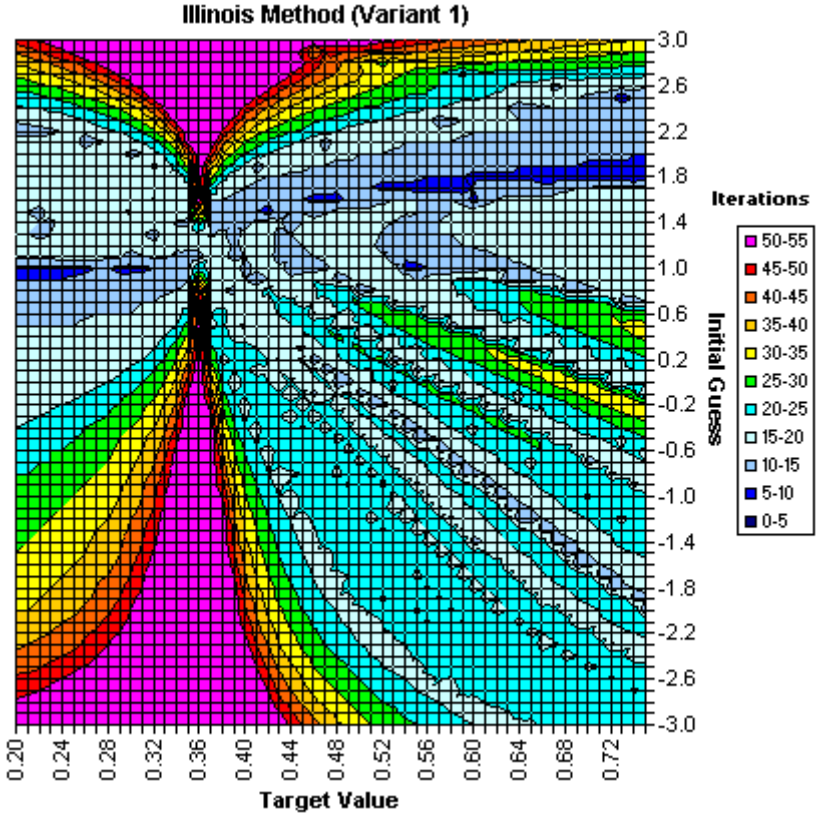
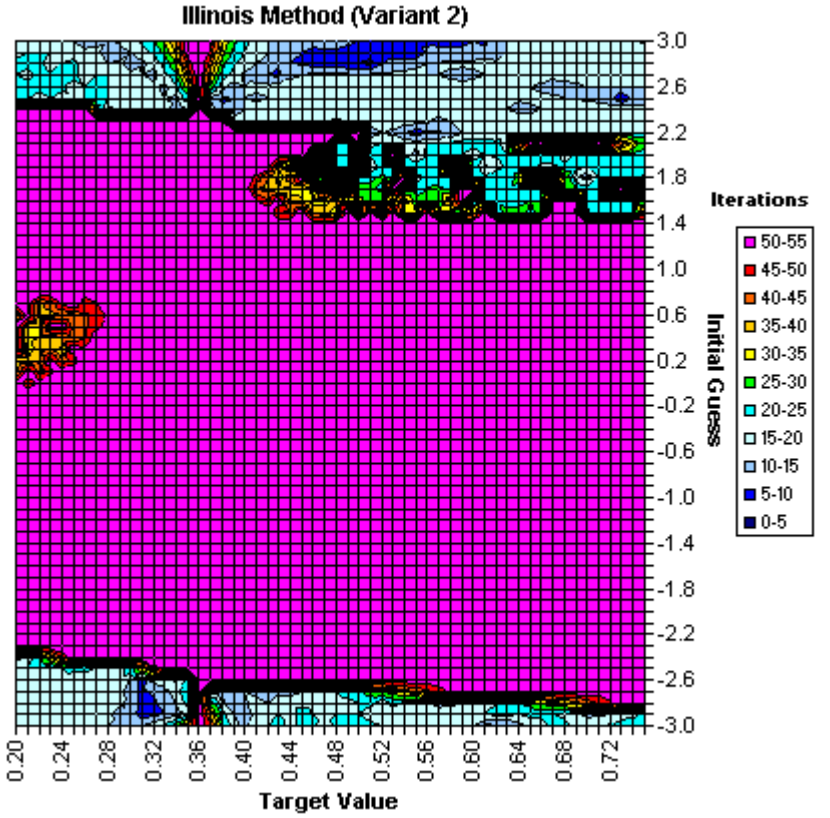
1.0
0.5
0.0
-0.5
-1.0
-1.5
y
-3
-2
-1
0
1
2
3
x
dy/dx

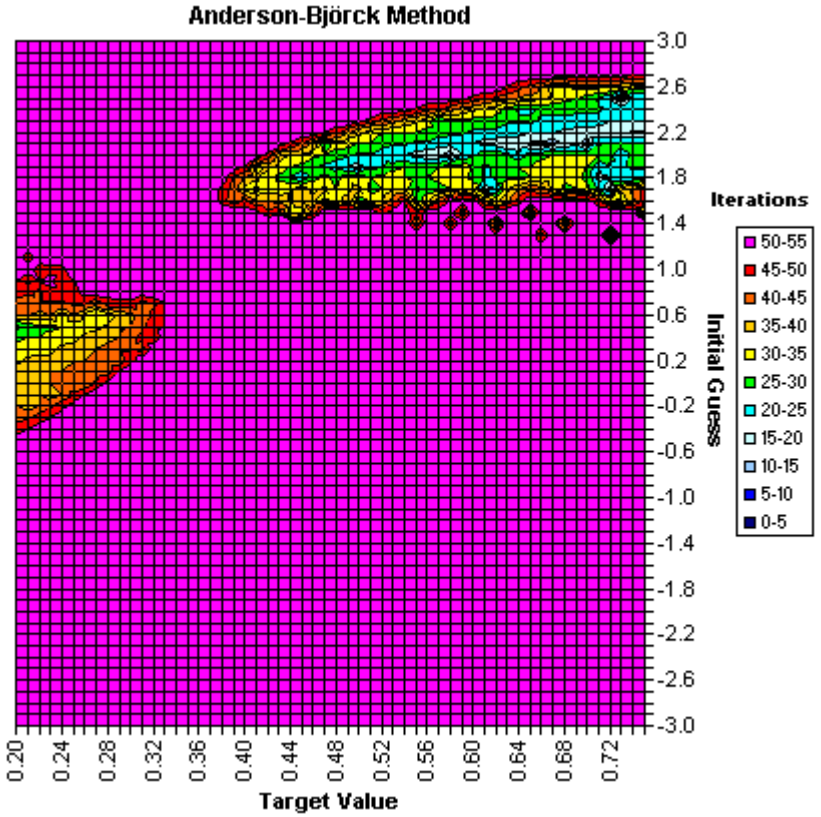

Newton-Raphson Method
Iterations
20-22
18-20
16-18
14-16
12-14
10-12
8-10
6-8
4-6
2-4
0-2
Initial Guess
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-1.4
-1.8
-2.2
-2.6
-3.0
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
0.52
0.56
0.60
0.64
0.68
0.72
Target Value

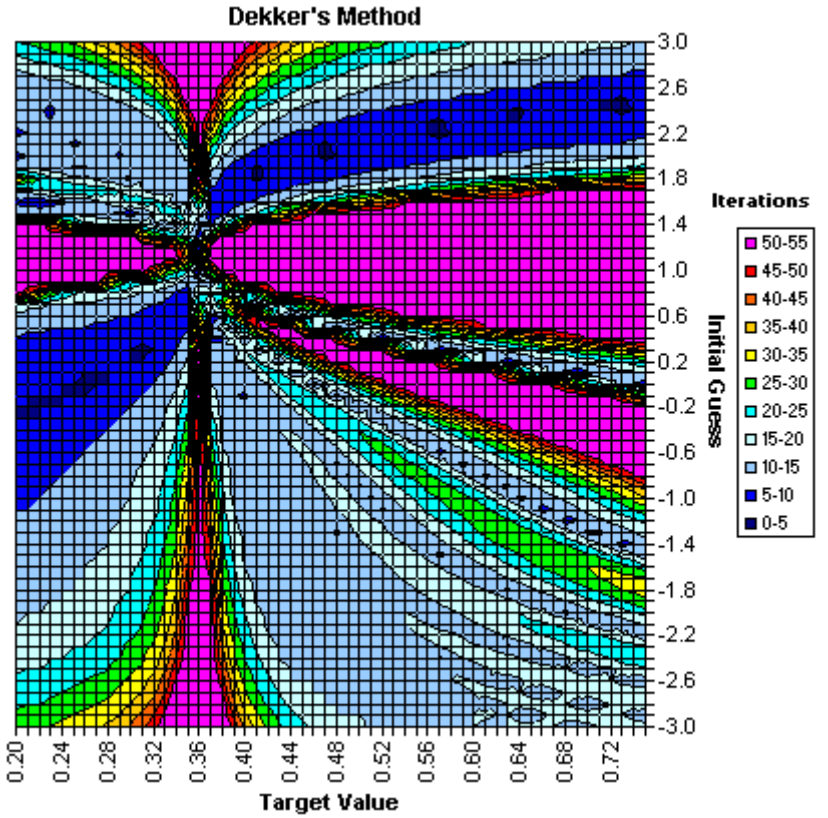

Secant Method
Iterations
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
Initial Guess
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-1.4
-1.8
-2.2
-2.6
-3.0
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
0.52
0.56
0.60
0.64
0.68
0.72
Target Value


Regula Falsi Method
Iterations
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
Initial Guess
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-1.4
-1.8
-2.2
-2.6
-3.0
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
0.52
0.56
0.60
0.64
0.68
0.72
Target Value

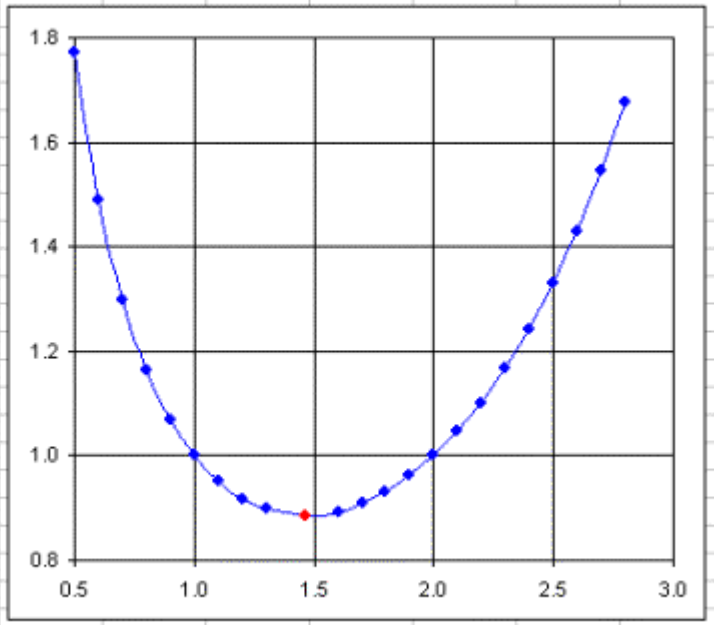

Illinois Method (Variant 1)
Iterations
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
Initial Guess
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-1.4
-1.8
-2.2
-2.6
-3.0
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
0.52
0.56
0.60
0.64
0.68
0.72
Target Value

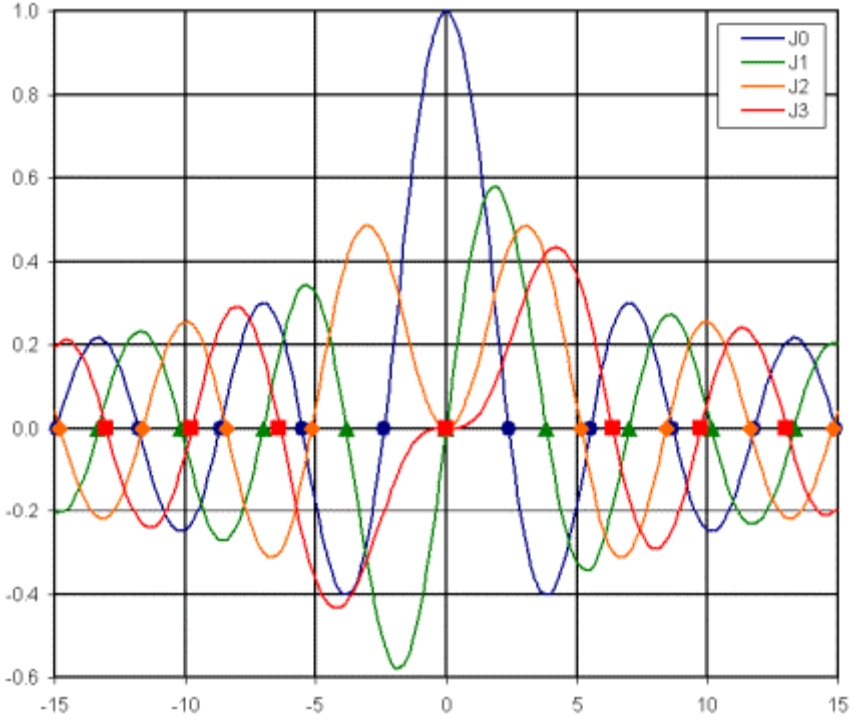

Illinois Method (Variant 2)
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-1.4
-1.8
-2.2
-2.6
-3.0
Initial Guess
Iterations
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
0.52
0.56
0.60
0.64
0.68
0.72
Target Value

