## Numerical Methods:

## Nonlinear Equations, Numerical Calculus, \& Differential Equations

...just the figures for those who got the B\&W text...
by D. James Benton

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FIGURE 2. CONJUGATE-GRADIENT





## Newton-Raphson Miethod




Regula Falsi Miethod


Illinois Method (Variant 1)


Illinois Miethod (Variant 2)


## Anderson-Björck Miethod



## Dekker's Miethod



Bisection Search






















## TOP VIEW (XY)


$8=0.1$
$\mathrm{Y}=0.2$
$\mathrm{z}=0.3$
FRONT VIEW (XZ)

$x=0.1$
$Y=0.2$
$2=0.3$








FRONT VIEW (XZ)
$x=0.1$
$Y=0.2$
$z=0.3$




FRONT VIEW (XZ)
$x=0.1$
$Y=0.2$
$z=0.3$


SIDE VIEW (YZ)
$x=0.1$
FRONT VIEW (XZ)
$1=0.2$
$2=0.3$


FRONT VIEW (XZ)
$x=0.1$
$\mathrm{y}=0.2$
$2=0.3$
















diffusion through a granulated media

| radius | relotive concentration, ClCO , at various times |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| cm | $\mathrm{t}=0$ | $\mathrm{t}=0.001$ | $\mathrm{t}=0.01$ | $\mathrm{t}=0.1$ | $\mathrm{t}=1$ | $\mathrm{t}=10$ | $\mathrm{t}=100$ |
| 0.000000 | 1.000000 | 0.991233 | 0.963563 | 0.883600 | 0.722751 | 0.496415 | 0.272451 |























|  |  |  |  |  |  | Tw |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ta |  |  |  |  |  | 148 | 148 | 148 | 148 | 148 |
| 78 | 102 | 116 | 124 | 128 | 132 | 115 | 115 | 124 | 130 | 131 |
| 78 | 92 | 99 | 108 | 115 | 118 | 97 | 104 | 108 | 113 | 121 |
| 78 | 86 | 92 | 98 | 103 | 108 | 88 | 96 | 99 | 104 | 110 |
| 78 | 82 | 88 | 92 | 96 | 101 | 84 | 89 | 93 | 98 | 102 |
| 78 | 80 | 84 | 87 | 91 | 94 | 81 | 85 | 89 | 92 | 97 |
|  |  |  |  |  |  | 11 approach |  |  |  | 89 |
|  |  |  |  |  |  | Hw |  |  |  |  |
| Ha |  |  |  |  |  | 260.1 | 260.1 | 260.1 | 260.1 | 260.1 |
| 41.6 | 74.7 | 107.4 | 131.5 | 149.2 | 165.8 | 104.7 | 105.8 | 132.7 | 157.3 | 162.5 |
| 41.6 | 59.3 | 70.8 | 86.9 | 103.7 | 114.1 | 66.8 | 79.0 | 87.5 | 100.9 | 122.9 |
| 41.6 | 50.5 | 58.7 | 68.0 | 77.6 | 88.2 | 53.6 | 64.4 | 69.2 | 79.0 | 93.4 |
| 41.6 | 46.0 | 52.8 | 58.6 | 64.5 | 73.4 | 48.0 | 54.5 | 59.9 | 68.0 | 74.5 |
| 41.6 | 44.0 | 48.0 | 51.8 | 57.5 | 62.2 | 45.2 | 49.3 | 54.6 | 59.1 | 66.2 |