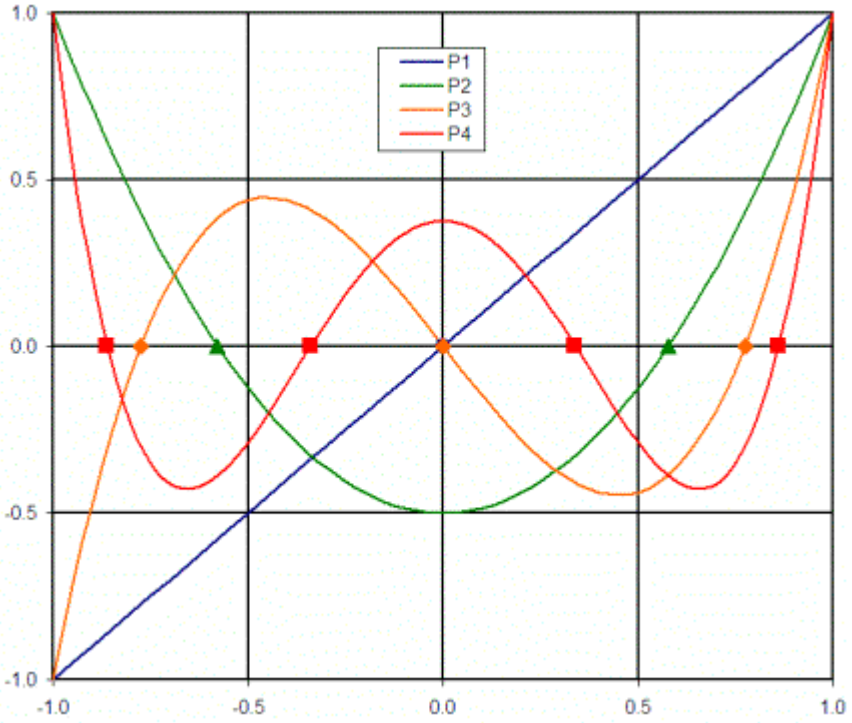

Anderson-Björck Method
Iterations
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
Initial Guess
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-1.4
-1.8
-2.2
-2.6
-3.0
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
0.52
0.56
0.60
0.64
0.68
0.72
Target Value

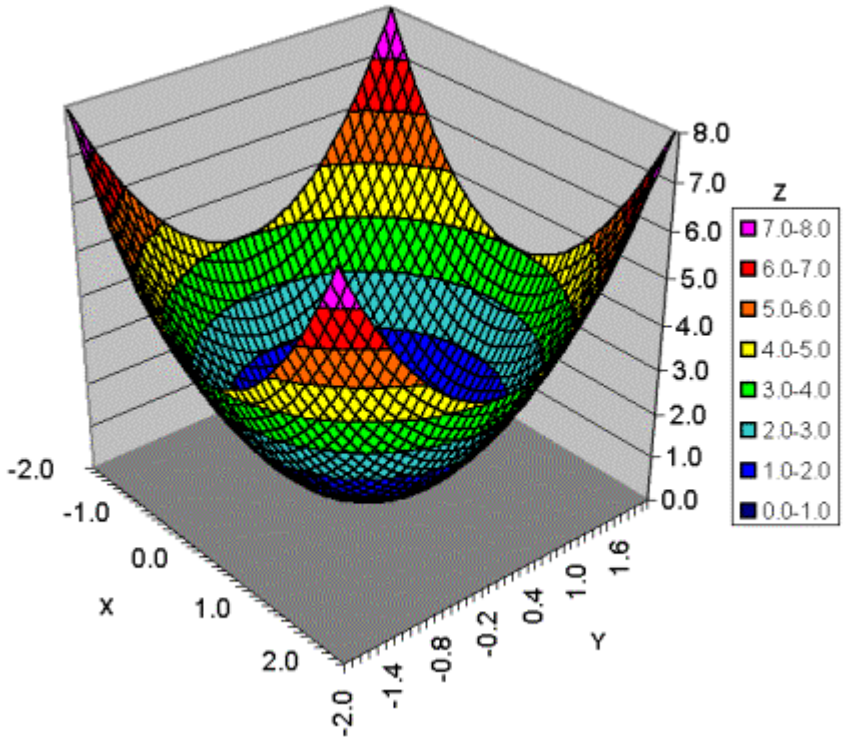

Dekker's Method
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-1.4
-1.8
-2.2
-2.6
-3.0
Initial Guess
Iterations
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
0.52
0.56
0.60
0.64
0.68
0.72
Target Value

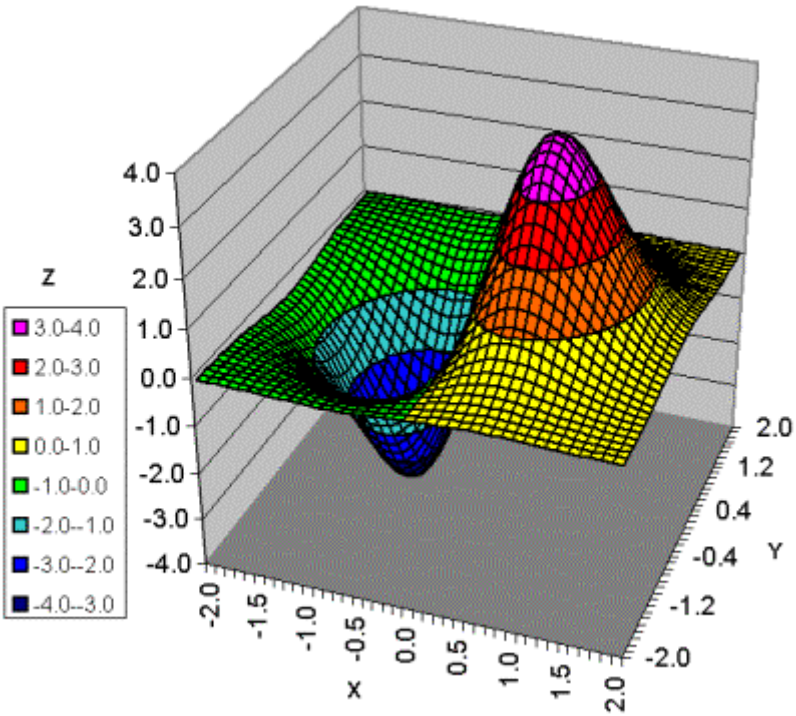
x	y
0.5	1.77
0.6	1.49
0.7	1.30
0.8	1.16
0.9	1.07
1.0	1.00
1.1	0.95
1.2	0.92
1.3	0.90
1.4616	0.8856
1.6	0.89
1.7	0.91
1.8	0.93
1.9	0.96
2.0	1.00
2.1	1.05
2.2	1.10
2.3	1.17
2.4	1.24
2.5	1.33
2.6	1.43
2.7	1.54
2.8	1.68

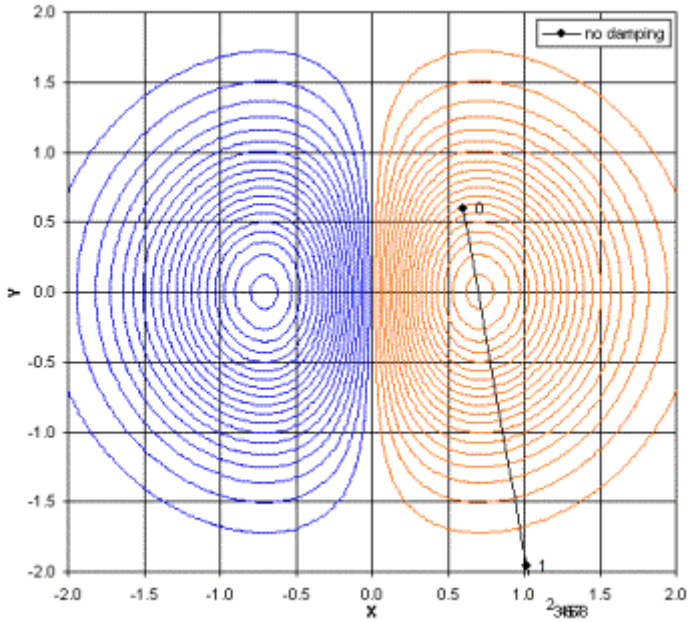


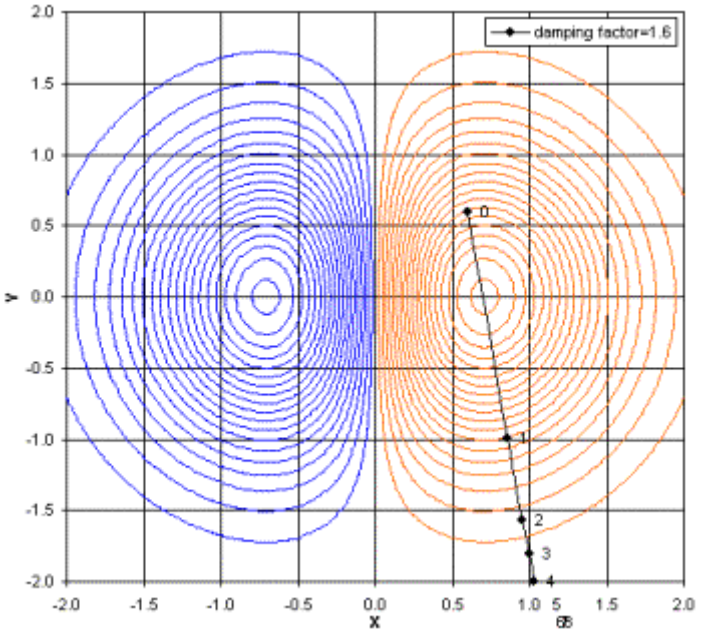

J0
J1
J2
J3
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-15
-10
-5
0
5
10
15

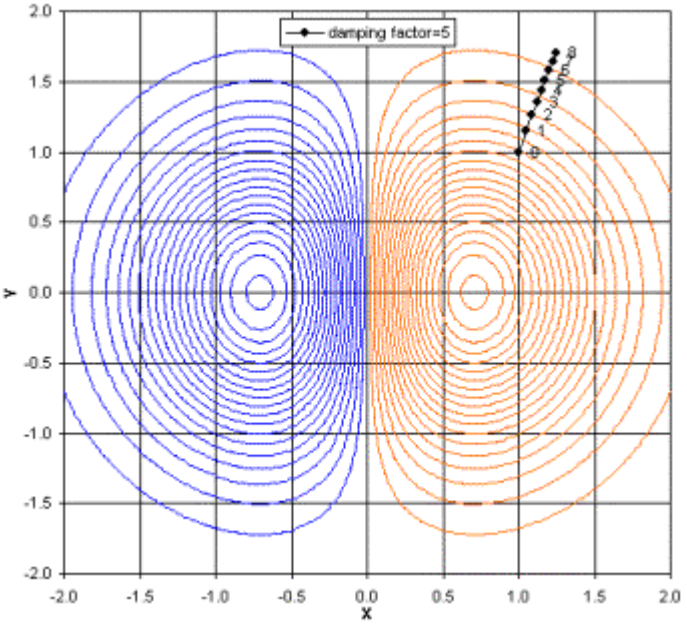

P1
P2
P3
P4
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0

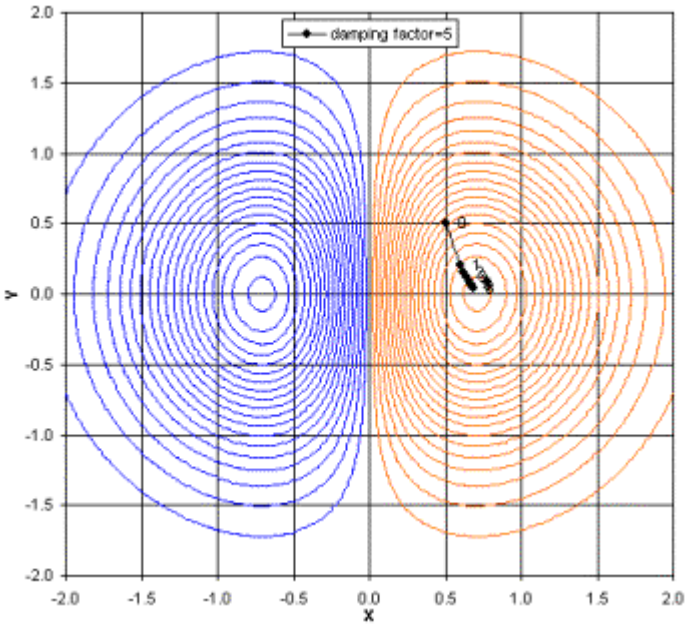

8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Z
7.0-8.0
6.0-7.0
5.0-6.0
4.0-5.0
3.0-4.0
2.0-3.0
1.0-2.0
0.0-1.0
-2.0
-1.0
0.0
1.0
2.0
X
-2.0
-1.4
-0.8
-0.2
0.4
1.0
1.6
Y

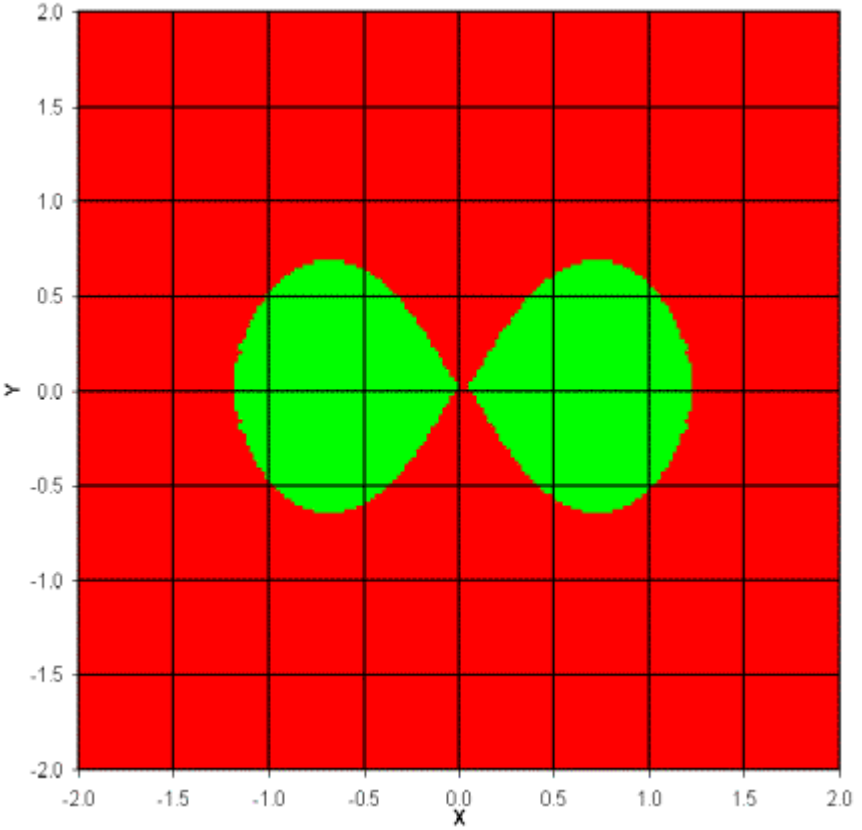

z
3.0-4.0
2.0-3.0
1.0-2.0
0.0-1.0
-1.0-0.0
-2.0--1.0
-3.0--2.0
-4.0--3.0
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
x
2.0
1.2
0.4
-0.4
-1.2
-2.0
Y

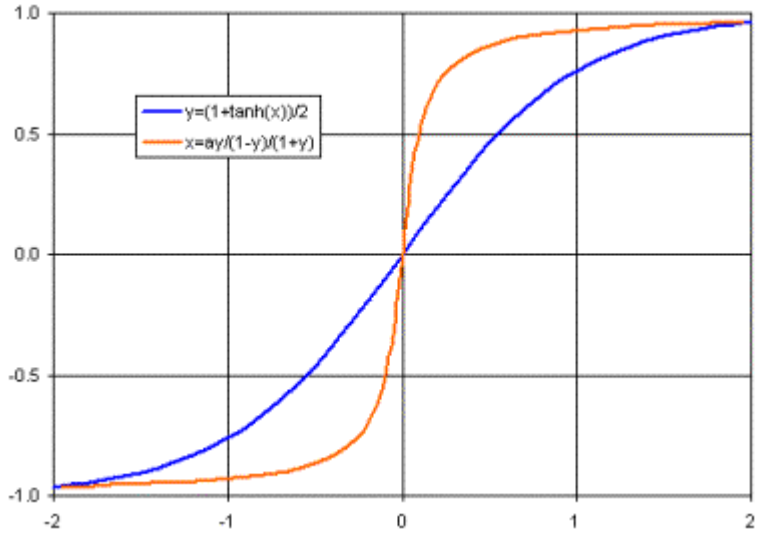

no damping
0
1
2
3
4
x
y

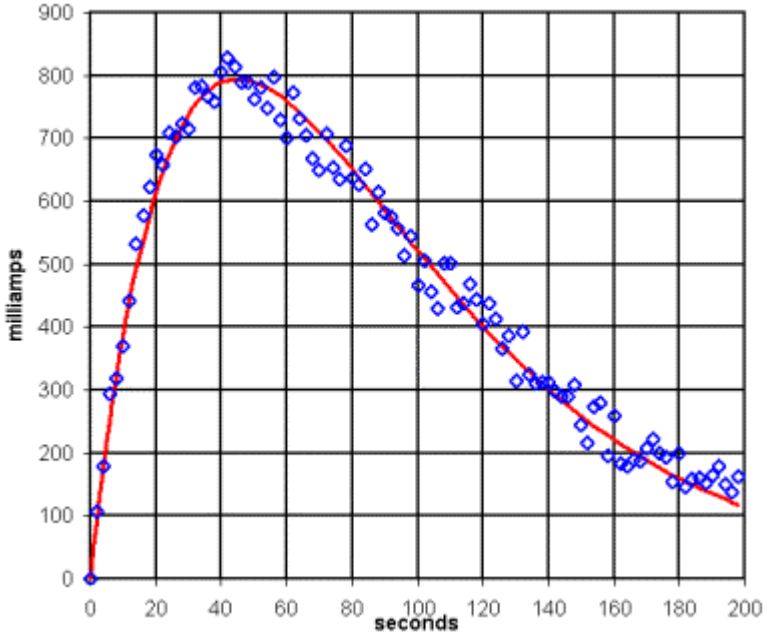

damping factor=1.6
0
1
2
3
4
x
y

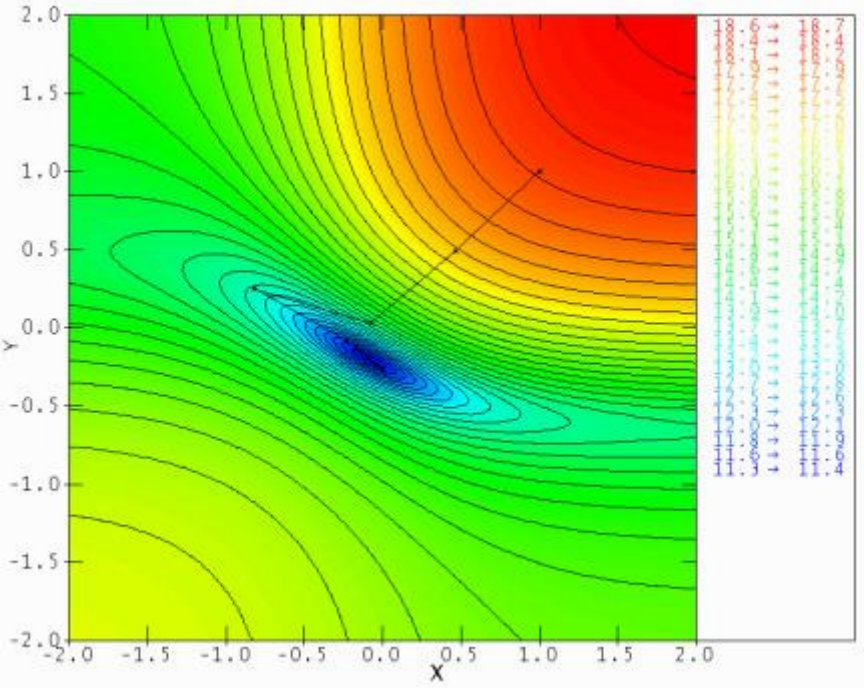

damping factor=5
0
1
2
3
4
5
6
7
8
x
y

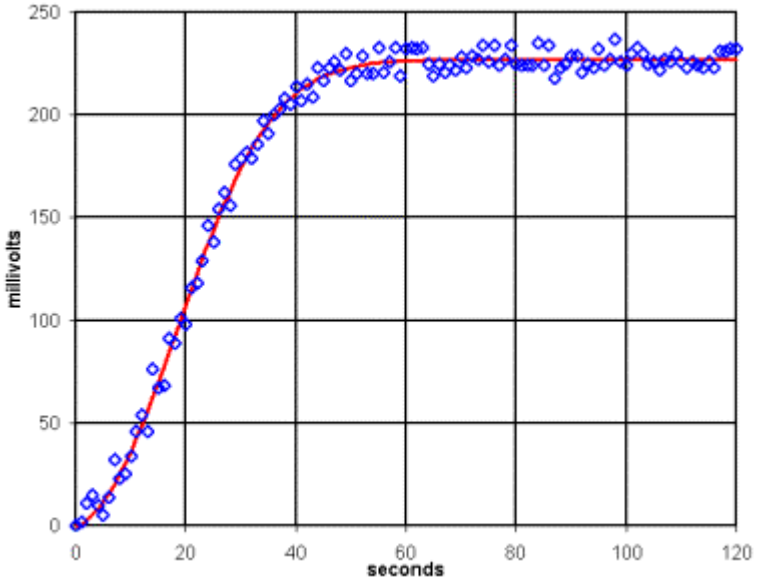

damping factor=5
0
1
2
3
4
5
6
7
8
x
y

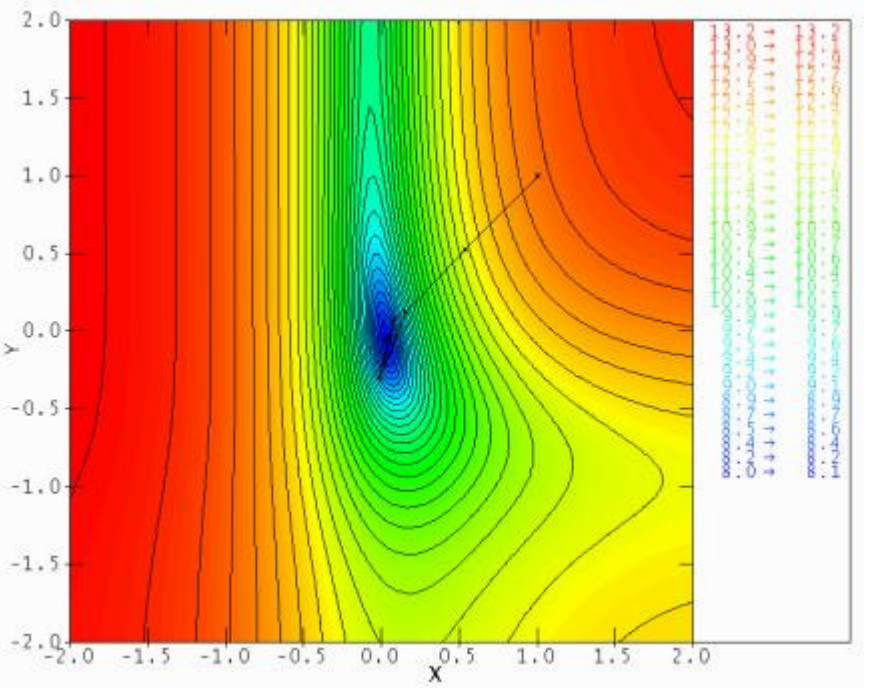

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
X

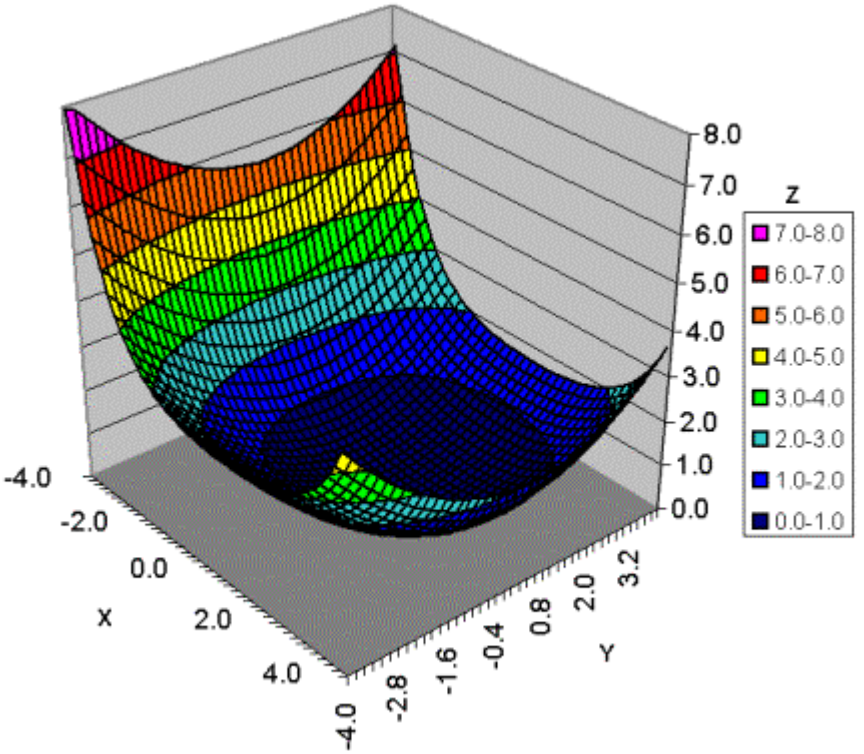

y=(1+tanh(x))/2
x=ay/(1-y)/(1+y)
1.0
0.5
0.0
-0.5
-1.0
-2
-1
0
1
2

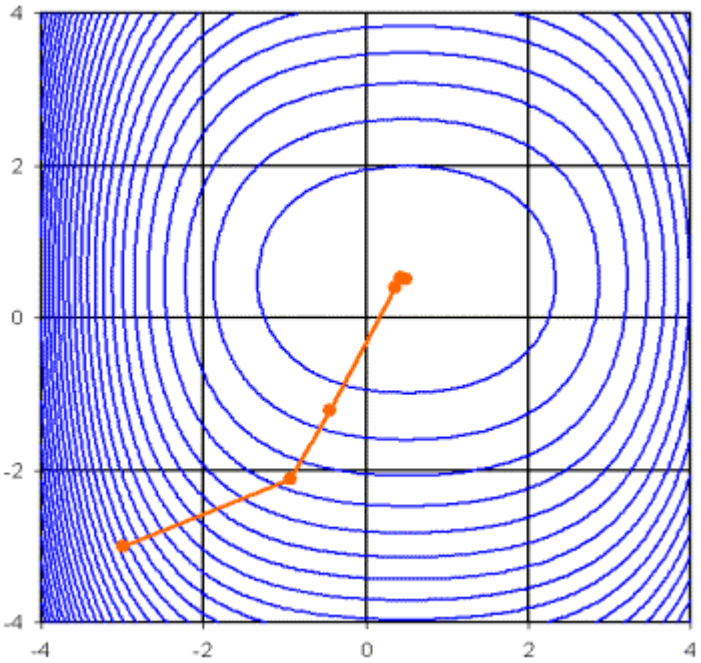

milliamps
seconds
900
800
700
600
500
400
300
200
100
0
0
20
40
60
80
100
120
140
160
180
200