Bogacki-Shampine Method



Cash-Carp Method



|  | A | B | c | D | $E$ | F | G |  | H | 1 | 3 | K |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\times$ | T | k | $1000$ <br> 800 |  |  |  |  |  |  |  |  |
| 2 | 0.0 | 1000 | 0.50 |  |  |  |  |  |  |  |  | 1.0 |
| 3 | 0.1 | 863 | 0.55 |  |  |  |  |  |  |  |  |  |
| 4 | 0.2 | 737 | 0.60 |  |  |  | -- temperature |  |  |  |  |  |
| 5 | 0.3 | 622 | 0.65 |  |  |  |  |  |  |  | 0.9 |
| 6 | 0.4 | 515 | 0.70 |  |  |  |  |  |  |  |  |  |
| 7 | 0.5 | 415 | 0.75 |  |  |  |  |  |  |  |  |  |
| 8 | 0.6 | 322 | 0.80 |  |  |  |  |  |  |  |  |  |
| 9 | 0.7 | 235 | 0.85 |  |  |  |  |  |  |  |  | 0.8 |
| 10 | 0.8 | 152 | 0.90 | $\underbrace{600}$ |  |  |  |  |  |  |  |  |
| 11 | 0.9 | 74 | 0.95 |  |  |  |  |  |  |  |  | $-0.7 \frac{\frac{3}{3}}{\frac{2}{2}}$ |
| 12 | 1.0 | 0 | 1.00 |  |  |  |  |  |  |  |  |  |
| 13 |  |  |  | ${ }^{\frac{E}{2}} 400$ |  |  | - |  |  |  |  |  |
| 14 |  |  |  |  |  |  |  |  |  |  |  |  |
| 15 |  |  |  | 200 |  |  |  |  |  |  |  |  |
| 16 |  |  |  |  |  |  |  |  |  |  |  |  |
| 17 |  |  |  |  |  |  |  |  |  |  |  |  |
| 18 |  |  |  |  |  |  |  |  |  |  |  |  |
| 19 |  |  |  |  |  |  |  |  |  |  |  |  |
| 20 |  |  |  |  |  |  |  |  |  |  |  |  |
| 21 |  |  |  |  | 0.2 |  | 0.4 | position ${ }^{0.6}$ |  | 0.8 |  |  |
| 22 |  |  |  |  |  |  |  |  |  |  |  |  |  |


|  | A | B | C | D | E | F | G | H | 1 | J | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Temperatures |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  | 1000 | 900 | 800 | 700 | 600 | 500 | 400 | 300 | 200 | 100 | 0 |  |
| 3 | 1000 | 911 | 816 | 722 | 629 | 537 | 446 | 357 | 270 | 185 | 105 | 37 | 0 |
| 4 | 950 | 847 | 751 | 661 | 573 | 489 | 406 | 325 | 248 | 174 | 106 | 47 | 0 |
| 5 | 900 | 794 | 699 | 612 | 529 | 450 | 374 | 301 | 231 | 165 | 103 | 49 | 0 |
| 6 | 850 | 746 | 654 | 571 | 493 | 419 | 348 | 281 | 217 | 156 | 99 | 48 | 0 |
| 7 | 800 | 701 | 614 | 536 | 462 | 393 | 327 | 264 | 204 | 148 | 95 | 46 | 0 |
| 8 | 750 | 659 | 578 | 505 | 436 | 371 | 309 | 250 | 193 | 140 | 90 | 44 | 0 |
| 9 | 700 | 618 | 545 | 477 | 413 | 352 | 293 | 237 | 184 | 133 | 86 | 42 | 0 |
| 10 | 650 | 579 | 514 | 453 | 393 | 335 | 280 | 226 | 175 | 127 | 82 | 39 | 0 |
| 11 | 600 | 543 | 487 | 432 | 376 | 322 | 269 | 217 | 167 | 120 | 77 | 37 | 0 |
| 12 | 550 | 511 | 466 | 415 | 363 | 311 | 260 | 209 | 160 | 114 | 71 | 32 | 0 |
| 13 | 500 | 490 | 452 | 405 | 355 | 304 | 253 | 200 | 154 | 107 | 62 | 23 | 0 |
| 14 |  | 500 | 450 | 400 | 350 | 300 | 250 | 200 | 150 | 100 | 50 | 0 |  |
| 15 | Thermal Conductivities |  |  |  |  |  |  |  |  |  |  |  |  |
| 16 |  | 0.50 | 0.5 | 0.5 | 0.62 | 0.66 | 0.71 | 0.76 | 0.81 | 0.87 | 0.93 | 1.00 |  |
| 17 | 0.50 | 0.53 | 0.57 | 0.61 | 0.65 | 0.69 | 0.73 | 0.78 | 0.83 | 0.88 | 0.93 | 0.98 | 1.00 |
| 18 | 0.52 | 0.56 | 0.59 | 0.63 | 0.67 | 0.71 | 0.75 | 0.80 | 0.84 | 0.89 | 0.93 | 0.97 | 1.00 |
| 19 | 0.54 | 0.58 | 0.62 | 0.65 | 0.69 | 0.73 | 0.77 | 0.81 | 0.85 | 0.89 | 0.93 | 0.97 | 1.00 |
| 20 | 0.56 | 0.60 | 0.64 | 0.67 | 0.71 | 0.75 | 0.79 | 0.82 | 0.86 | 0.90 | 0.93 | 0.97 | 1.00 |
| 21 | 0.57 | 0.62 | 0.65 | 0.69 | 0.73 | 0.76 | 0.80 | 0.83 | 0.87 | 0.90 | 0.94 | 0.97 | 1.00 |
| 22 | 0.59 | 0.63 | 0.67 | 0.70 | 0.74 | 0.77 | 0.81 | 0.84 | 0.87 | 0.91 | 0.94 | 0.97 | 1.00 |
| 23 | 0.62 | 0.65 | 0.69 | 0.72 | 0.75 | 0.78 | 0.82 | 0.85 | 0.88 | 0.91 | 0.94 | 0.97 | 1.00 |
| 24 | 0.64 | 0.67 | 0.70 | 0.73 | 0.76 | 0.79 | 0.82 | 0.86 | 0.89 | 0.92 | 0.95 | 0.97 | 1.00 |
| 25 | 0.66 | 0.69 | 0.71 | 0.74 | 0.77 | 0.80 | 0.83 | 0.86 | 0.89 | 0.92 | 0.95 | 0.98 | 1.00 |
| 26 | 0.68 | 0.70 | 0.72 | 0.75 | 0.78 | 0.81 | 0.84 | 0.87 | 0.90 | 0.92 | 0.95 | 0.98 | 1.00 |
| 27 | 0.71 | 0.71 | 0.73 | 0.76 | 0.78 | 0.81 | 0.84 | 0.87 | 0.90 | 0.93 | 0.96 | 0.98 | 1.00 |
| 28 |  | 0.71 | 0.73 | 0.76 | 0.79 | 0.81 | 0.84 | 0.87 | 0.90 | 0.93 | 0.97 | 1.00 |  |