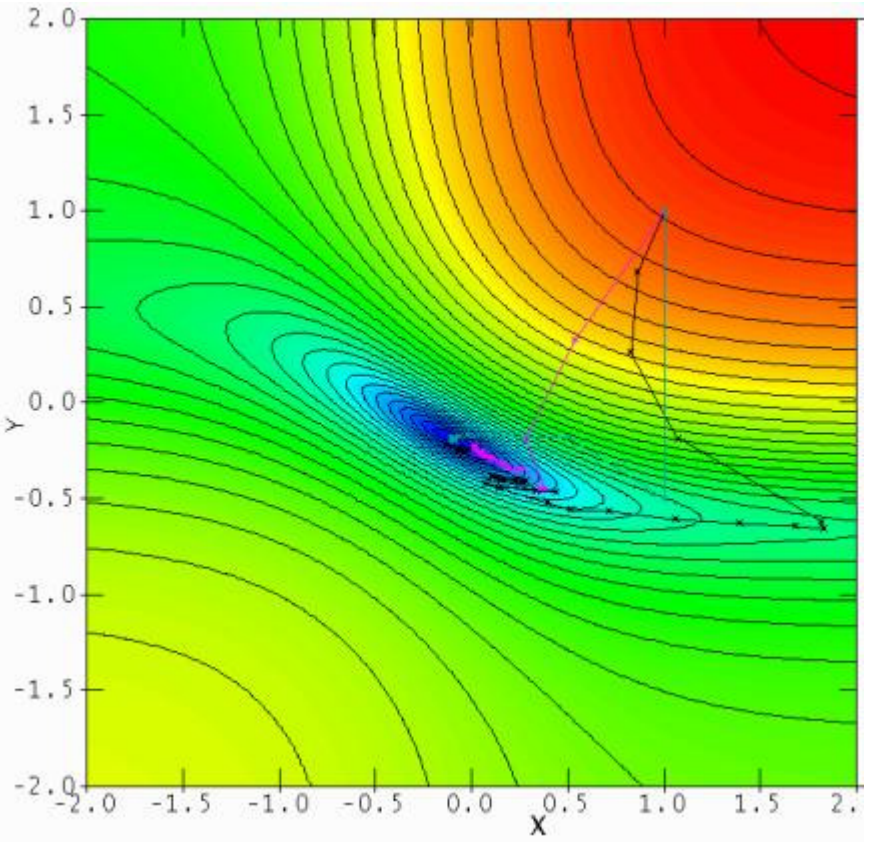

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0
X

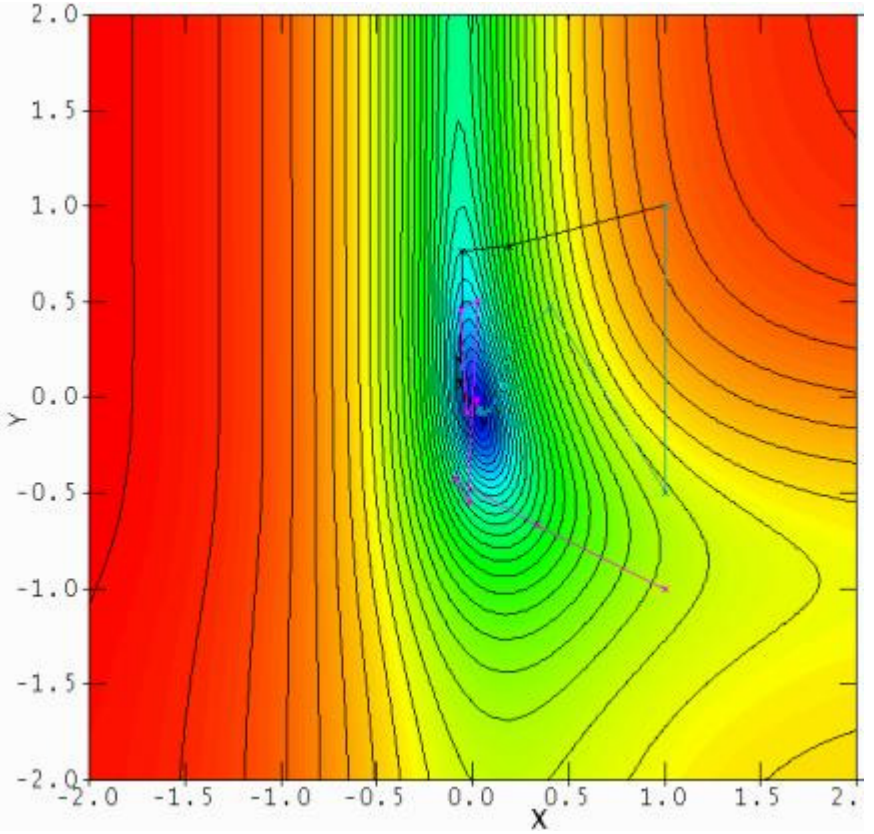

250
200
150
100
50
0
millivolts
0 20 40 60 80 100 120
seconds

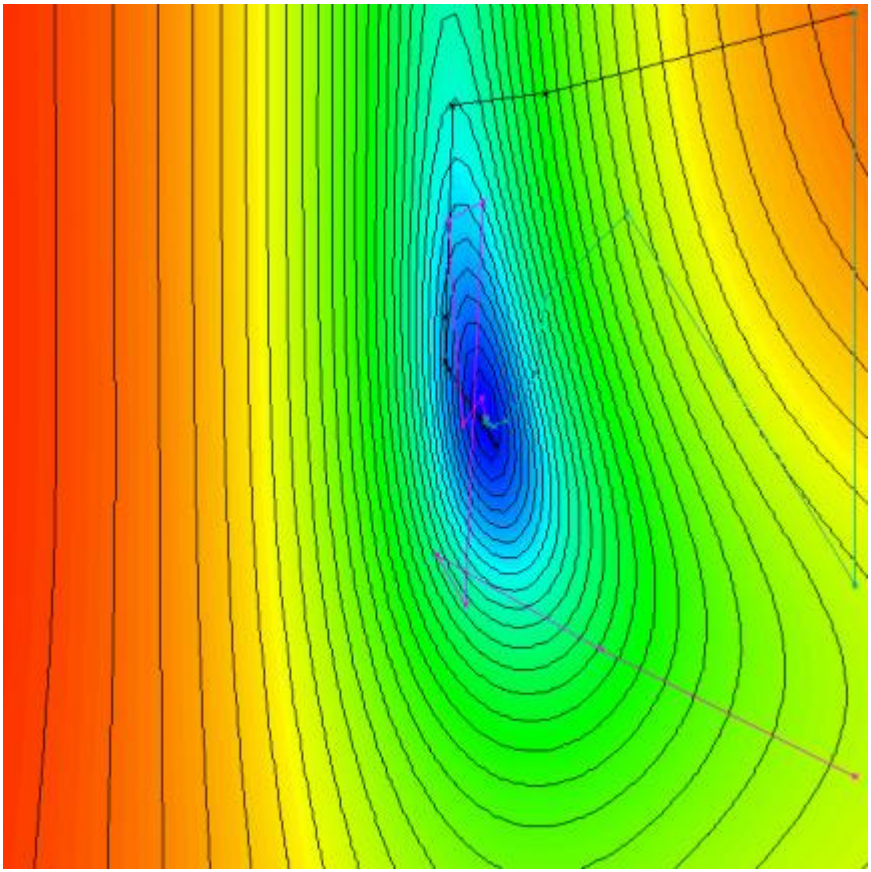

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
x

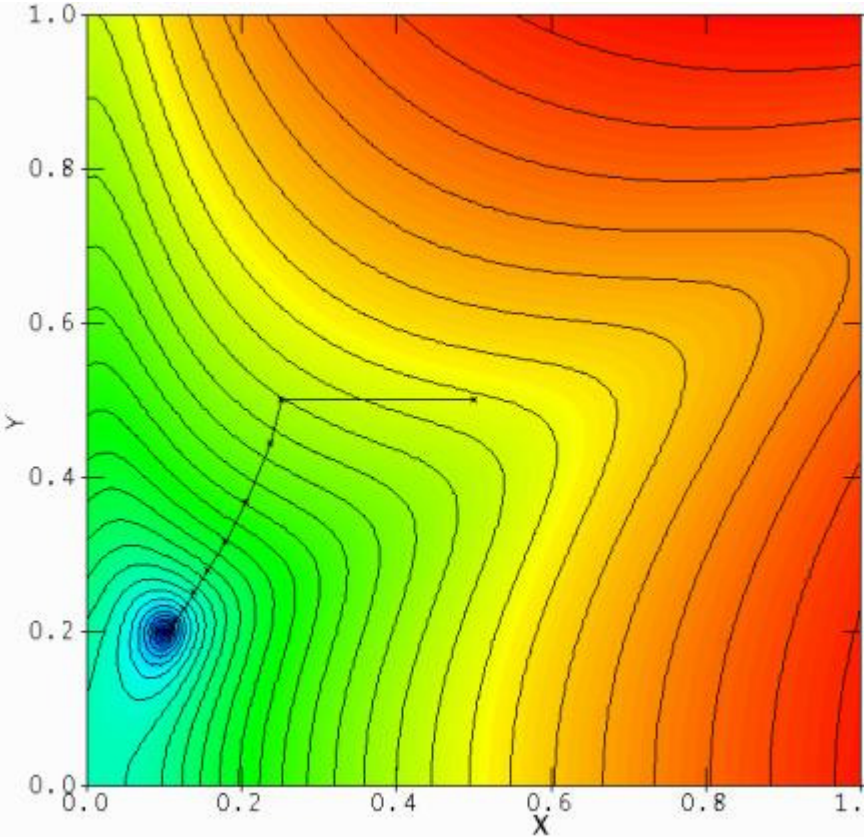

8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Z
7.0-8.0
6.0-7.0
5.0-6.0
4.0-5.0
3.0-4.0
2.0-3.0
1.0-2.0
0.0-1.0
-4.0
-2.0
0.0
2.0
4.0
X
-4.0
-2.8
-1.6
-0.4
0.8
2.0
3.2
Y

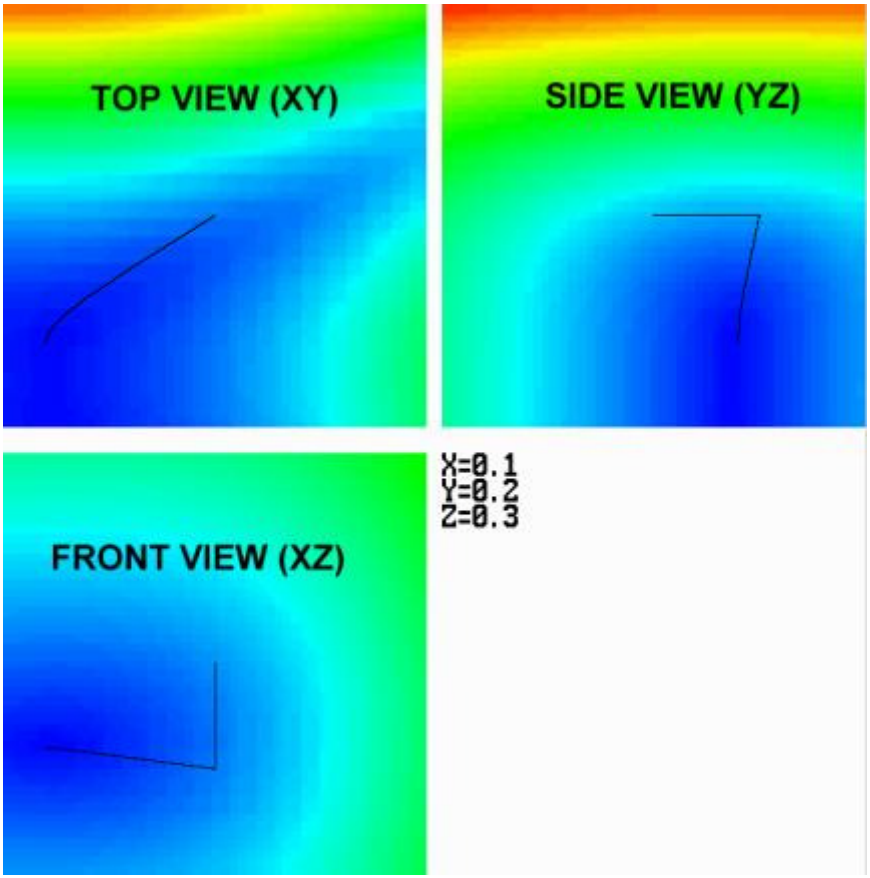

4
2
0
-2
-4
-4
-2
0
2
4

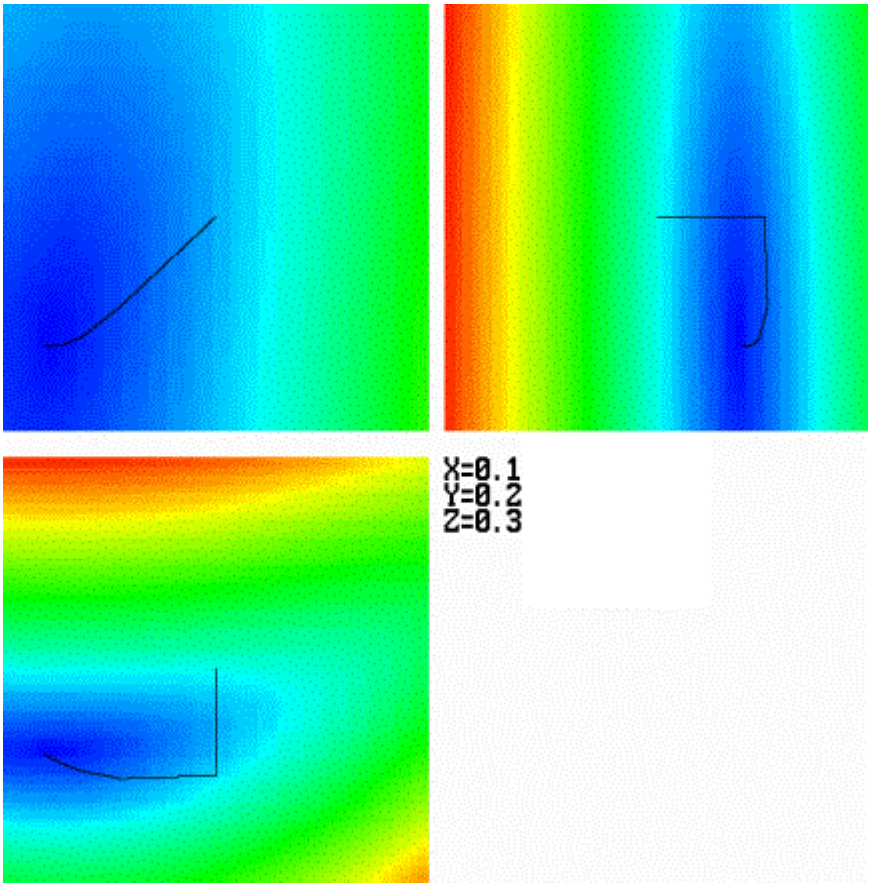

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.
X

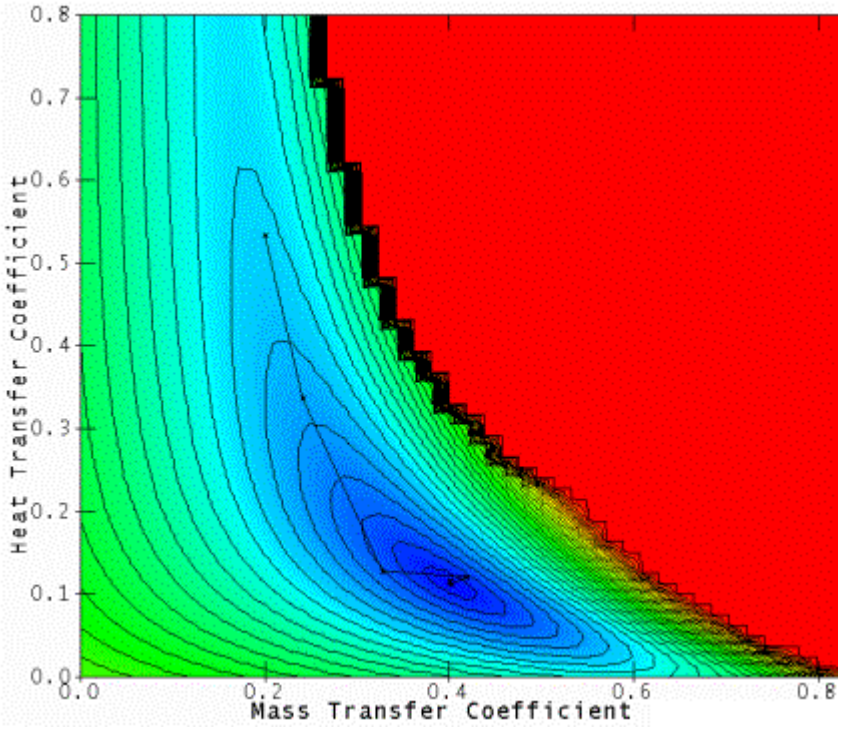

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.
X

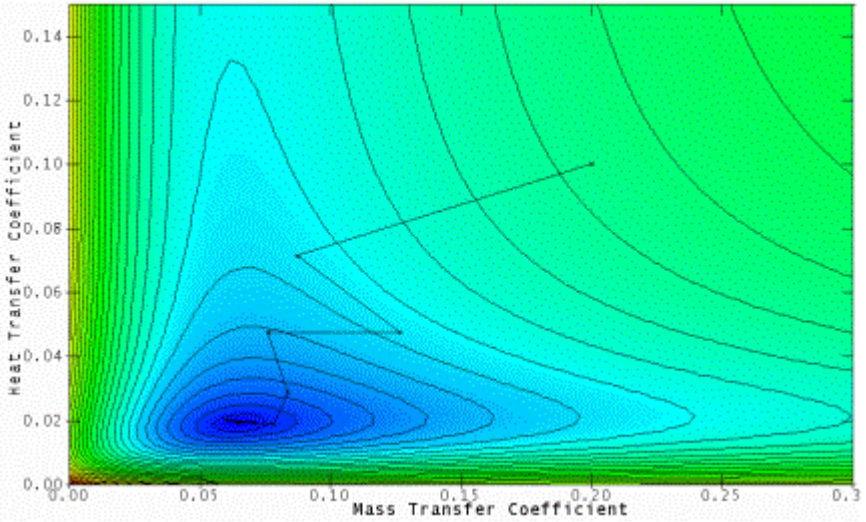


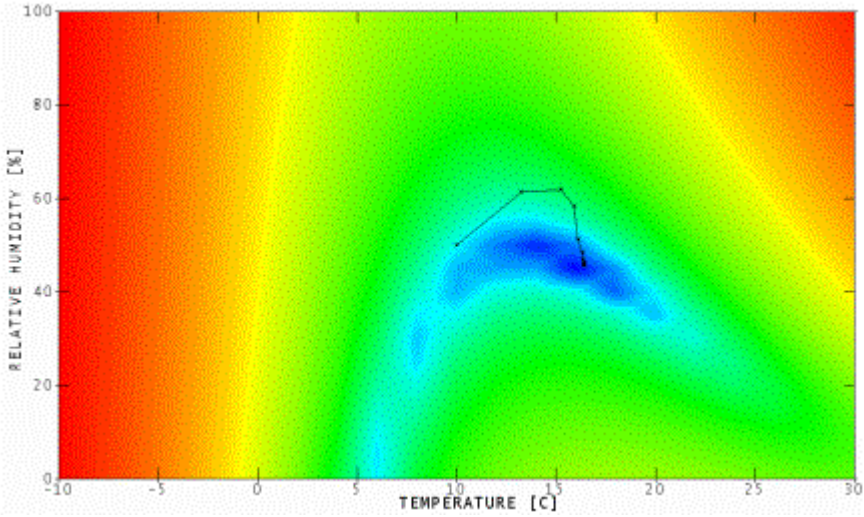

1.0
0.8
0.6
0.4
0.2
0.0
Y
0.0
0.2
0.4
0.6
0.8
1.
X

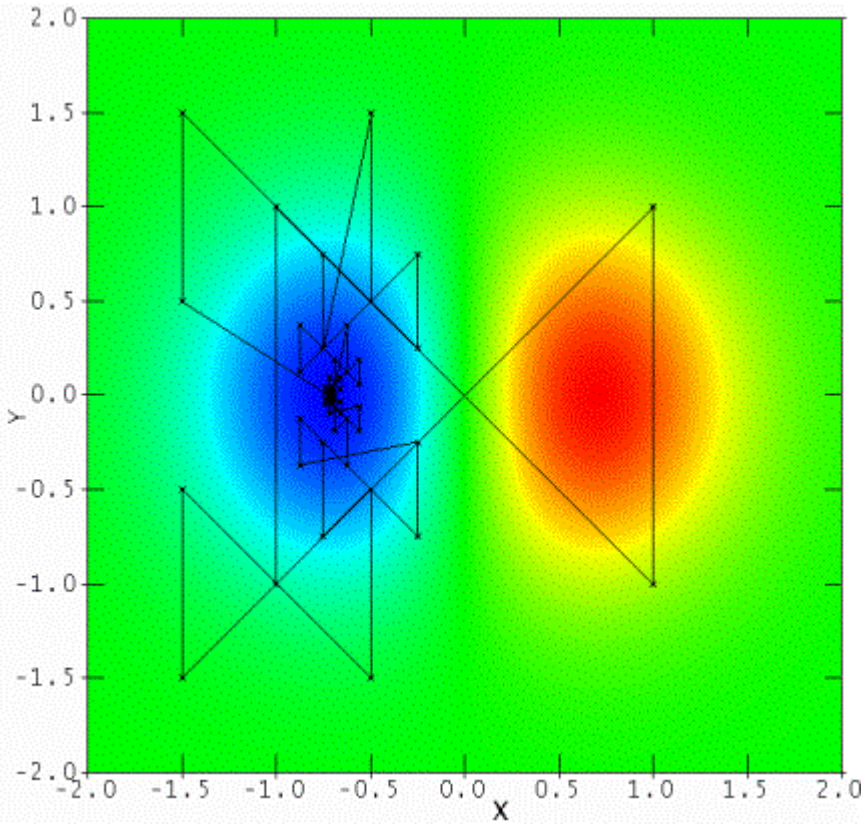

TOP VIEW (XY)
SIDE VIEW (YZ)
FRONT VIEW (XZ)
X=0.1
Y=0.2
Z=0.3

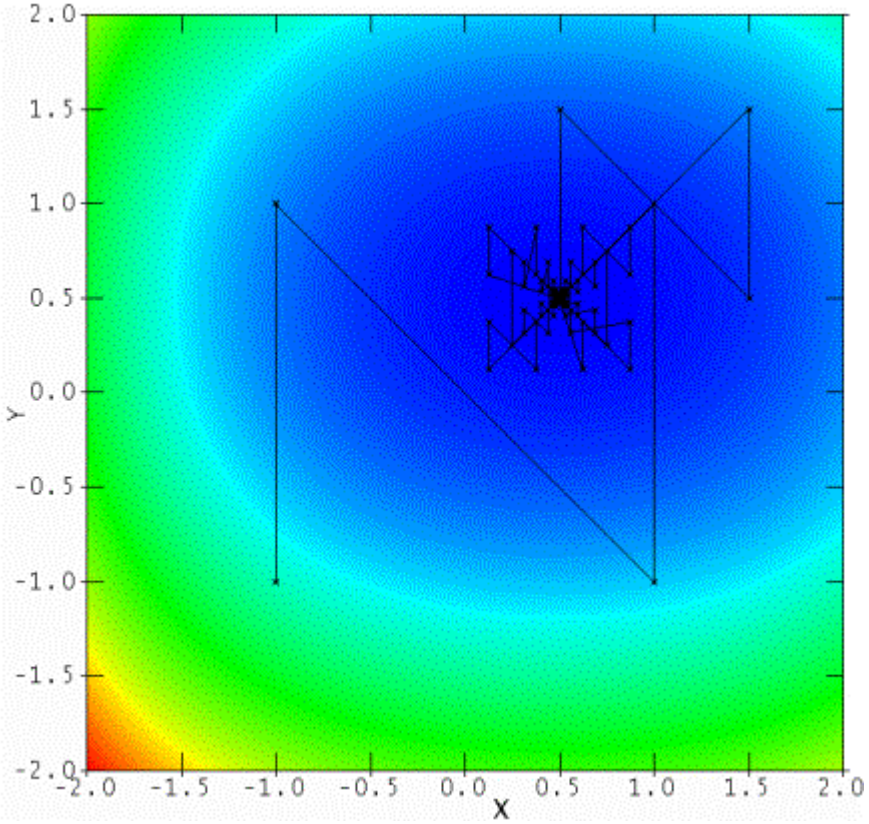

X=0.1
Y=0.2
Z=0.3

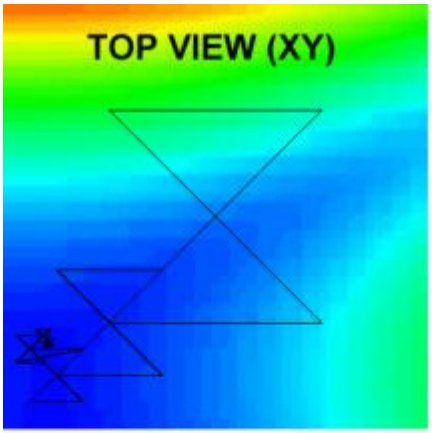

Heat Transfer Coefficient
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.0
0.2
0.4
0.6
0.8
Mass Transfer Coefficient