|  | -950-1000 |
| :---: | :---: |
|  | - $900-950$ |
|  | -850-900 |
|  | -800-850 |
|  | -750-800 |
|  | -700-750 |
|  | - $650-700$ |
|  | -800.650 |
|  | -550.600 |
|  | -500.550 |
|  | -450-500 |
|  | -400-450 |
|  | - $350-400$ |
|  | -300.350 |
|  | - 290.300 |
|  | -200-250 |
|  | -150-200 |
|  | -100-150 |
|  | -50-100 |
|  | -0.50 |



Crank-Nicholson Matrices
properties

| matrix A |  |  |  | matrix $B$ |  |  |  | $\Delta t$ | 1.00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0 | -0.5 | 0.0 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 | $\Delta$ | 1.00 |
| -0.5 | 2.0 | -0.5 | 0.0 | 0.5 | 0.0 | 0.5 | 0.0 |  | 1.00 |
| 0.0 | -0.5 | 2.0 | -0.5 | 0.0 | 0.5 | 0.0 | 0.5 | C | 1.00 |
| 0.0 | 0.0 | -1.0 | 2.0 | 0.0 | 0.0 |  | 0.0 | k | 1.00 |
|  |  |  |  |  |  |  |  |  | 1.00 |

Crank-Nicholson time steps
calculated temperatures at each time step

| $\mathrm{t}=0$ | $\mathrm{t}=\Delta \mathrm{t}$ | $\mathrm{t}=2 \Delta \mathrm{t}$ | $\mathrm{t}=3 \Delta \mathrm{t}$ | $\mathrm{t}=4 \Delta \mathrm{t}$ | $\mathrm{t}=5 \Delta \mathrm{t}$ | $\mathrm{t}=6 \Delta \mathrm{t}$ | $\mathrm{t}=7 \Delta \mathrm{t}$ | $\mathrm{t}=8 \Delta \mathrm{t}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 100 | 46.4 | 38.0 | 30.8 | 26.3 | 22.5 | 19.3 | 16.5 | 14.2 |
| 100 | 85.6 | 66.3 | 56.7 | 48.3 | 41.5 | 35.6 | 30.6 | 26.2 |
| 100 | 95.9 | 85.1 | 73.2 | 63.1 | 54.1 | 46.5 | 39.9 | 34.3 |
| 100 | 97.9 | 90.5 | 79.1 | 68.1 | 58.6 | 50.3 | 43.2 | 37.1 |

1-D RESERVOIR ROUTING NODEL



star.fEM: IUPUT FILE: STAR.BOU


