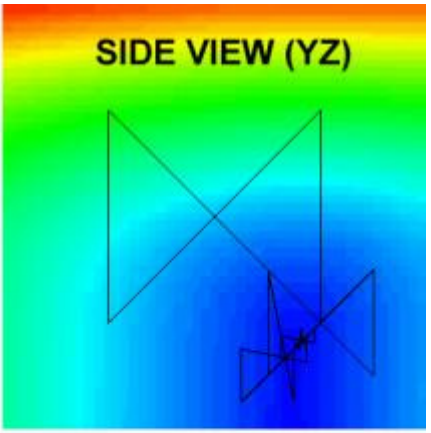

Heat Transfer Coefficient
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.00
0.05
0.10
0.15
0.20
0.25
0.3
Mass Transfer Coefficient

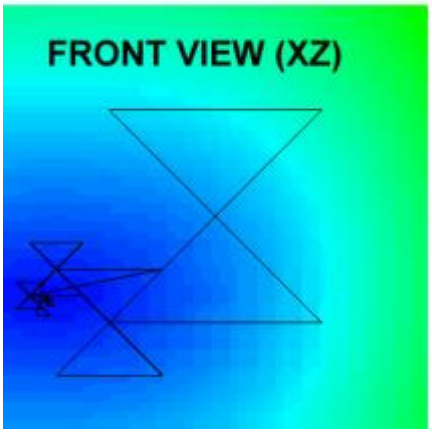

RELATIVE HUMIDITY [%]
100
80
60
40
20
0
-10
-5
0
5
10
15
20
25
30
TEMPERATURE [C]


2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
X

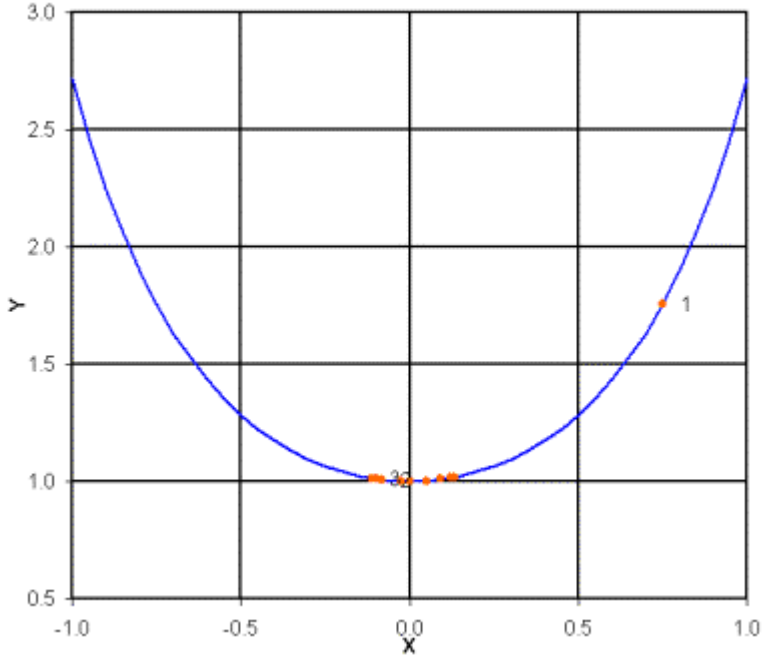

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
X

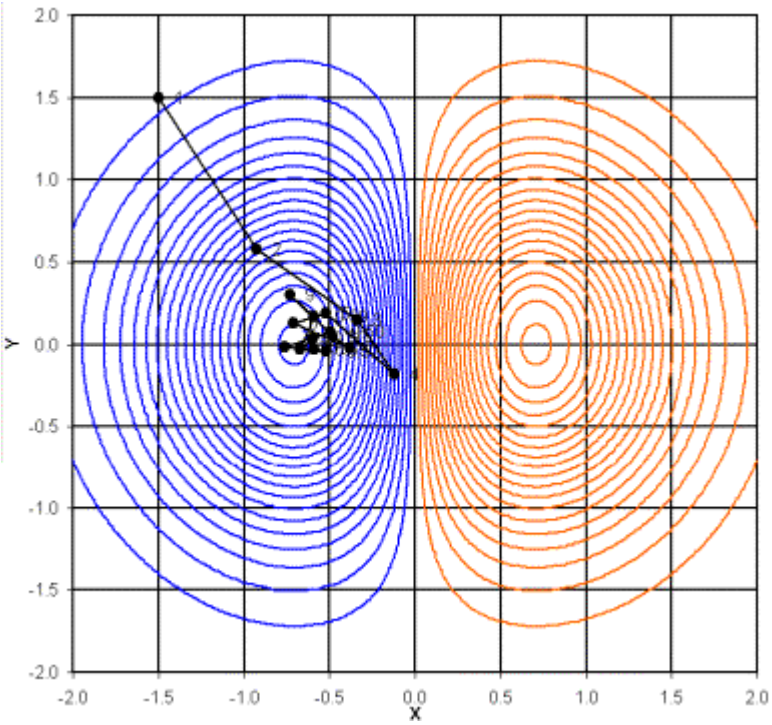


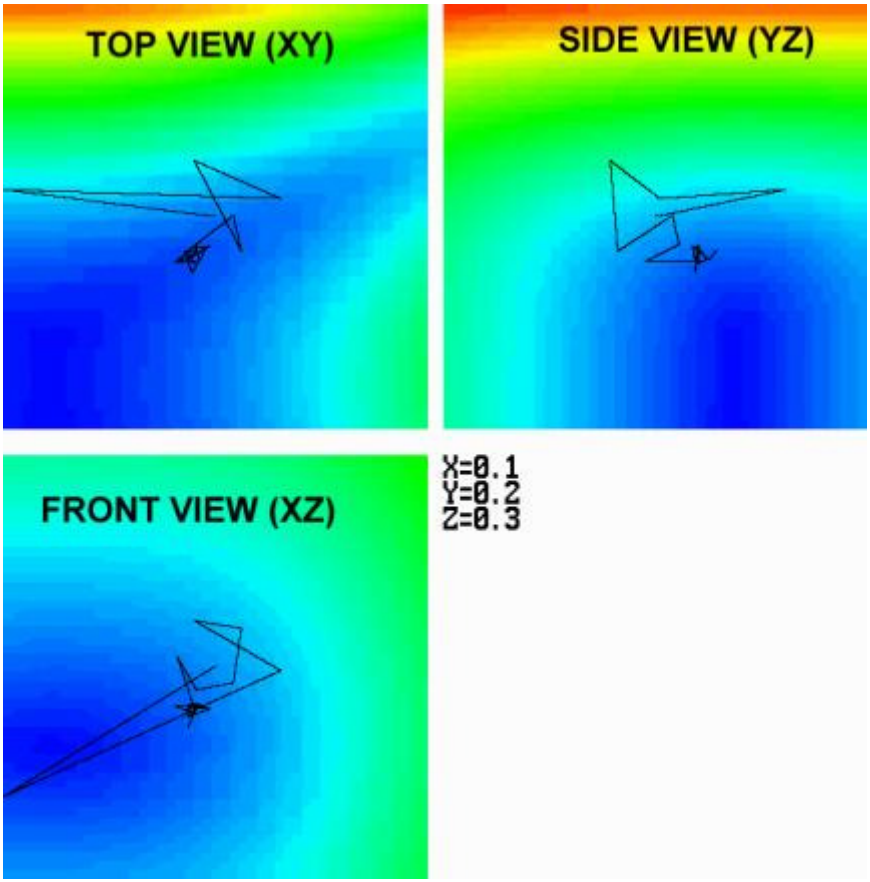


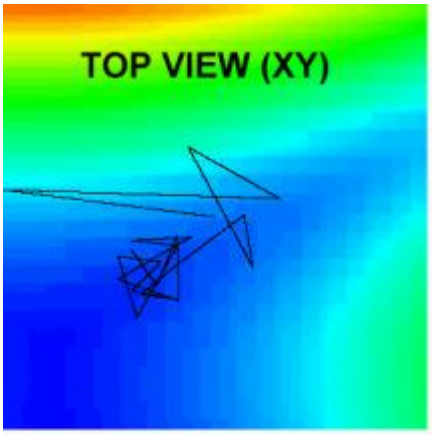


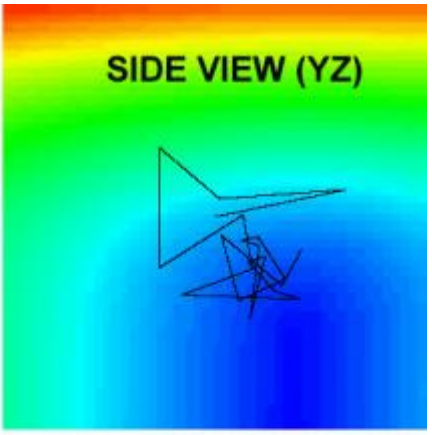
X=0.1
Y=0.2
Z=0.3

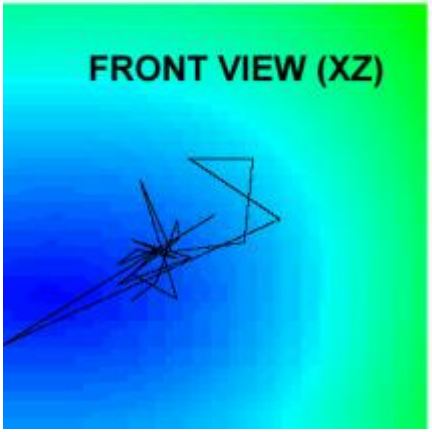

3.0
2.5
2.0
1.5
1.0
0.5
Y
-1.0
-0.5
0.0
0.5
1.0
X
1
32


2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Y
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
X

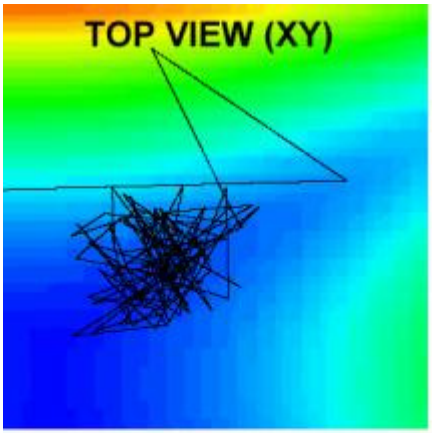

TOP VIEW (XY)
SIDE VIEW (YZ)
FRONT VIEW (XZ)
X=0.1
Y=0.2
Z=0.3

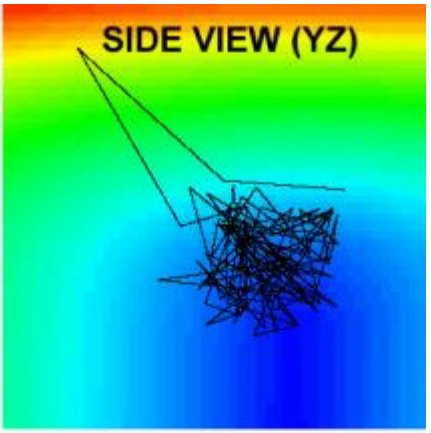


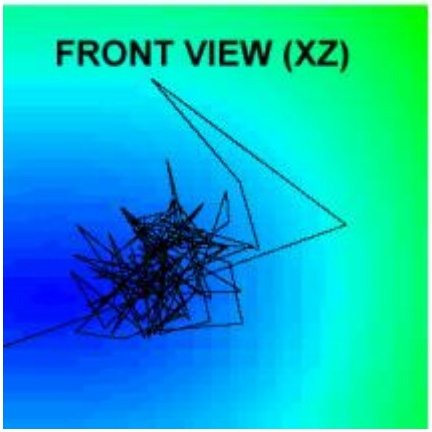




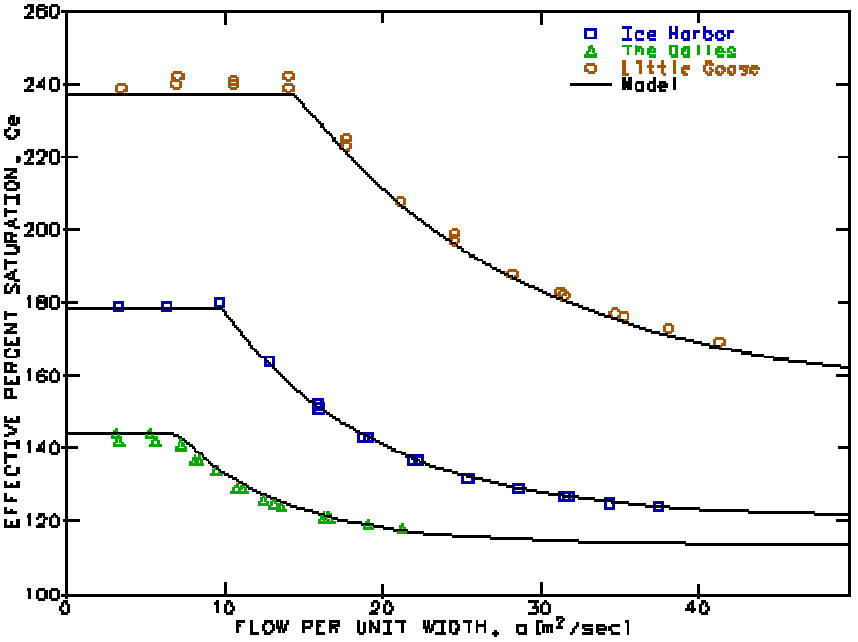
X=0.1
Y=0.2
Z=0.3

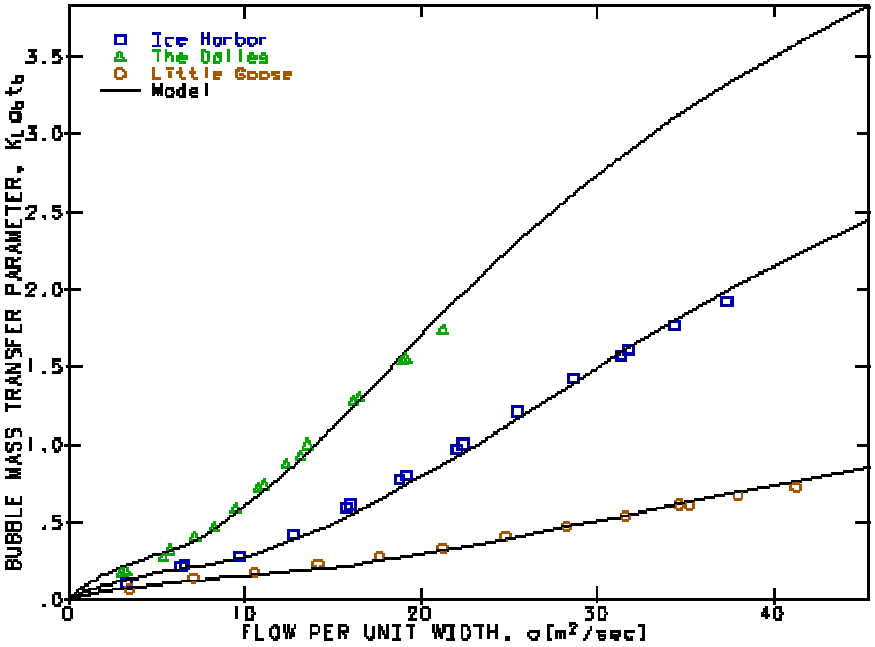


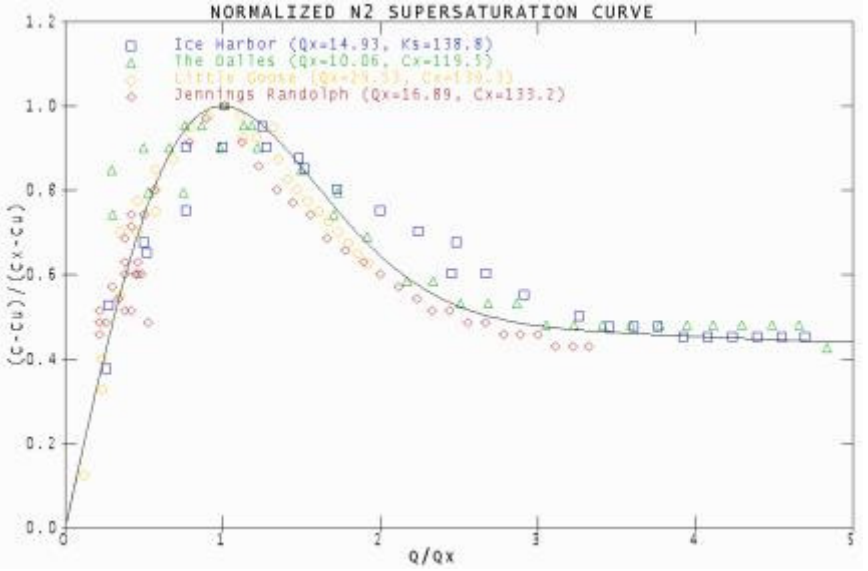


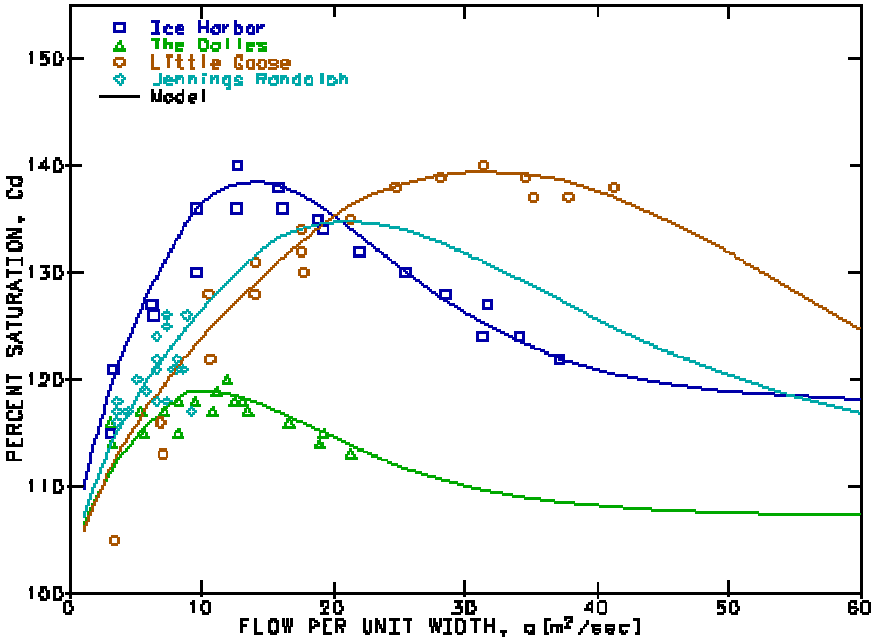


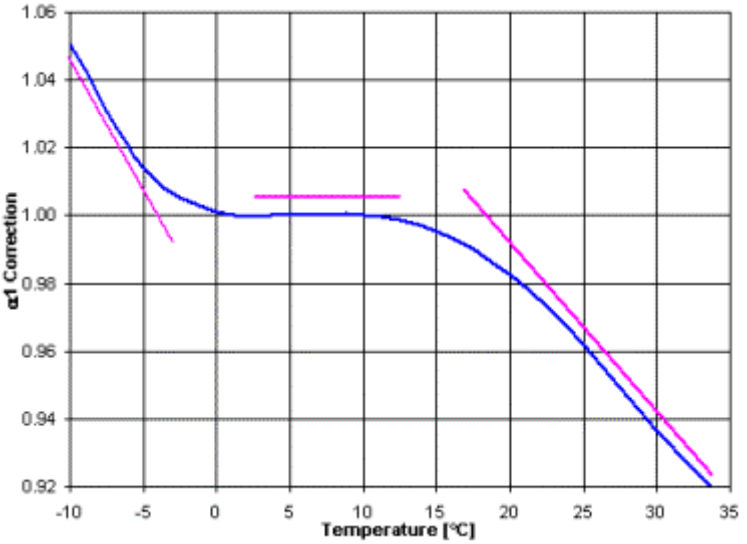
X=0.1
Y=0.2
Z=0.3

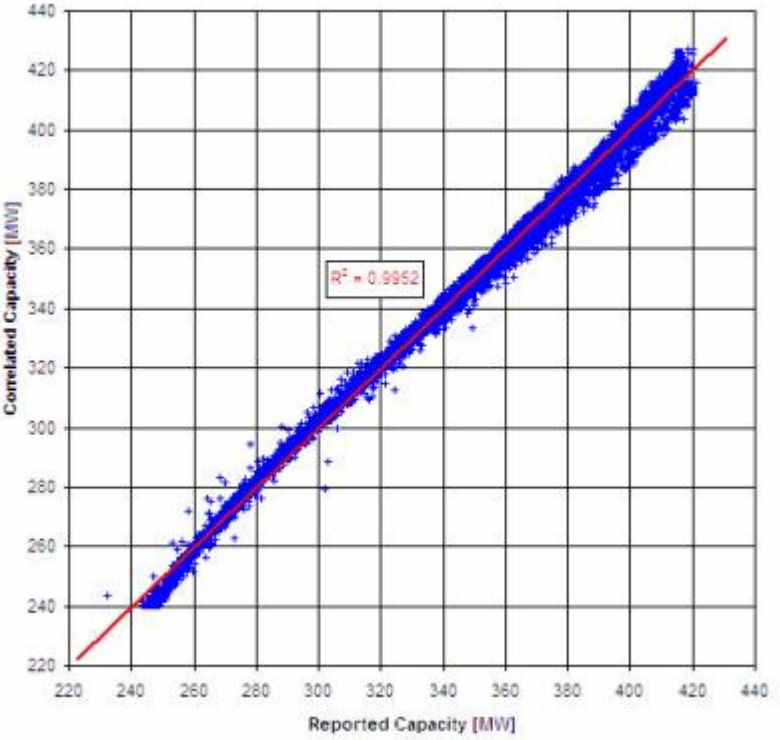

Ice Harbor
The Dalles
Little Goose
Model
EFFECTIVE PERCENT SATURATION, Ce
FLOW PER UNIT WIDTH, q [m²/sec]
260
240
220
200
180
160
140
120
100
0
10
20
30
40

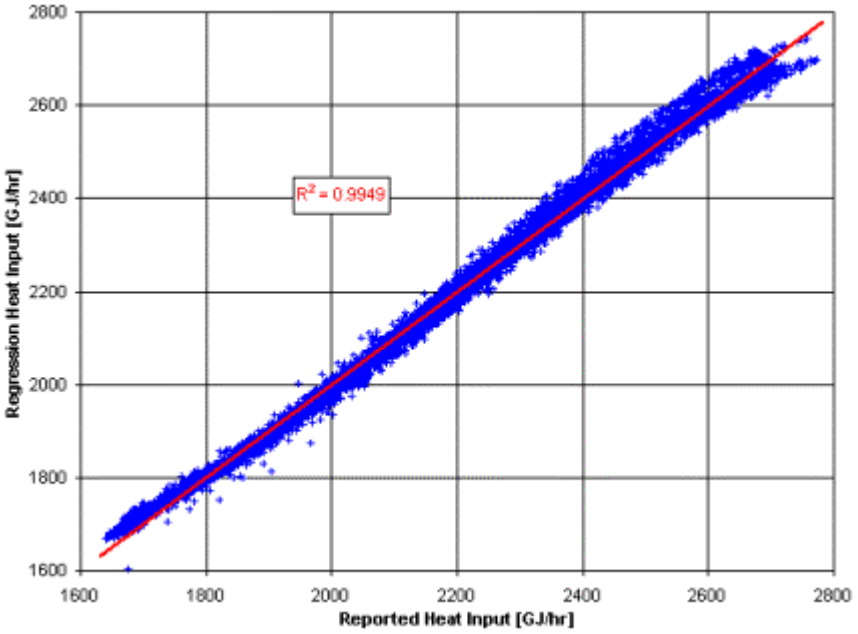

Ice Harbor
The Dalles
Little Goose
Model
BUBBLE MASS TRANSFER PARAMETER, $K_L a_b t_b$
FLOW PER UNIT WIDTH, q [m²/sec]
0
10
20
30
40
.5
1.0
1.5
2.0
2.5
3.0
3.5

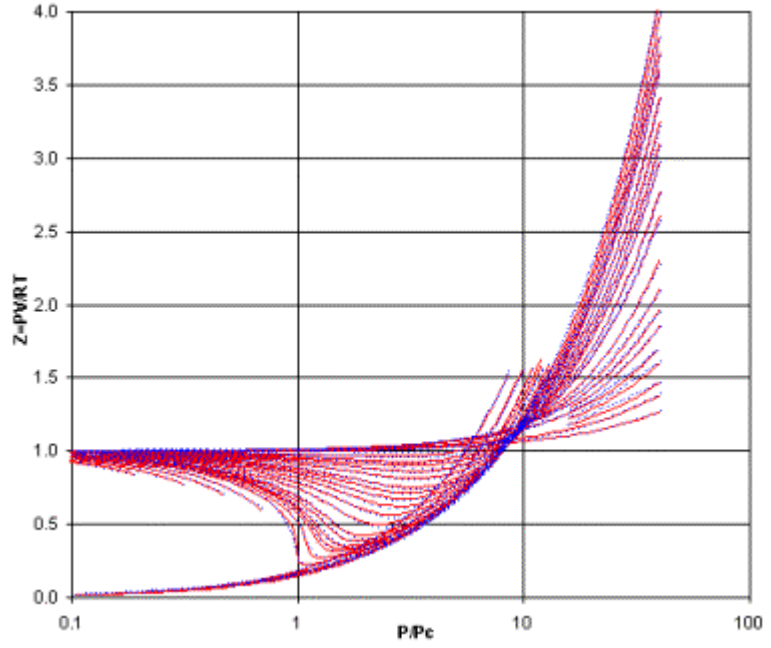

NORMALIZED N2 SUPERSATURATION CURVE
Ice Harbor (Qx=14.93, Ks=138.8)
The Dalles (Qx=10.06, Cx=119.5)
Little Goose (Qx=24.53, Cx=130.3)
Jennings Randolph (Qx=16.89, Cx=133.2)
(c-cu)/(cx-cu)
Q/Qx
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
1
2
3
4
5

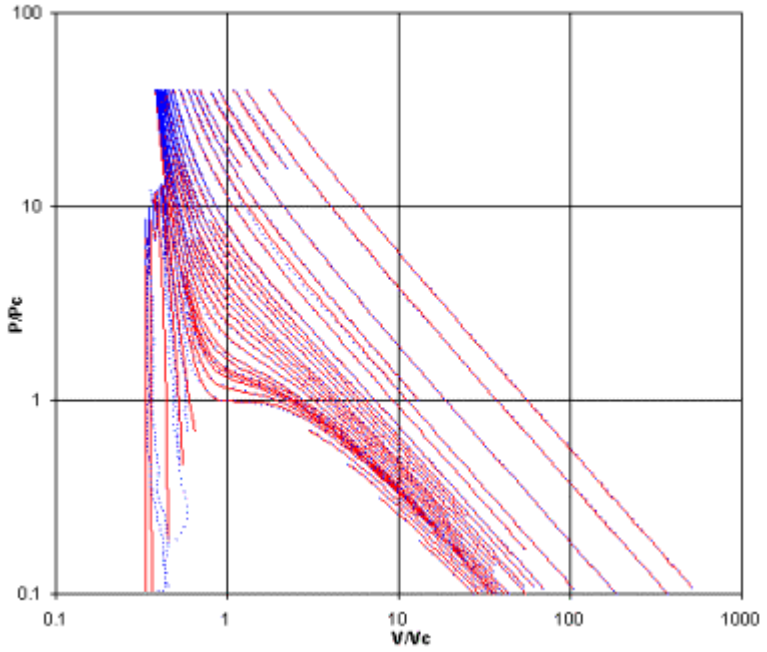

Ice Harbor
The Dalles
LTtle Goose
Jennings Randolph
Model
PERCENT SATURATION, Cd
150
140
130
120
110
100
0
10
20
30
40
50
60
FLOW PER UNIT WIDTH, q [m²/sec]

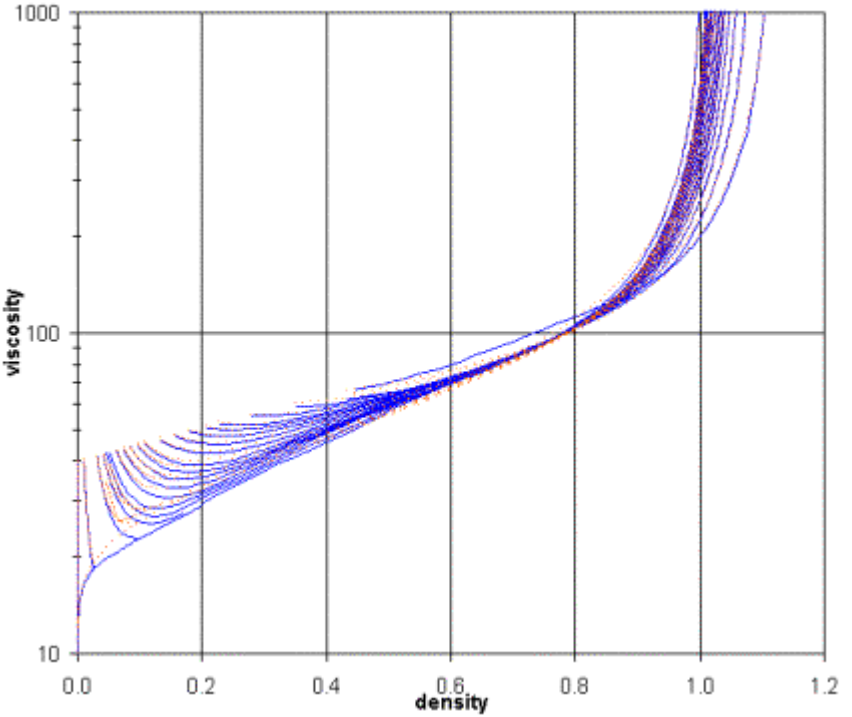

α1 Correction
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
-10
-5
0
5
10
15
20
25
30
35
Temperature [°C]

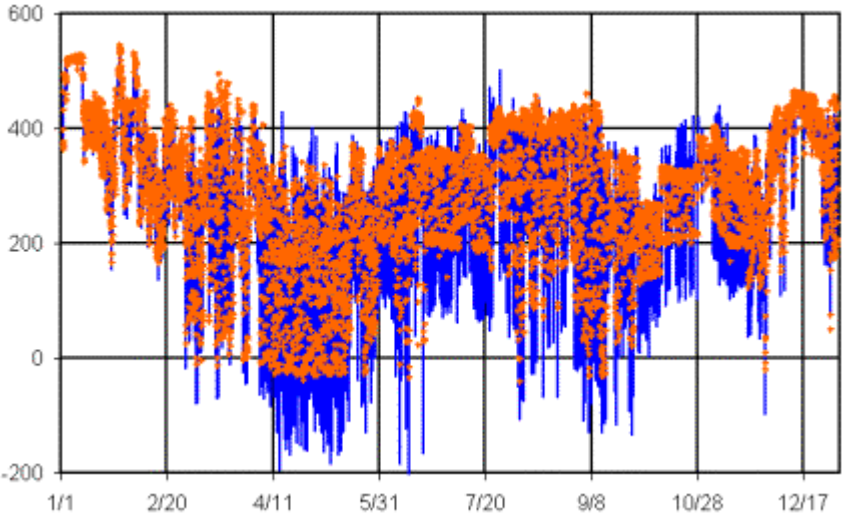

Correlated Capacity [MW]
Reported Capacity [MW]
R^2 = 0.9952
440
420
400
380
360
340
320
300
280
260
240
220
220
240
260
280
300
320
340
360
380
400
420
440

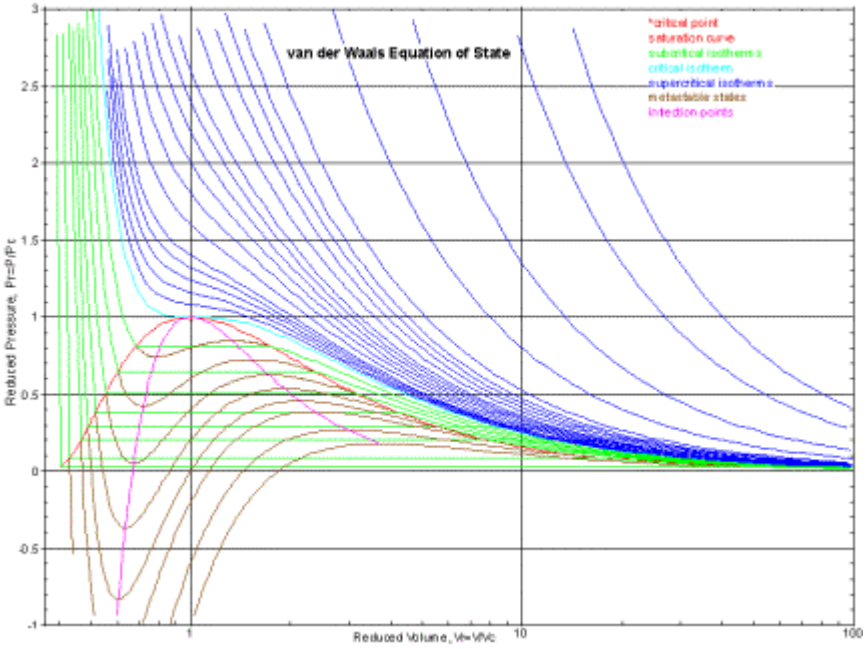

2800
2600
2400
2200
2000
1800
1600
Regression Heat Input [GJ/hr]
R² = 0.9949
1600
1800
2000
2200
2400
2600
2800
Reported Heat Input [GJ/hr]

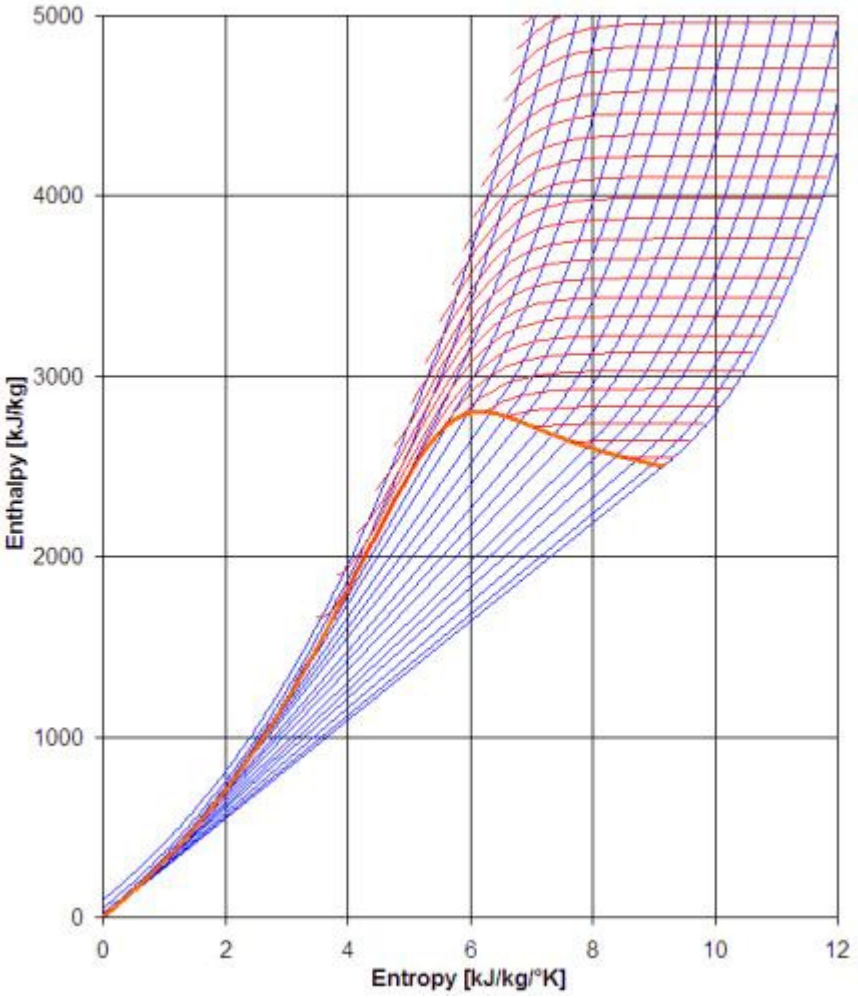
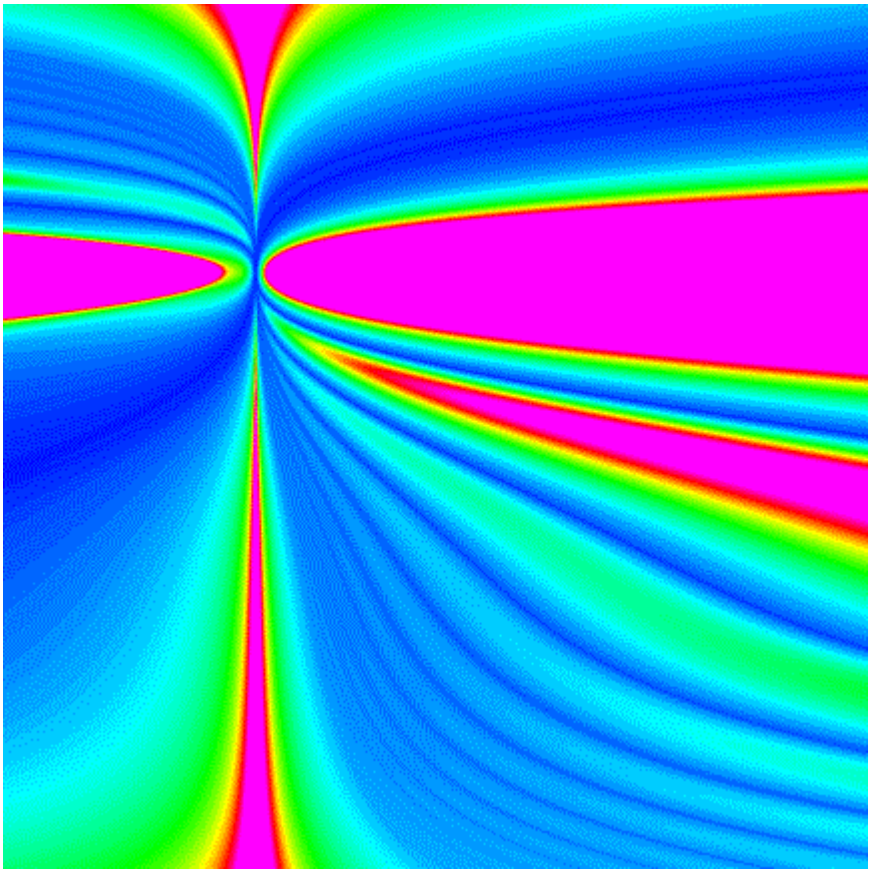

4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Z=PV/RT
0.1
1
10
100
P/Pc


100
10
1
0.1
P/Pc
0.1
1
10
100
1000
V/Vc


1000
100
10
viscosity
0.0
0.2
0.4
0.6
0.8
1.0
1.2
density


600
400
200
0
-200
1/1
2/20
4/11
5/31
7/20
9/8
10/28
12/17


van der Waals Equation of State
critical point
saturation curve
subcritical isotherms
critical isotherm
supercritical isotherms
metastable states
inflection points
Reduced Pressure, Pr=P/Pc
Reduced Volume, Vr=V/Vc
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
10
100


5000
4000
3000
2000
1000
0
Enthalpy [kJ/kg]
0
2
4
6
8
10
12
Entropy [kJ/kg/°K]